Sources of Variability in Magnitude Estimates

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Abstract

Data from two magnitude estimation experiments were analyzed to obtain estimates of the coefficient of variation (σ/m) both for individual responses and for ratios of successive responses. The coefficients of variation for ratios of successive responses were markedly lower than would be predicted on the basis of the analysis of individual responses. This suggested that the successive responses were correlated. An evaluation of the several sources of correlation indicated that events occurring on the previous trial played a more important role than long-term drift in producing the disparity between the predicted and obtained coefficients of variation. The relative magnitude of the previous response was found to play a more important role than the magnitude of the previous stimulus in predicting the present response.

Introduction

This study concerns the variability of magnitude estimates which we hope may reveal something of the nature of the judgmental process used. It may also tell us

some ways in which one subject differs from another and hence something about the variability between subjects which, as is well known, is often sizeable.

In most orthodox studies of magnitude estimation, one plots the mean (or median) judgment to signals of the same intensity against that intensity to analyze the psychophysical relation. Our interest is not so much in the mean of the judgments as in the character of their distribution. For this purpose, a statistic of considerable interest is the coefficient of variation σ/m: the standard deviation, σ, of a distribution divided by its mean, m. We have found, for example, that the coefficient of variation of the ratio of successive responses depends both upon the observer and upon the difference in dB between the corresponding signals (Green and Luce 1974). We concentrate here on further explorations of the local events prior to each judgment and how they appear to influence it.

Three experiments were run. The first and third were conventional magnitude estimation ones in which a single signal was presented on every trial and the subject was asked to assign numbers so that the ratio of his numbers to successive tones corresponded to their subjective loudness ratios. The second experiment, which we do not discuss here, involved presenting pairs of tones and asking the subject to assign a pair of numbers which were proportional to the ratio of the loudness of these two tones.

For theoretical reasons, discussed in Luce and Green (1972) and Green and Luce (1974), we have explored the hypothesis derived from the timing model that the reciprocals of the magnitude estimates are distributed as the gamma distribution:

$$\frac{\mu^k x^{k-1} e^{-\mu x}}{k!}.$$

The order of the distribution, k, can be directly estimated from the coefficient of variation by the relation

$$k = \frac{1}{(\sigma/m)^2}. \quad (1)$$

In the timing theory, k is one less than the number of pulses upon which the magnitude estimate is allegedly based. This theory, while not adequate to account for the data in detail, does provide a reasonable first approximation to the distribution of judgments.
As is known from other data (Cross 1973, Garner 1953, Holland and Lockhead 1968, Ward 1972, 1973, Ward and Lockhead 1970, 1971), there are consistent sequential effects present in magnitude estimation. This led us (Green and Luce 1974, Luce and Green 1974) to analyze not the individual responses, R_i, but rather the ratio of successive ones, R_i/R_{i-1} to the same pair of successive signals. On the hypothesis of the timing model that the sensory information derived on each trial is a random variable independent of those on other trials and distributed as a kth order gamma, it can be seen that the ratio of successive responses must be distributed as a beta distribution of the second kind with 2k, 2k' degrees of freedom. The degrees of freedom correspond to the orders of the gamma distributions on the two successive trials. For that case, the coefficient of variation for the ratio of successive responses is given by

$$(\sigma/m)^2 = \frac{k + k' - 1}{k'(k-2)} . \qquad (2)$$

Although the fit of the gamma to the reciprocals of the individual responses was not perfect, it was not too bad. So it should be possible to use Eq. 1 to estimate k and, assuming $k' = k$, to predict σ/m for the ratio analysis by Eq. 2. The important assumptions being tested here are, of course, essentially those of stationarity and independence. The sensory factors leading to successive responses must be independent and any other processes influencing successive responses must be stationary for the prediction to work. The data we collected using the two conventional magnitude estimation experiments allow us to test this prediction. Before presenting the results, we describe the procedure.

