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The Evolution of Shared Concepts in Changing Populations

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Abstract

The evolution of color categorization systems is investigated by simulating categorization games played by a population of artificial agents. The constraints placed on individual agent's perception and cognition are minimal and involve limited color discriminability and learning through reinforcement. The main dynamic mechanism for population evolution is pragmatic in nature: There is a pragmatic need for communication between agents, and if the results of such communications have positive consequences in their shared world then the agents involved are positively rewarded, whereas if the results have negative consequences, then involved agents are punished. A mechanism for changing the composition of the population due to agents' birth and death is also investigated. This birth-death mechanism is found to effectively move an established population color naming system toward a theoretically more optimal one. Specifically, our use of birth-death dynamics suggests that (*i*) agent populations do reach stable, consistent color categorization solutions, (*ii*) population categorization solutions derived from categorization game encounters can change even though each individual agent's solutions tend to remain unchanged, (*iii*) the number of color categories in observed solutions varies with fluctuations of agents' lifespans, and (*iv*) categorization solutions are subject to change until they reach theoretical optimality. The simulation results of this article provide insights regarding mechanisms that may constrain universal tendencies in human color categorization systems observed in the linguistic and anthropological literatures.

Introduction

Because there is no necessary connection between the sound of a word and its meaning, meanings of words are considered conventional. For example, there is no principled reason why the meanings of the words "blue" and "yellow" could not have evolved so that "blue" refers to the color of a ruby gemstone and "yellow" to the color of an emerald. A related issue is described in the biology literature as the evolution of signaling systems used to communicate information within and across species.

The formulation of theories and explanatory mechanisms that capture how words and signals acquire meaning is a major area of study which spans a wide variety academic disciplines, ranging from philosophy and linguistics in the humanities, to psychology and anthropology in the social sciences, to artificial intelligence and robotics in computer science and engineering. This article focuses on the evolution and communication of concepts, emphasizing its most studied, cross-disciplinary case: the evolution and communication of color concepts.

The formal study of the evolution of meaning began in 1969 with the publication of the seminal book *Convention* by the philosopher David Lewis. [20] At that time, the evolution of linguistic meaning was held to be a special kind of convention with other conventions being formulated and communicated through an already established conventional linguistic meaning system. [26–28] Lewis held an alternative viewpoint, and formulated a general notion of convention as an equilibrium achieved in a coordinated game, which Lewis illustrated using ideas involving signaling games and their equilibria. He argued that linguistic meaning evolved by means similar to his analysis of simple signaling games. This started a tradition in philosophy that has been systematically developed further by Brian Skyrms. [33–35] Another, independently developed, game-theoretic approach to animal signals contemporaneously appeared in biology (e.g., see [23]). Both of these approaches generally consider situations among individuals in a population for which specific actions are communicated by signals.

Despite being valuable in signaling research, the above mentioned approaches are less amenable to communicating concepts, because many concepts resemble "natural kind" categories and are comprised of sets of multiple items which are related by an underlying similarity structure that is not present in signaling games, and this similarity structure adds a psychological component to the evolutionary process. This kind of psychological similarity has been much studied in psychology, especially in studies involving color similarity and color naming.

There are many articles on the game-theoretic evolution of color naming, and there are many reasons color naming is a core case-study in the area: Namely, it has a rich history of scientific study; its similarity structure has been much investigated and modeled; and there is a large literature on how to convert color similarity into geometric structures that express increases in similarity between two given colors as decreased distance between the two particular colors. In addition, there is a large literature on the evolution of color lexicons and how such evolution varies with language and culture. Because of this, color is a natural starting point for developing models of the evolution of shared concepts, and it is the domain employed throughout this article.

The basic principle underlying this article's theory of categorization is that colors that are perceived as perceptually similar, or close, will likely be described by the same color name, and those that are appear perceptually dissimilar, or distant, are likely to be described by different names. This principle is used to formulate the evolutionary dynamics for a color naming game that is played by a population of communicating agents, for which the emerging (stochastic) equilibria of the game yields population naming strategies for categorizing colors.

Most naming and signaling evolutionary game-theoretic research employ either a form of Darwinian algorithms (e.g., replicator dynamics) or a form of learning algorithms (e.g., reinforcement learning). For the purposes of this article's simulations, these two kinds of algorithms, although conceptually different, often yield similar results (for example, see [5]). The present article's simulation results are based on a form of reinforcement learning.

Our previous work has shown this simulation approach to be a useful and informative method for analyzing how communication and pragmatic constraints combine to produce wide variety color categorization systems exhibiting key regularities seen throughout the color naming literature (e.g., [4, 15]). This line of research also established that reinforcement algorithms, employed for populations of homogeneous agents tasked with categorizing sets of homogeneous color stimuli, could be easily modified to investigate a range of simulated situations: Such as (1) color stimuli that varied in pragmatic importance [19]; (2) populations of agents based on models of varying color perception [13, 14, 18]; and (3) populations where interactions varied according to structures imposed by a communication network. [24].

This article further explores investigations involving modified situations of simulated category learning similar to those reported in Komarova, et al. [19]

Simulated population category learning scenarios investigated in [19] tended to produce near optimal categorization solutions. A confluence of factors gave rise to those solutions: The use of small sized populations (usually between 30 and 100 agents); the use of a small number discriminable color chips in the training set; and the convenient setting of parameters in learning algorithm. The present investigations modify these factors to examine when simulated learning does not lead to near-optimal solutions. This allows for simulation investigations of new and interesting evolutionary phenomena found in diachronic studies of color naming in written languages, including, for example, the dropping out of an established category without its disappearance, due to “splitting” into subcategories.

This article’s methods show that stochastically stable population naming strategies can become near optimal by gradually changing the composition of the agent population through the random introduction of new members with random, *naive* categorization strategies (analogous to a “Birth” mechanism) and the random elimination of an equivalent number existing members (analogous to “Death”). This kind of process resembles the routine lifecycle present in societies of human learners, in which information evolves and is shared as communicating members cycle through a population — as is the case where individuals in a population die and are replaced by the birth of new members. Key properties of this cycle — e.g., population size variations due to warfare or catastrophe, and subsequent regrowth, etc. — are also approximated here using simulation processes. The findings of our present simulation studies suggest that Birth-Death dynamics are a natural process in the evolution of color lexicons, which extend the previous category learning solutions that have been observed, permitting a population’s non-optimal categorization solution to further develop into an optimal one.

Materials and Modeling Methods

An evolutionary game-theoretic modeling framework is used to investigate the impact of specific realistic constraints on color categorization and inter-individual communication. In this framework, individual agents learn to do the following:

- categorize simulated colors through reinforcement learning by playing “discrimination-similarity games,”

and

- communicate the meaning of categories to each other. [18, 19]

The main components of this approach consist of (*i*) the color stimulus space, (*ii*) color observer models, (*iii*) the structure of evolutionary game, and (*iv*) the evolutionary dynamics employed. (Articles [13, 14, 24] also incorporate these four components in both modeling and theories used to examine substantive issues contributing to color categorization phenomena.)