Procedure

Four female students were paid $ 2.25 per hour to serve as subjects. The stimuli were 1000-Hz pure tones, 500-msec in duration. There were 27 intensities ranging from 36 to 88 dB SPL in 2-dB steps. Each subject listened to the stimuli in 60-trial blocks and made a loudness judgment after each stimulus presentation. They typed their responses on a computer display keyboard located in a large sound-treated room. A new stimulus was presented about two seconds after the preceding response was recorded.
Standard magnitude estimation instructions were used. To acquaint subjects with the loudness range used in the experiment, they were required at the beginning of the first test session to turn an attenuator knob to produce tones with different loudness ratios (method of production). They then ran about 10 blocks of trials (600 observations) as practice. The data we report were collected after these practice sessions.
Each of the experiments reported here required three to four sessions in a one or two week period. An average of 600 responses were made in each two-hour test session. The two experiments were separated in time by about five weeks.

Coefficient of variation results

Table 1 shows the coefficients of variation obtained for four subjects in Experiments 1 and 3. The same data are analyzed in two different ways. The values in the column labeled "individual responses" were computed by collecting together all the responses given to each stimulus, calculating the ratio of the standard deviation to the mean of the reciprocal responses for each stimulus and then forming a weighted average of this statistic for the 27 different stimuli. The values in the next column labeled "ratio of successive responses"

Table 1: Coefficients of variation (σ/m) for reciprocals of magnitude estimates

Subject	Individual responses	Ratio of successive responses	Prediction
CS - Exp. 1	.580	.604	1.308
Exp. 3	.424	.676	.715
DM - Exp. 1	.398	.436	.653
Exp. 3	.437	.647	.747
PM - Exp. 1	.646	.598	1.998
Exp. 3	.486	.581	.889
BE - Exp. 1	.522	.543	1.017
Exp. 3	.423	.575	.712

$$\Sigma \frac{(E-O)^2}{E} = 1.801$$

were computed as follows. The ratios of successive responses were collected together for each possible difference in decibels between successive stimuli. Since there are 27 stimuli spaced at 2 dB intervals, there are 53 distinctive differences (26 positive, 26 negative, and zero). For each stimulus difference, we computed the ratio of the standard deviation to the mean of the response ratio distribution. Again, we computed a weighted mean of these coefficients of variation over all 53 stimulus differences.

The data in Table 1 show that the average coefficient of variation is somewhat lower for the individual response after several weeks of practice (Experiment 1 versus Experiment 3). This is not surprising.

What is surprising is that the average coefficient of variation is about the same whether one analyzes the ratio of responses or simply the individual responses themselves. In one case, the value for the ratio is

actually smaller. Statistically, this is quite unusual. One generally expects the ratio of two random variables to show considerably more variability than either separately. Specifically, the third column shows the expected coefficient of variation for the ratio analysis calculated from the observed coefficient in the analysis of individual responses and Equations 1 and 2. The striking deviations from these predictions suggest that successive responses must be highly correlated. The remainder of the paper is devoted to exploring the sources of this correlation. Once these sources are discovered, we should be able to devise transformations to render the correlation near zero. These transformations will also produce "corrected" responses that should be useful in determining the true forms of the response distribution. Hence, these transformed distributions may provide clearer information about the judgment process.

Long-term drift

The most obvious source of the correlation is a drift in modulus between blocks of trials or between sessions. Such a drift will increase the variability observed in individual responses, but will not alter values of σ/m obtained in the ratio analysis. Changes in the slope of the magnitude estimation function should also have a greater influence on the individual-response measure than on the ratio measure.

To assess the role of slope and modulus changes, all of the data for a given subject in each experiment were transformed so that the slope and intercept of the magnitude estimation function for every 60-trial block were equal to the slope and intercept for that subject for the experiment as a whole. In the transformed data, then, a subject looked perfectly consistent from block to block. A reanalysis of the data after this transformation showed a 9 % reduction, on the average, in σ/m for the individual response analysis and no change in

σ/m for the ratio analysis. This reduction greatly improved the accuracy of the predictions for the ratio analysis, but in all cases the obtained ratio σ/m was still smaller than the predicted value.