Stimulus space

Similar to previous investigations, e.g., [13, 14, 18, 19], this article employs a hue circle stimulus space. This is a natural subspace of color appearances that preserves similarity relations among hue categories, regardless of the normal individual variation that may occur within a given population for salient hue point locations within a color order system. It can also be used to model artificial agent observer groups by having each agent use hue circle color perception data (e.g., the Farnsworth-Munsell 100 hue test) obtained from a human tasked with evaluating standardized color systems. Such color perception data can be used to create a population of heterogeneous color perceiving agents. [13, 14],

The hue circle stimulus model we use is similar to the hue gradient of constant value and chroma found in the Farnsworth-Munsell 100 hue test, abbreviated *FM100*. [7] The FM100 contains 85 color chips, or “caps,” designed to form a perceptually smooth gradient of hue, ostensibly at a fixed level of brightness and a fixed level of saturation. [22] Advantages of using the FM100 stimulus set include that it is well understood colormetrically, and is in common use as a diagnostic for human color vision deficiencies. Such features permit the present use of the FM100 for modeling varying discrimination capabilities in our artificial observer models. Also, because the hues used form an approximate continuum of uniform Munsell *Chroma* and comparatively uniform Munsell *Value* levels ([22] p. 2239, Figure 1), it provides a standard for comparisons between our simulated population category solutions and existing human color category partitioning results on a Munsell Book of Color stimulus set. [30]

In the investigations described here, populations of simulated agent observers engage in color communication games to establish how to categorize sampled colors from a hue circle. For our purposes, this provides enough perceptual color space variation to capture the primary factor of any hue-based color categorization system.

The Pragmatic Discrimination Factor k_{sim}

Different color chips a and b of equal brightness and saturation are said to be *just-noticeably different in hue*, or more briefly, *one "jnd" in hue,* if and only if the typical subject is 75% correct in saying whether or not a and b are the same color. Colors c and d on a hue circle of equally bright, equally saturated colors are said to be m jnds apart if and only if m is the smallest positive integer such that there is a sequence $c = c_1, c_2, \ldots, c_m, c_{m+1} = d$ such that c_i and c_{i+1} are 1 jnd apart for $i = 1, \ldots, m$. Color chips of equal brightness and saturation on a circular arrangement are said to be *equally spaced in hue* if and only if every pair of adjacent chips are the same number of jnds apart.

Jnds are used in defining the important evolutionary pragmatic concept, k_{sim}. Following [19], [24] motivates evolutionary use of concept of k_{sim} for color categorization as follows:

> The evolutionary link we use between perception and naming is based on the following idea: Because there are considerably fewer color names than colored patches, two colors within a relatively small number of jnds will have a very strong tendency to be given the same name. This idea is an obvious consequence of the following three principles: (i) categorization is important; (ii) to be useful, categorization should attempt to minimize ambiguity, and (iii) objects of a kind with highly perceptually-similar colors tend to represent similar properties within the kind. The concept of *similarity range*, k_{sim}, formalizes these three principles for a circle of color patches.
>
> By definition, for color patches a and b, $k_{sim}(a, b)$ is the minimum number of colored patches between a and b for which it becomes important to treat a and b for *pragmatic purposes* (and not for *perceptual purposes*) as belonging to different color categories. Pragmatically speaking (see principle (i) above), it is beneficial to assign colors outside their k_{sim}-range to different color categories (principle (ii)), and colors within their k_{sim}-range to the same color category (principle (iii)).
>
> $k_{sim}(a, b)$ is interpreted as being related to the utility of categorizing a and b as the same or different colors. It is defined by the environment and the life-styles of the individual agents. It is used to reflect the notion of the pragmatic color similarity of the patches. For instance, suppose one individual shows another a fruit and asks her to bring another fruit "of the same color." It is a nearly impossible task to bring a fruit of a color perceptually identical to the first, because different lighting, different color background and slight differences in

fruits' ripeness contribute to differentiating its perceived color from the comparison fruit. Therefore to satisfy "of the same color" of a fruit's ripeness in practical terms, the individual must be able to ignore such unimportant perceptual differences and bring a fruit that is "of the same color" practically. It may also be as important to be able to distinguish ripe, edible, "red" fruit from the unripe, "green" ones.

It is important to note that k_{sim} is not a large scale jnd concept: Unlike the jnd concept, it is generally easy to discriminate colors within k_{sim}.

Agents' knowledge

In the simulation of artificial agents, each agent has a reinforcement matrix that is used to decide which color term the agent will use in color-naming games. The more reinforcement unit in a given cell of the matrix there is, the more likely an agent will use the color term associated with that cell in a game. Table 1 is a visual representation of the matrix.

Table 1. Knowledge matrix of an agent for 8 chips and 4 categories.

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Orange	16	15	1	0	0	0	0	2
Red	0	1	15	16	0	1	0	0
Green	0	0	0	0	1	0	16	14
Blue	0	0	0	0	15	15	0	0
Total	16	16	16	16	16	16	16	16

An example of a knowledge table of an agent when the number of chips is 8 and the number of given color terms is 4. This agent ended up using all the four terms in the partitioning of its color space.

Notice that the total number of reinforcement units per color chip is fixed to be $4 \cdot$ (the number of color terms). This remains invariant after agents have been reinforced (or punished) in the context of the color categorization game.

Networks

Most of the color naming simulations in the literature have been based on games where in each round of the simulation the players are randomly selected. ([24] considers situations that are exceptions to this.) A more general and realistic assumption is to have the population engage in games on a communication network, where network nodes correspond to agents, and agents only participate in games involving their connected "neighbors" — that is, agents only play color-naming games with those agents with which they have an edge in common. In such cases, in a given round an agent plays with a randomly selected neighbor. Of course, different forms of communication networks can be adopted: For example, a *complete network* is a network in which every two agents are neighbors. Thus for this kind of network, the random selection of two neighbors from the network is the same as the random selection of two players from the population.

In this article the emphasis is on the development of evolutionary methods that lead to optimal categorization systems. For convenience and comparison purposes, this is mostly done for complete networks. An example involving a real world network is mentioned in the Discussion section later, to illustrate that the results are not strictly tied to scenarios where agents randomly interact.

The discrimination-similarity game

Throughout simulations presented here, artificial agents play numerous color categorization games in order to achieve a population categorization system. These games are called "discrimination-similarity games".

Given two color chips, i and j, an agent is said to make a *coherent* choice of names for them if and only if it assigns the same name to them if the chips are within k_{sim} and different names to them the jnd distance between i and j is greater than k_{sim}. Two agents are said to make a *congruent* choice of names for i and j if and only if they make the same choice of name for i and the same choice of name for j. Discrimination-similarity games are based on networked agents. In this article, the network is taken to be a complete network. This is equivalent to agents randomly meeting other agents and playing a discrimination-similarity game. Two network neighbors, A_1 and A_2, and two color chips, i and j, are randomly chosen. Based on their individual knowledge tables (e.g., a matrix like that shown in Table 1) each agent has a probabilistic choice of names for chips i and j. A_1's probability for its choice of name for i is its entry for chip i divided by the sum of i's entries. Similarly for A_1's probability for its choice of name for j and A_2's probabilities for choices of names for i and j. Based on these probabilistic choices, the entries for these names are either enhanced by adding 1 if pragmatically effective or diminished by subtracting 1 if ineffective. When the value in a cell reaches a maximum, instead of an increment by 1, it is incremented by 0, and when the value is 0, it is decremented by 0 instead of being decremented by 1.

If the choices of names for i and j are coherent and congruent, then the entries for those names for i and j for those agents are enhanced. If choices of names by each agent are not coherent, then the entries for those names are diminished for each agent. If both agents are coherent but not congruent, then one agent is chosen randomly to be "the teacher" and the other agent adopts the teacher's naming strategy for i and j and the entries for those names for i and j are enhanced for both agents. If one agent is coherent and the other is not, the coherent agent is chosen to be "the teacher" and the teacher's entries for i and j are enhanced. The other agent adopts the teacher's naming strategy for i and j by enhancing the entries for teacher names for i and j. Except for this last case involving the simultaneous occurrences of coherence and non-coherence, whenever and entry of i is enhanced by adding 1, another entry of i is randomly chosen to be diminish by subtracting 1 and vise versa, thus leaving unchanged the sum of entries for i. For the case of the simultaneous occurrences of coherence and non-coherence, the teacher's entries for i and j are enhanced by 1 with other randomly chosen entries diminished by 1, and the other agent's entries for i and j are each enhanced by 1 (it learned from the teacher) and its non-coherent choices for i and j are each diminished by 1.

Optimal number of color categories

For given number of equally spaced color chips and a given k_{sim}, there are optimal naming strategies for the color categorization game. "Optimality" here means that the number of agreeing categorizations of the population of agents is maximal.

According to [19] the optimal number of color categories for a given k_{sim} and a given number of color chips is the natural number closest to

$$\frac{\text{number of color chips}}{\sqrt{2 \cdot k_{sim} \cdot (k_{sim}+1)}}\,. \tag{1}$$

Table 2 shows some examples of the optimal number of color categories according to Eq 1.

In general, in an optimal solution the sizes of named categories will be near equal. Because of this, in our simulations we view movement towards the number

Table 2. Knowledge matrix of an agent for 8 chips and 4 categories.

# color chips	k_{sim}	n
24	3	5
24	5	3
24	8	2
48	3	10
48	5	6
48	8	4

Example optimal number of color categories for given number of color chips and k_{sim}.

of categories given by Eq 1 and/or equal size as movement towards a more optimal naming strategy.

Note that although for simplicity the modeling in this article assumes a constant value for k_{sim}, the theory presented here can be modified to accommodate variable k_{sim}. (See [19] for examples using variable k_{sim}.)

Implementing Birth-Death Dynamics

In our modeling, the population of agents changes by permitting new agents to enter and other agents to leave. We refer to this as the "Birth-Death" dynamic, because an agent leaving the population functions like a "death" since their ability to engage in naming game interactions with other agents in the population ceases, thus eliminating their direct influence on an evolving population categorization solution. Similarly, in our model, a new agent entering the population functions like a "birth", because such an agent enters the population with no implicit knowledge of the categorization strategies of the other members of the population. Specifically,

- *birth of agents:* introducing new agents with random categorization strategies after a certain number of iterations,

and

- *death of agents:* having each agent leave the population after playing certain number of games.

In general, in complete networks we observe that in order to maintain a constant size for the population, the parameter corresponding death rate must be two times larger than the parameter corresponding birth rate. That is, when for every n iterations an agent is introduced, that agent will play on average $2n$ games before leaving the population. For non-complete networks a similar process will maintain constant size.

Note that in many ways Birth-Death is similar to *Migration*—where, over time, a fixed number of agents enter the population and the same number of agents leave. The two concepts have slightly different influences on the evolutionary process: In Birth-Death, a new agent has a random strategy. By comparison, in Migration, a new entering agent comes with an already fixed categorization strategy. If at a given time, the number of new agents is relatively small, then one would expect that Migration would also result in fixed systems converting to optimal ones, similar to what is seen with Birth-Death dynamics. We have not, however, investigated this through simulations.

Another property affecting the evolutionary dynamics is the notion of a "shock" to evolving processes. Here "shock" implies significant events that enter and impact the structure of an evolving process. We consider two forms of shock. First, the occasional introduction of abrupt, substantial, blocks of changes, implemented by modifying all values in the knowledge matrices of all agents in a population by a fixed change, and thereby altering all the probabilities in a population's knowledge matrices by a single secular event. Depending on the simulation under consideration, these blocked changes were introduced to a population at various frequencies, between 800 and 10,000 iterations. As a second form of "shock", we introduced continuous small streams of changes. This second kind of shock was implemented by modifying the probabilities of a single randomly-chosen agent's knowledge matrix. This was done by making a fixed small change in the values contained in a single column in the chosen knowledge matrix. This more modest form of shock was implemented as a systematic trickle of change occurring continuously during the evolutionary process. To further investigate the role of shock on Birth-Death evolutionary dynamics, we also implement a"*noise dynamic*" where an amount of randomness is uniformly distributed across the population corresponding to the amount of randomness introduced through a Birth-Death dynamic.

In this article, the above mentioned evolutionary features are used in a series of simulation studies to investigate the constraints they place on the formation of stable population categorization solutions.

1 Simulations

In a discrimination-similarity game, a population naming strategy is said to be in a *stochastic equilibrium* if and only if the continued evolution of the game stays close to that strategy for a very long time. Here a "very long time" means much longer than it takes to achieve that naming strategy. For example, in a discrimination-similarity game on a complete graph, if a naming strategy appears to be in a stochastic naming equilibrium after individual agents play on average 500 games, and is observed to stay in the equilibrium with individual agents playing on average an additional 100,000 games, then we would consider the additional 100,000 games to be a very long time. In other studies we have tested up to 100,000,000 games to define a "very long time" for a population of agents. Essentially we test for "a very long time" which is practical with desktop computing resources.

The evolution of the discrimination-similarity games in [19] used complete networks that generally led to stochastic equilibria. The number of achieved categories were either generally optimal or occasionally missing optimality by one category. This result was achieved by starting with more than the optimal number of names and observing during system evolution that support for some names diminished to zero as the solution obtained an optimal or near optimal number of categories. In all cases, the sizes of resulting categories tended to be similar across solutions, with minor degrees of variation one might expect from a stochastic process. Komarova et al. describe their observed stochastic equilibria as "near-optimal" [19].

Both Komarova et al.'s [19] and the present discrimination-similarity game simulations use only a reinforcement algorithm on complete networks; however, unlike [19] the present investigations do not always lead to optimal solutions. This was especially the case when the maximum reinforcement values were increased, when the number of color chips used were increased, and larger numbers of disjoint k_{sim}'s were used compared to those previously employed in [19]. In many of the present simulations, the evolved color categorization systems included solutions with categories that substantial differed in size and which had non-optimal numbers of categories. Fig 1 exemplifies this kind of situation. In other cases overlapping, or discontinuous, categories were observed, for example, in Fig 2.

The present investigation's increased computational power, compared to [19], permitted simulation solutions like those seen in Fig 1 and Fig 2 from briefer computation durations compared to those used by Komarova et al. [19]. As a practical matter, this permitted greater freedom to investigate varying reinforcement values and increased numbers of color chips and number color terms. In addition, we shifted our focus from trying to find combinations of these parameter values to aiming to obtain optimal results for a given value of k_{sim}, and exploring other features of the evolutionary dynamics. In interpreting our agent-based simulations to actual color-naming pragmatics, we view k_{sim} as an analogue of the environment, with the hue circle with its *jnd* structure and its maximum reinforcement value as built-in features of human psychology. This leaves the number of categories and their sizes, and other categorization solution features, to be determined by the evolutionary dynamics.

Results

Under the conditions used in the present simulations, our findings reveal the following general pattern.