Stimulus factor

One short-term correlative factor is an assimilation process (C r o s s 1973; W a r d 1973) related to the stimulus value on the previous trial. A statistic that reveals this correlation is

$$\Omega(s, s') = \frac{E(\underline{R}_n/\underline{S}_n = s \text{ and } \underline{S}_{n-1} = s')}{E(\underline{R}_n/\underline{S}_n = s)}, \qquad (3)$$

where E is the expectation operator, $\underline{R}_n$ is the response on trial n, $\underline{S}_n$ is the stimulus on trial n, and s and s' are particular stimulus values.
Fig. 1 shows this statistic, averaged over s, denoted simply as $\Omega(s')$ and plotted as a function of stimulus level in dB for the preceding trial. In these coordinates there is a strong linear relation between $\log \Omega(s')$ and the intensity of the previous stimulus measured in decibels. To make this trend in the data clear, the log units on the ordinate have been greatly enlarged with respect to the abcissa. We will not dwell on this finding to any extent since it confirms what was found in previous papers.

Response factor

Consider the successive responses in the experiment normalized in the following way. Divide each response by the expected value of the response to that stimulus presentation, and then take the logarithm of that ratio. Dividing the response by its expected value produces a number that varies about unity. Taking the

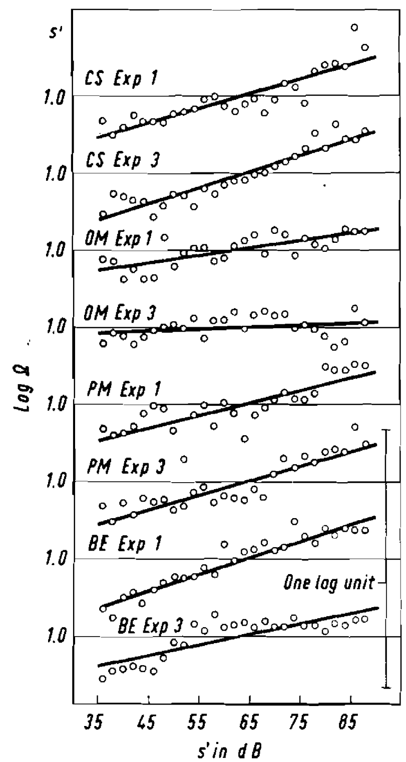

Fig. 1

logarithm makes the statistic largely independent of the size of the stimulus. Thus the log of the relative response has an expected value of nearly zero and variability that is roughly the same for all stimuli.
We now compute the lag-one correlation of the relative response on trial n versus the relative response on trial n-1. The scatter graphs for each subject for Experiment 1 are shown in Fig. 2. The correlations are sizeable for all subjects; the correlation coefficients for subjects CS, DM, PM and BE are .55, .45, .57, and .39.

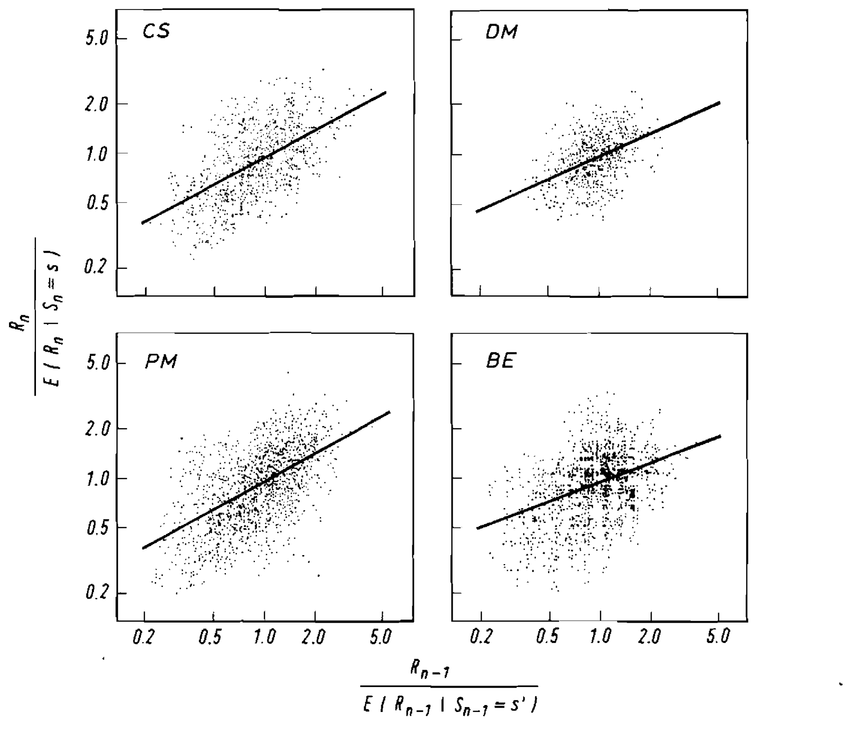

Fig. 2

The correlation is approximately the same size if we use raw data or if we first treat the data to remove the effects of the previous stimulus value (as discussed earlier), that is, if we transform the data to remove the assimilation effect.
Like the stimulus factor, the response factor can be described in terms of a linear relation in log coordinates, as shown by the lines in Figure 2.