1. Population birth-death dynamics are an integral feature of communication within populations, are likely to exert varying degrees of influence across both populations and time, and represent factors which, to our knowledge, have not been previously modeled as part of color categorization phenomena. One interesting finding suggested by the present results is that a birth-death dynamic enables color categories to compete with one another in ways that produce more optimal solutions than those seen in the absence of birth-death dynamics.
2. Color categorization solutions that contain category partitions with nonuniform numbers of color chips have a different "evolutionary force" with respect to k_{sim}. This difference in evolutionary force produces a dynamic that tends to move categorization systems towards solutions involving categories with similar numbers of chips, which in turn drives systems towards more optimal solutions.
3. When initial parameter settings are held constant, simulations converge to systems of color categorization that have stochastically stable categories with approximately the same number of chips.
4. Varying the lifespan of agents can produce changes in the number of categories in an otherwise stable solution. We observed solutions in which changes in lifespan during evolution led to optimal categorizations, as well as solutions resulting in increased numbers of categories, and others in decreased numbers of categories.

Birth-Death dynamics for complete networks

Initial investigations by Komarova et al. [19] observed stochastically stable, near optimal solutions for populations between 10 to 100 agents engaged in discrimination similarity games for a small number of chips and employing an evolutionary dynamic very similar to that used in here. Komarova et al.'s results yielded color categorization systems involving from 3 to 7 optimal number of categories. Elsewhere it was found that when population size was greatly increased, or other parameters modestly changed, stochastic stability was still observed, although without optimality ([24]). Given these results, it is natural to ask which evolutionary mechanisms contribute to driving solutions toward optimality for equilibria that differ from those described in [19]. The present investigation was designed to investigate this issue.

In the simulations presented here, the following general trends were observed: Stable solutions can become unstable by introducing random changes to a population, and stability losses can be in a direction toward optimality when the amount of random change introduced is in a specific interval. In addition, the present simulation results strongly support the following hypothesis: Solutions with categories of adequate size have an evolutionary advantage over those with diffuse or small categories, and similarly have an advantage over categories that are too large. Random changes to the population that lead to "adequate category sizes" can occur in several ways. The two investigated here are (1) agents being randomly replaced by other agents (referred to here as a "Birth-Death" dynamic), and (2) the introduction of noise to a population's naming strategy implemented as random changes in each individual's naming strategies. Specifically, our use of "noise" functionally resembles "mutation" dynamics used in other kinds of evolutionary simulations. We also used "noise" as a control variable to investigate whether Birth-Death variations led to superior optimal solutions, and found that it did not. That is, in comparisons between Birth-Death with noise variations, we found that (while other processes and features held fixed) the two processes yielded about the same levels of optimality, with Birth-Death contributing to optimality only slightly better than noise.

Simulations without Birth-Death dynamics

Regier et al. (2007) investigated ideas developed by Jameson and D'Andrade [10] and observed a universal pattern found in across languages in the color categorization data of the World Color Survey. As a result, Regier and colleagues suggested that the investigated color lexicons optimally partition the empirical color space employed by the WCS. [29]. Results of both [19] and the present investigations are also consistent with this view. This subsection presents two typical simulation solutions from the present investigations showing results involving fixed, unchanging populations. The subsection's aim is to explore reasons why the findings described in Figs 1 and 2 did not achieve optimal solutions, and to describe how alternative modeling may clarify the dynamics underlying those findings.

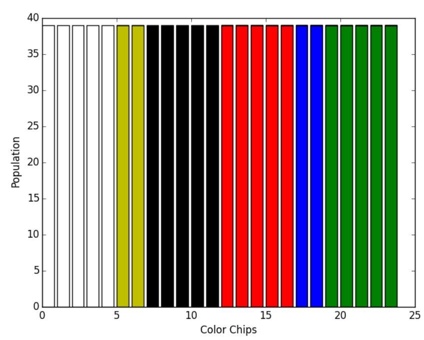


Figure 1. Color categorization solution reached by a fixed population of 49 agents using complete graph, 24 color chips, 8 color terms and $k_{sim} = 3$. Each column in this kind of histogram represents a stimulus of a particular color appearance (or a "color chip") and the pseudo-colored vertical bars shown represent frequency with which a population used one or more color term(s) to categorize a chips. Thus, for example, the five white bars seen at chips 0–5 convey that the results of the population simulation solution assigned those 6 color appearances to the same single categorical partition are represented by the 100% white bars at those locations. In this solution, the number of color categories is suboptimal (6 instead of an optimal 5) and uneven. The solution was convergent at the $1,080,000^{th}$ iteration and remained unchanged to $3,000,000^{th}$ iteration when the simulation was stopped. Note because the 24 chips form a color circle, the white chip between 0 and 1 is the same chip as between 24 and 25.

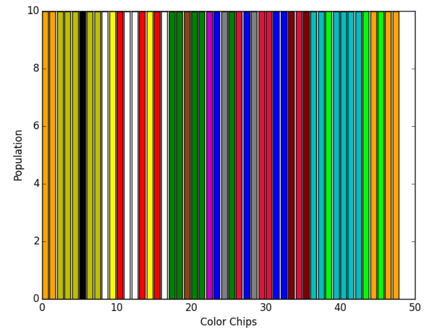


Figure 2. Color categorization solution reached by a fixed population of 49 agents using complete graph, 48 color chips, 16 color terms and $k_{sim} = 5$. In this suboptimal solution, most of color categories observed in the categorization solution do not manifest as continuous sets of color chips. (The optimal solution has 6 categories.) The solution was convergent at the $1,080,000^{th}$ iteration and remained unchanged to $1,400,000^{th}$ iteration when the simulation was stopped.

Fig 1 illustrates how suboptimal categorizations can occur without Birth-Death dynamics. The suboptimality in Fig 1 occurred because the simulation algorithm lacked mechanisms to provide enough variation to overcome a solution arising from prematurely established agreement. In particular, the rate at which randomness was used while reaching a solution was lower than the rate at which the agents recovered an already established solution.

Fig 2 illustrates another way suboptimality can be observed. The noticeable lack of contiguous color categories in Fig 2 occurred because the evolving concept of color similarity did not guarantee that each color category partition manifested as a continuum of neighboring, perceptually similar, color chips. Fig 2's solution illustrates a color categorization solution in equilibrium, but which exhibits disjoint category members and no common categorical boundary. Real-world examples of disjoint category structures are rare in human color categorization literature, and are not consistent with widely-held defining ideas suggesting that categorical exemplars mostly share features in common. In addition, in Fig 2 color categories emerged consisting of only a single chip. Such categories remained in a stochastic equilibrium because the rate at which they were "corrected" by adjacent, more consistent categories was slower than the rate that they were able to "recover" themselves—where "recover" means that once a population agrees on a particular solution, even though that solution might be temporarily disturbed with small amount of variation, the population quickly "corrects" such variation by returning to the previously established solution, that is, a return to the previous state of agreement without variation.

The apparent mechanism for phenomena like those in Fig 2 is that results from games that maintain the current equilibrium occur much more frequently than the results that contribute to equilibrium change. This produces an extremely low probability for changing a converged stochastic solution into a different one.

As an example, consider the situation illustrated in Fig 3 consisting 24 color chips, 8 terms, and k_{sim} of size 3 and color categories having instances of a *single color chip* (green and white color coded categories at chips 17 and 24) that are in an equilibrium state. In order to move toward a more optimal equilibrium state, a one-chip category must either enlarge itself to become an equivalent size as other categories or it must disappear. However, most of the games played by the population will produce results that prevent this from happening. One possible explanation for this is the following:

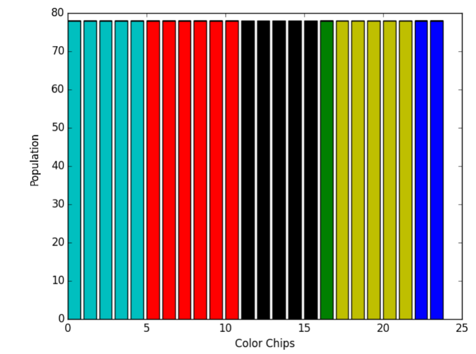


Figure 3. Color categorization solution reached by a fixed population of 49 agents using complete graph, 24 color chips, 8 color terms and $k_{sim} = 3$. Results shown converged at $1,080,000^{th}$ iteration and remains unchanged until $3,000,000^{th}$ iteration. Optimal number of categories under this setting is 4.899

Let the chip between 16 and 17 be denoted by G and its category be called "green". For it to disappear from the categorization solution, it must be played repeatedly against other chips different from G that are within k_{sim} of G's members. This results in categorization game failures. On average such failures decrease the probability that an agent engaged in a naming game for G will name G "green" the next time it appears in a contest for that same agent where G is one of two compared chips. For related reasons, on average the probability that G will be named "black" (the category to the left of "green") will be incremented in that next game that G appears for that agent, and similarly for the category "yellow" that is just right of the category "green". However, this decrement of G for "green" is outweighed by the increment in the probability that G is named "green" that is due to the chance of G being chosen to be played with one of the many chips that is at least k_{sim} distant from it. It is the result of this kind of outweighing dynamic that permits "green" to remain a stable one-chip category.

A similar line of reasoning applies to the suboptimal "blue" category between chips 22 and 24.

In other words, in the case of the simulation parameters described, the variation provided by the evolutionary dynamics is not sufficient to perturb a converged solution. Thus, to achieve optimality, a different evolutionary algorithm, or different dynamic, must be applied to Fig 3. A general procedure for accomplishing this is described in the following subsection.

Simulations using Birth-Death dynamics

For our study, and, more generally for social evolution, Birth-Death is a useful mechanism to investigate, because, unlike genetic mutation, society can develop mechanisms to alter its rate. Changing birth-death rates over time is a key feature in our simulations that allows for investigations of movement toward optimality. In human societies, changes in birth-death rate can be caused by a variety of factors such as marriage rules, warfare, plague, and so forth. Also for many human societies there exist independent data about the birth-death rate that can be incorporated into evolutionary modeling methods. Such data doesn't exist for noise. And, even ignoring noise data, it is harder to find relevant social-psychological mechanisms for the introduction of noise or mutation-like processes for the evolutionary color categorization scenarios addressed in this article, or, more generally, for human social evolution.

Both Birth-Death and Noise can be viewed as providing "shocks" to an established naming system. Such shocks, if not cumulatively too minimal or too excessive, may provide means for a naming system to leave a local, non-optimal equilibrium and move toward a more optimal one. This has been repeatedly and systematically observed in our simulations. Depending on the current situation, both increases in shock rates, decreases in rates, and changes from increases to decreases and vice versa can lead to optimality. Our simulations reveal that there is an optimal rate change for moving toward an optimal color naming strategy. In the discussion below we call this rate Ω and describe how it depends on the current naming situation. Its definition involves the following cases that are summarized below and illustrated through simulations in the Supporting Information (SI) section.

(1) Ω remains constant. In this case, the amount of change is great enough to eliminate narrow categories while retaining categories that are wide enough. S3 Fig shows three examples of the kind of change that results from constant Ω.

(2) Birth-Death rates are sufficiently lower than Ω. In this case the amount of change is insufficient to eliminate suboptimal categories or to produce changes in boundary configurations. An implication of this is that once a solution has converged to a stochastic equilibrium, rates significantly below Ω will preserve an existing form of stability, and thus, to move toward optimality, rates at Ω or greater are needed. This idea is illustrated in S2 Fig

(3) When Birth-Death rates exceed the value of Ω, convergence to non-optimal stochastic equilibria are seen. Thus, for (1) and (2) to apply, the birth-death rate needs to be decreased below Ω in order to achieve a more optimal stochastic equilibrium.

(4) Increases toward optimal numbers of color categories. By applying (1)-(3), equilibria can be arrived at with n near-equal size categories. If the optimal number of categories is $> n$, $> n$ (usually $n + 1$) number of equal size categories can be achieved by introducing fluctuations in the birth-death rate. S5 Fig shows this.

Simulations and explanations describing items (1)–(4) are presented in *Supporting Information* material below.

Discussion

The evolution of human color categorization systems was developed as a topic of scientific investigation following the publication of a book by Gladstone in 1858 [8] which includes a chapter devoted to the semantics of ancient greek color terms. (Besides being an eminent Greek scholar, Gladstone was also four-time Prime Minster of England.) There was subsequent work by linguists and anthropologists, but the modern theory took a dramatic turn in 1969 with the publication of Berlin & Kay's cross-cultural study and theory about the universal and systematic development of human color categorization systems [4]. This advance led to a number of empirical investigations and generated a number of theories about the cultural evolution of categorization systems. And this, in turn, ultimately led to game-theoretic modeling and artificial agent computer simulations aimed at better formulating and testing theoretical ideas surrounding the sharing and use of categorization systems. In addition to the present study, our previous work systematically examines simulated color category learning and evolution in artificial agent populations. This research is designed to emphasize pragmatic constraints that are generally believed to be important to the formation and stabilization of population categorization systems seen in human societies.

This article systematically explores a new pragmatic constraint which is an analogue of the birth-death dynamic naturally occurring in human societies. The birth-death cycle (or modal lifespan) in human societies is known to vary with technological development, which in turn is believed to be linked to the size of a society's color lexicon. Our game-theoretic simulations show that variation in birth-death rates in simulated color categorization games can be a critical factor in the production of more efficient color categorization systems.

To our knowledge, Birth-Death dynamics have not been considered previously in the human color categorization literature, nor has it been modeled or investigated in simulation studies for its role in color categorization universals. In fact, our review of the literature has found very little use of this kind of dynamic in the social sciences, although its impact is well-investigated in the biological sciences. An exception is [2] where it is used to investigate the evolution of ethnocentric behavior. For the case of complete networks the birth-death dynamics utilized by [2] produces a contrary result from the one we observed: For their setup, [2] finds for a complete network no stable equilibria. However, for a network consisting of grid on a torus where players only play with neighbors, and offspring can only be produced when there is an empty space next to the potential parent, [2] find stable population equilibria. In our simulations, we find stable equilibria for both the complete network case as well as for the Baarabaási-Albert construction of scale-free networks. (This construction is described in [1].) In general, we find parallel simulation results for complete and scale-free networks.

The 2008 findings of the *United Nations World Population Prospects Report* suggest birth-death rates — which are correlated with measures of life expectancy — are comparatively higher for more technologically advanced societies (compared to those that are less technologically advanced). [36] Moreover, the color lexicon literature suggests that underdeveloped societies comprised of small linguistic populations are also likely to be technologically under-developed, and consist of populations of individuals who generally have briefer lifespans (compared to those from more technologically advanced societies). [6, 8]. Color lexicon survey data show that societies that are less technologically developed are ones that tend to exhibit fewer lexical categories in their shared color naming systems. [12, 15] This is consistent with our simulation results which show that the rate of birth-death, in conjunction with correlated measures of population size, do contribute to variation in stable color category solutions. This result may help in explain the variation in color lexicon size seen across societies worldwide. Although the present findings are suggestive, and real-world investigations of the issue are complicated by increasing globalization even among small isolated linguistic communities, the relevance of the Birth-Death dynamic needs further exploration to demonstrate it's relevance in both the simulation and the empirical human color category literature.