Multiple regression analysis

The strong correlation between the present response and the previous response, as well as the previous stimulus, suggests that we try to predict the present response on the basis of three factors: (1) the present stimulus intensity I_n, (2) the previous stimulus intensity I_{n-1}, and (3) the previous response R_{n-1}. The individual correlations suggest that the expected log R_n is related to the several factors as

$$E(\log R_n \mid I_n, I_{n-1}, R_{n-1}) = C + \gamma \log I_n + \alpha \log I_{n-1} + a \log R_{n-1}$$

This equation is the same as one suggested by Cross (1973) except it adds the term $a \log R_{n-1}$. Estimates of the four parameters are given in Table 2. The parameter of substantive interest is γ, the exponent of the power law with the two sequential factors removed. There is still an appreciable scatter in γ and subject BE shows a large change (20 %) in the course of the experiment.

Table 2: Parameters and multiple correlation coefficient for prediction of R_n on the basis of I_n, I_{n-1}, and R_{n-1}

Subject		Parameters γ	α	a	C	Multiple R
CS -	Exp. 1	.314	-.085	.478	-.429	.907
	Exp. 3	.299	-.024	.301	-.407	.904
DM -	Exp. 1	.266	-.073	.381	-.359	.933
	Exp. 3	.290	-.051	.196	-.500	.910
PM -	Exp. 1	.234	-.082	.591	-.376	.890
	Exp. 3	.253	-.048	.435	-.466	.918
BE -	Exp. 1	.298	-.036	.353	-.262	.940
	Exp. 3	.234	-.031	.338	.351	.906

The parameters indicated in Table 2 can account for approximately 83 % of the variance in the logs of the individual responses. To gain some appreciation of the relative importance of the sequential stimulus and sequential response effects in predicting the logarithm of the responses, we perform multiple regressions omitting one or the other factor. Omitting the previous stimulus factor still allows us to account for 82 % of the variance in the logarithm of the responses, a reduction of 1 %. Omitting the previous response factor allows us to account for 79 % of the variance, a reduction of 4 %. The previous response factor appears to be about four times as important as the previous stimulus factor. We might also assess the relative importance of these two factors via a beta weight analysis. The mean ratio of β^2 for the previous response compared with the previous stimulus is 4.73.
There is, of course, a large positive correlation between the two factors. This is responsible for the negative parameter value in Table 2 associated with the previous stimulus level. In fact, the relation between the present response and the previous stimulus is a positive one, if the previous response is ignored, as is shown in Figure 1.

Coefficient of variation analysis

Given the above model, we now wish to test the hypothesis that by removing the effects of both previous stimulus and response we can predict the coefficient of variation of the ratio of successive responses from that of the reciprocal of individual responses. Thus we transformed the data by simply subtracting the proportion predicted by the previous stimulus and response and computed new coefficients of variation for both individual responses and the ratio of successive responses. We then used Eq. 2 to predict the coefficient of variation for the response ratios from the coefficient of variation for the individual responses. The obtained and predicted values in Table 3 show the improvement in these predictions resulting from the transformation. The predicted minus obtained values are half positive, half negative. The relative mean squared error is approximately .1.

Table 3: Coefficients of variation (σ/m) for data transformed to remove effects of I_{n-1} and R_{n-1} on R_n

Subject	Individual responses	Ratio of successive responses	Prediction
CS - Exp. 1	.428	.804	.725
Exp. 3	.394	.803	.644
DM - Exp. 1	.351	.537	.554
Exp. 3	.432	.762	.735
PM - Exp. 1	.497	.746	.925
Exp. 3	.389	.621	.633
BE - Exp. 1	.404	.566	.667
Exp. 3	.342	.574	.536

$$\Sigma \frac{(E-O)^2}{E} = .1022$$

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