A second issue of investigated here is understanding how (1) color category systems evolve category partitions, and (2) how the dynamics that lead to losing a category (or the "lumping" of two categories into one, for example, the commonly found *grue* category) versus those leading to the formation of a new category (or the "splitting" of a category area into two distinct categories — for example, *siniy* (синий), or dark blue, and *goluboy* (голубой) or light blue, seem in Russian color naming). In the literature, progressive subdivision or "splitting" of basic color terms is viewed as the natural evolutionary progression occurring across human societies. In principle, however, as category subdivisions progressively increase, population consensus about category denotata and robustness of meaning for color terms will decrease and depreciate the categorization system's utility as a shared communication code. This article's research examines the evolutionary processes that seem to drive "splitting" and "lumping," and identify the process features in order to predict and explain the formation (or loss) of new categories in a given society's color lexicon. [16, 32]

Conclusion

There has been much research suggesting that there are universal tendencies in human color naming across linguistic societies. Berlin and Kay [4] proposed that each language has a subset of 11 basic color terms. A Basic Color Term (BCT) is described as " ... a color word that is applicable to a wide class of objects (unlike *blonde*), is monolexemic (unlike *light blue*), and is reliably used by most native speakers (unlike *chartreuse*). The languages of modern industrial societies have thousands of color words, but only a very slender stock of basic color terms. English has 11: *red, yellow, green, blue, black, white, gray, orange, brown, pink, and purple*." [9] Except for very few cases, languages with larger number of terms tend to contain translation equivalents of the terms seen in languages with fewer BCTs. Others claim that color categories are universal due to innate, biological features of humans [17, 21, 30], while still others argue that those universalities are due to cultural causes [3, 31]. Our simulations suggest that color naming systems can develop through evolutionary processes that reflect the pragmatic necessity for naming and communicating color appearances, while additionally incorporating psycholinguistic requirements that colors that are perceptually similar ought to receive the same name, while those that are perceptually distinct should each be referred to by different names. The present study's emphasis on a pragmatic basis for color naming is aligned with cultural theories of color naming universalities because pragmatic necessities tend to vary across linguistic societies and cultures. Moreover, unlike the Berlin-Kay evolutionary modeling that proceeds by way of a successive splitting of evolved color categories, the evolutionary modeling presented here proceeds using both evolutionary features of "splitting" and "lumping" of categories.

Unlike our previous simulations [13, 14, 18, 19, 24], simulations parameters in the present investigations were selected so that non-optimal stochastically stable solutions would result. This allowed us to explore mechanisms that would permit evolution to extend from such non-optimal solutions toward more optimal ones. We showed that birth-death was one such mechanism, and that it has some very useful evolutionary features. Besides being susceptible to various biological and environmental factors, e.g., plague, drought, etc., it is susceptible to many social factors, e.g., warfare, marriage rules, political rules (taxation, limits on family size, etc.), which allow birth-death rates to be manipulated by society. Our research shows that such modulations play an important role in allowing cultural evolutionary processes break out of suboptimal equilibria thereby speeding up cultural evolution, and in our case of simulated color evolution, driving toward more optimal solutions, and resulting in more efficient social transmission of color categorization system information.

Supporting Information

Supporting Information provided here describes systematic tendencies observed in categorization solution simulations that are attributable to changes in Birth-Death parameters. These are described qualitatively using series of progressions that summarize numerous simulations. As shown below, four scenarios, or features, of Birth-Death dynamics (mentioned earlier in the Results section) are illustrated by the SI figures presented. In particular, S1 Fig and S3 Fig images depict progressions in simulation observed results as solutions evolved and stabilized, and should be read, starting at the top-left panel, from left to right and continuing from left to right on subsequent rows of each figure. Specifically, all SI figures illustrate four scenarios, or features, of Birth-Death dynamics mentioned earlier in the Results section.

That is, S1 Fig to S5 Fig illustrate how fluctuations in the birth-death rate with respect to Ω can effect stochastically stable color categorization systems. Items (1)–(4) below describe the logic underlying the four scenarios involving features emphasized previously in the Results section, where Ω is defined.

(1) The value of Ω remains constant. In this case, the amount of change is high enough to eliminate narrow categories but keep categories that are wide

enough. S1 Fig shows the kind of change that can be seen under a constant value of Ω.

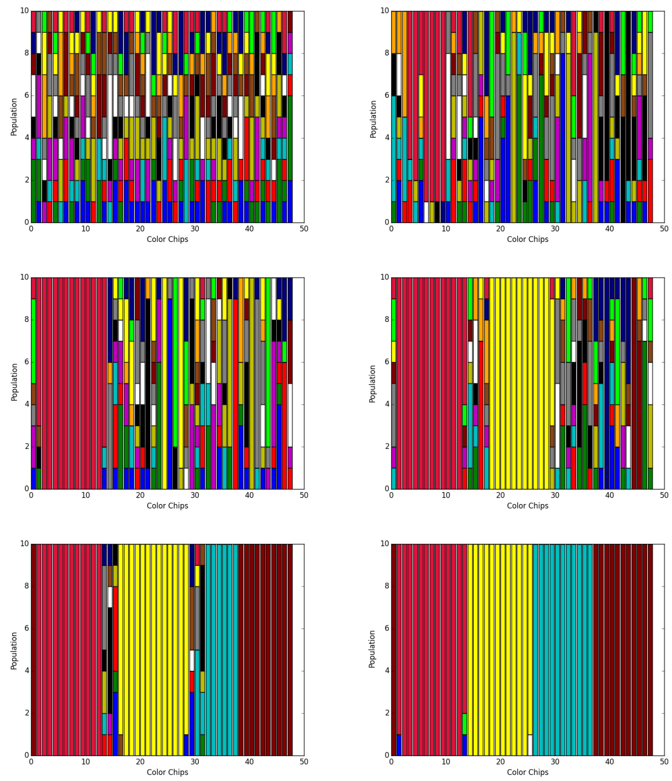
(2) The birth-death rate is sufficiently less than the value of Ω. In this case the amount of change is insufficient to eliminate suboptimal categories or produce changes in boundary configurations. An implication of this is that once a solution has converged to a stochastic equilibrium, rates significantly below Ω preserve this form of stability, and thus, to move toward optimality, rates either equal to or exceeding the value of Ω are needed. This idea is illustrated in S2 Fig.

(3) When the birth-death rate exceeds Ω, it converges to a non-optimal stochastic equilibrium. Then, so that (1) and (2) can apply, the birth-death rate needs to be decreased below the value of Ω in order to achieve a more optimal stochastic equilibrium.

(4) Increase toward an optimal number of categories. By applying (1)-(3), an equilibrium can be arrived at having n near-equal sized categories. If the optimal number of categories is $> n$, $> n$ (usually $n + 1$) number of equal size categories can be achieved through introducing fluctuations in the birth-death rate. S5 Fig shows this.

S1 Fig. The value of Ω remains constant. The level variability produced by the birth-death rate is high enough to eliminate categories that are too narrow and low enough to retain sufficiently wide categories. Through such dynamics, a solution will accumulate wide, or robust, categories until all chips belong to one of them. Three examples of such progressions occuring at different points along the evolutionary path are shown by the series depicted in S1 Fig. For example, the top-left panel shows that early in a categorization system evolution all color chips (denoted along the horizontal axis) are associated by the population of agents with a considerable number of color names, as shown by the multi-color codes depicted in the vertical bars at each chip location. The top-right panel shows how subsequently some chips – in particular the ones denoted by the vertical bars that are color coded primarily "reddish" – exhibit the population's agreement on how to label some chips. Moreover, while chips where there is population agreement tend towards uniform color coding, categories of chips begin to arise from adjacent chips being given the same label by the population. Subsequently in the simulation naming agreement and a trend towards categorization further develops, as seen in the pair of panels in the middle row of S1 Fig. There the middle-left panel depicts where two different emerging categories (denoted by vertical bars that are color coded by two different, nearly homogeneous, colors) can be directly adjacent to each other but lack the critical mass (by only having one or two adjacent chip members) to achieve category robustness. The middle-left panel then suggests that such cases have simulation dynamics that cause either one category to absorb chips on the common boundary or the two categories will remain in an equilibrium state. Also observed is when the distance between the boundary chip and the center of each category differs, one category will be more likely to absorb the boundary chip. Namely, the category whose center is nearest the boundary chip will exhibit the highest probability of absorbing the color chip. For this reason, a smaller category has the greatest chance of absorbing a boundary chip and the larger category has less of a chance of absorbing a boundary chip. Over time, this dynamic leads to the two categories achieving approximating equal sizes. As the solution eventually moves to a more optimal equilibrium where the size of the categories are equal or nearly equal.

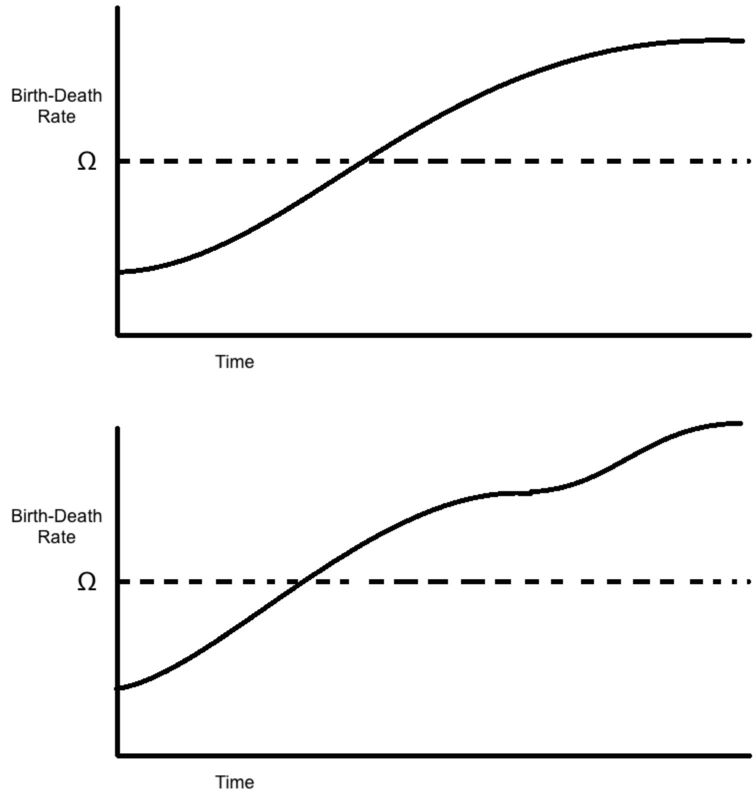
S2 Fig. S2 Fig illustrates how changes in the Birth-Death rate over time when values of Ω are constant. This illustrates the scenario described in item (2) where the Birth-Death rate is less than Ω and must be increased to exceed Ω in order to reach optimality. In S2 Fig's top panel, the amount of Birth-Death variability early



S1 Fig The value of Ω remains constant. Panel series illustrates item (1.) color categorization solution reached by a varying population of 10 agents using complete graph, 48 color chips, 16 color terms and $k_{sim} = 5$. Each agent plays 9300 games before it dies out of the population. From the chaotic state in starting in the top-left panel, the evolution continues in the top-right panel, followed by the middle-left, middle-right, bottom-left and is stable in the bottom-right panel. In this series it is apparent that the color categories of near-optimal length gradually emerge and adjust their length to be optimum under these dynamics.

in the simulation (at the origin of the "Time" axis) is insufficient to eliminate suboptimal categories or encourage boundary chips to change categorization. Once a non-optimal solution is reached, then in order to move towards a more optimal categorization, the level of Birth-Death variability must be increased to exceed the value of Ω. If the simulation began with a level of Birth-Death variability greater than the value of Ω, then it is likely that no color categories would form. However, once a widespread color category is formed which has multiple consecutive color chips, then, if the level of variability is greater than Ω, that category can persist in future generations. But those categories that do not possess (enough) consecutive chips will not survive in later generations, and they will most likely be categorized into one of the surviving categories. When surviving categories become larger, they have the capability of persisting with even higher level of Birth-Death variability. (This is shown in S2 Fig's bottom panel). The solution will tend more towards optimality, if it wasn't already optimal. (It also follows that Birth-Death dynamics can show deviations from S1 Fig and S2 Fig while keeping a current solution.)

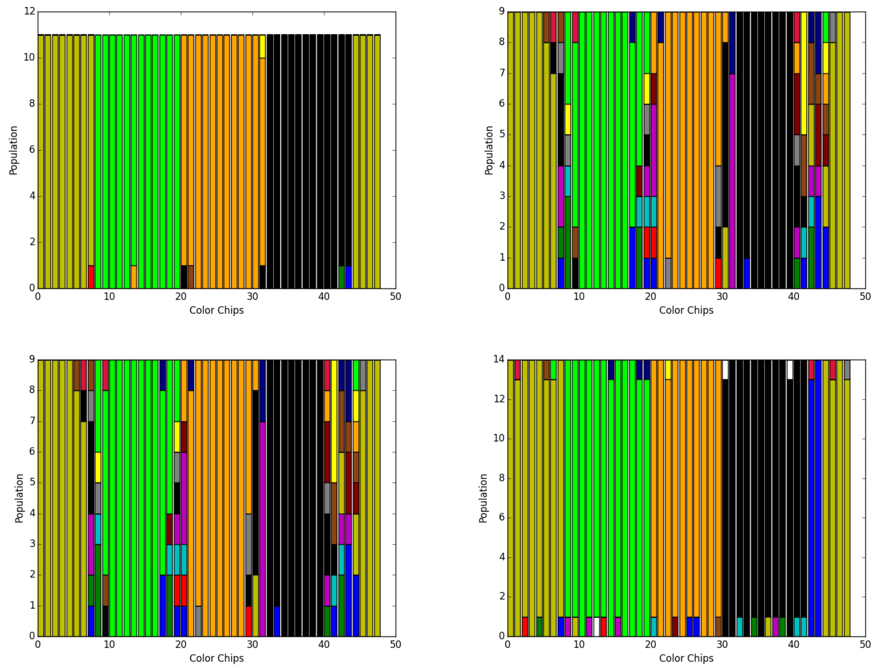
S3 Fig. S3 Fig top-left and top-right panels depict an alternative scenario as described by item (3) in which the birth-death rate is greater than the value of Ω. In the top-left panel the birth-Death rate is at Ω where the existing categories



S2 Fig Schematically representing Birth-Death dynamics when the value of Ω is fixed. S2 Fig depicts general trends observed across multiple birth-death simulation series and may not be accurate in scale. Beginning with a Birth-Death rate lower than Ω and then later increased higher than Ω, the figure illustrates the scenario described in item (2). The top panel illustrates a case where Birth-Death rate is increased once in order to reach a more optimal solution. There is a limit to the amount of increase in variability that can happen at once without eliminating categories. Bottom panel illustrates a situation where Birth-Death rate gradually increases multiple times to achieve a stochastically stable equilibrium that would not be stable if otherwise reached too quickly.

become equal in length, but the number of categories is not the optimal number. To move toward an optimal number of categories the birth-death rate must temporarily increase to provide the dynamics that create space for a new category to be formed. This is shown in the top-right panel where increases in birth-death rate produce a configuration conducive to the formation of a new category. Under these circumstances the scenarios described in S1 Fig and S2 Fig will apply.

Figure S3 Fig's bottom panels also depict an alternative scenario as described by item (4), illustrating an increase toward the optimal number of categories. From the bottom-left panel it is seen that in the bottom-right panel an increase occurs by creating a new category (the blue bars near chips 42-43) by the following fluctuation of the birth-death rate: First room needs to be made for a new category by increasing the birth-rate, yielding a result like shown in the top two diagrams of S3 Fig, where the birth-death rate was chosen high enough to destabilize the solution. If simulation is run long enough at that appropriate birth-death rate, the structure of the solution will disappear. However, before then, there will be an opportunity for a new category to emerge. When this happens the birth-death rate needs to decrease. As it decreases, a birth-death rate can be chosen so that it won't eliminate narrow categories. Then a new category can form and become stable.

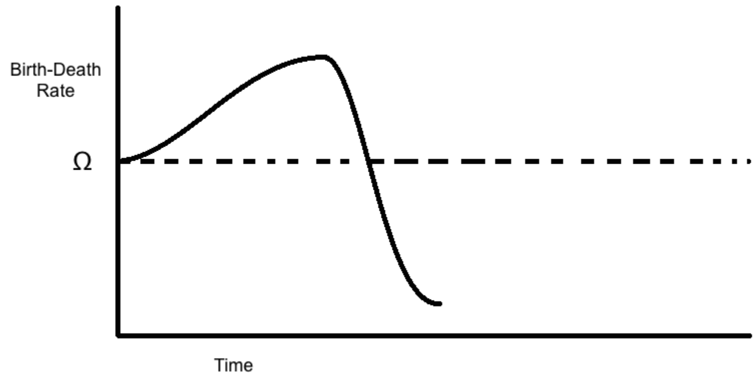


S3 Fig Illustrates scenarios described by items (3) and (4). Agent birth rate on top left= 9300 and on top right = 7200; agent birth rate on bottom left= 7200 and on bottom right = 12800.

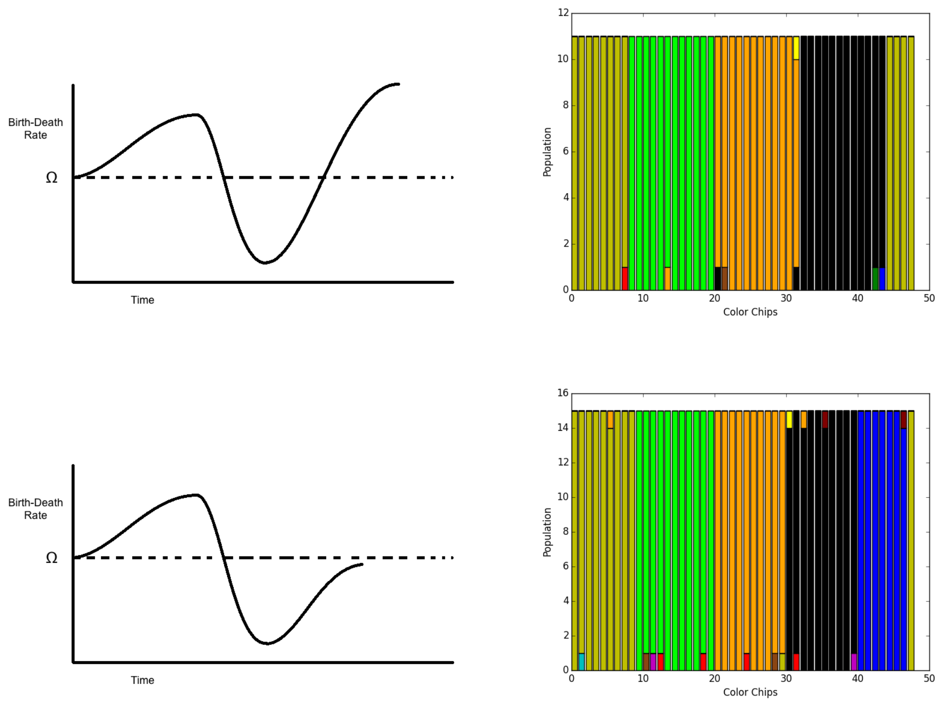
S4 Fig. Figure S4 Fig depicts the observed Birth-Death rate seen under a scenario like that shown in Figure S3 Fig. S4 Fig Schematically depicts results seen when birth-death rates start out equal to the value of Ω. In these cases the initial amount of change produced is insufficient to eliminate suboptimal categories or produce changes in boundary configurations. An implication of this is that once a solution has converged to a stochastic equilibrium, rates significantly less than the value of Ω preserve this form of stability, and, thus, to move toward optimality rates equal to, or in excess of, Ω are needed.

When the state in the bottom-right diagram of S3 Fig is reached, there are possible two options: Make the categories of that solution equivalent-length by either (a) removing the new blue category or (b) enlarging the blue category. The solution ended up in depends on the Birth-Death rate. (a) is shown in the top of S5 Fig and (b) in the bottom S5 Fig.

S5 Fig. Extending the results from the bottom-right panel of S3 Fig to the top panels of S5 Fig. The top panels of S5 Fig show what happens if the Birth-Death rate increases to exceed the value of Ω. The bottom panels of show what happens if the Birth-Death rate is less than Ω. The top panels show that under a new level of Birth-Death variability, the smaller category will disappear in a manner similar to that described in item (2). By comparison, the bottom panels of S5 Fig show that under a new level of Birth-Death variability, the capacity of the smaller category to retain color chips is larger than those of other categories. In this latter case the smaller category becomes equivalent in length as other categories, and the solution moves towards optimality.



S4 Fig Illustrates when the birth-death rate starts out at Ω.



S5 Fig Change toward optimality. Top-left panel depicts when the Birth-Death rate increases to exceed the value of Ω. Top-right panel depicts how a smaller category is eliminated according to item (2). Bottom-left panel depicts when the Birth-Death rate becomes less than Ω. Bottom-right panel shows a smaller category enlarging itself to become equivalent in length to other categories in a manner like described in item (4).

Note that in the top-right panel of S5 Fig, the smaller category is eliminated.

In the bottom-left of S5 Fig, the birth-death rate is less than Ω. Previously, when the birth-death rate was higher, categories had to be large enough to survive in the solution, and this was the case for the 4 categories in the top of S5 Fig, even though the optimal number of categories were larger than 4. With the new level of variability, the Birth-Death rate is not high enough to eliminate the small category while still providing enough variability to disturb solutions of populations. When there is a smaller category that moving towards its optimal length, that category's capacity to retain color chips is larger than those of other categories. As long as enough variability is provided to change an otherwise fixed solution, the smaller category becomes equivalent in length to other categories, and the solution reaches optimality.

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