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The Response Ratio Hypothesis for Magnitude Estimation[1]

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Assume that each presentation of a signal produces two independent random variable representations and that the ratio of responses on successive trials of a magnitude estimation experiment are proportional to the ratio of a representation from the present trial, which representation is then lost, to the remaining one from the previous trial. The mean response to a particular signal depends on the mean of the representation used, but in general exhibits drift over trials and sequential effects due to the preceeding trial; the mean response ratio does not exhibit drift, but it has a simple form only when there are no sequential effects; however, a modified mean ratio function has a simple form. A model suggested by D. V. Cross is a special case of this one. Simple timing and counting models for the representations fail to exhibit sequential effects, contrary to considerable data. However, data of the authors have suggested a version of the timing model in which the sample size of the representation varies by an order of magnitude depending on how close the signal is to the preceding one; this hypothesis accounts for the observed sequential effects and other aspects of the data.

1. INTRODUCTION

Ward (1973) has demonstrated for magnitude estimation (ME) the existence of significant sequential effects having a character similar to those found in absolute judgments (Garner, 1953; Holland & Lockhead, 1968; Ward, 1972; Ward & Lockhead, 1970, 1971). There appear to be two distinct aspects to these effects. One is an "assimilative" tendency in which the estimate given to the current signal is biased in

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the direction of the preceding signal. The other is a "drift" (Ward calls it a "time order error") in the scale being used. Cross (1973) has suggested a simple model which gives a good account of the assimilative effect and, in addition, explains the so-called regression effect in comparing magnitude production with magnitude estimation (Stevens & Greenbaum, 1968).

Our purpose is to generalize considerably Cross' formulation with an eye to understanding better which of his several assumptions is the key to explaining the assimilative tendency and to see whether the same model suggests the existence of drift.

2. The Response Ratio Hypothesis

Although ME instructions invariably place great emphasis on the point that responses should preserve the subjective ratios of the signals, experimenters have not usually taken these instructions seriously in their analysis of the data; an exception is Ward (1973). The mean, or geometric mean, of the responses is typically plotted as a function of signal intensity (or whatever physical parameter is varied) without regard to any ratios. The alternative is to try to find how the ratio of responses reflects the subjective ratio of the stimuli. If there is no standard signal, as has become standard practice in collecting magnitude estimates, then the observer has little option but to work with previous signals and responses. The simplest hypothesis, that proposed by Ward, is that the observer attends only to the immediately preceding signal and his response to it. The purpose of this paper is to provide an analysis of this hypothesis.

Let $\mathbf{S}_n$ be a *RV* representing the signal presentation on trial n and $\mathbf{R}_n$ the *RV* representing the ME response on trial n. Most psychophysical theories assume, in one form or another, that each signal produces an internal representation which serves as the basis of the response to that signal. Of course, since the responses are variable, the representation is assumed to be variable. Let $\mathbf{X}(s)$ denote the representation *RV* when signal s is presented. Note that $\mathbf{X}$ is a function only of s, not of the trial number n. Basically, what we wish to postulate is that the response ratio on successive trials is proportional to the ratio of the internal representations on successive trials; however, some subtlety is needed to account for the sequential dependencies actually observed.

Our basic postulate may be stated as follows. We assume that the internal representation of the signal is lost whenever that representation is used in a response process. Thus, if one signal plays an active role in m response processes, there must be m distinct, independent representations of that stimulus. The basic notion is that the internal representation is a short-term storage which is destroyed when read by the response process. Obviously this representation could also become processed in another way to yield a longer term representation of the signal, although in the latter case other factors might influence and distort the stored version of the signal. Because sensory signals are processed by several parallel channels in the nervous system, it does not strain our credulity to suppose that several independent representations are possible, provided that the variability of each representation is assumed to increase with m. For our present analysis, we postulate that *whenever signal s is presented in an* ME *experiment, there are two independent representation RVs $\mathbf{X}(s)$ and $\mathbf{X}^*(s)$. These two RVs are used in the following way to generate a response on trial n,*

$$\mathbf{R}_n/\mathbf{R}_{n-1} = C\mathbf{X}(\mathbf{S}_n)/\mathbf{X}^*(\mathbf{S}_{n-1}), \tag{1}$$

where C is constant. We refer to this as the *response ratio hypothesis.*

It is important to understand why we have assumed that a signal representation is destroyed when it is used in a response. If there were only one representation, so $\mathbf{X}(s) = \mathbf{X}^*(s)$, then it follows immediately from Eq. (1) that

$$\mathbf{R}_n = C^{n-1}[\mathbf{X}(\mathbf{S}_n)/\mathbf{X}(\mathbf{S}_1)]\,\mathbf{R}_1\,,$$

and so the dependence of $\mathbf{R}_n$ is on the internal representation on that trial and on the *first* signal and the *first* response. We might, in addition, assume that the internal representation changes over time, so that the chain of dependences weakens in time. This makes quantities similar to $\mathbf{X}(s)$ and $\mathbf{X}^*(s)$ partially correlated, and greatly complicates any analysis. In this paper we investigate the case where the independence between $\mathbf{X}(s)$ and $\mathbf{X}^*(s)$ is complete. As the theorem proved in the next section shows, there may be sequential effects when Eq. (1) holds even with $\mathbf{X}$ independent of $\mathbf{X}^*$.

3. Principal Result

Theorem. *Assume that the response ratio hypothesis, Eq.* (1), *holds and suppose that*

$$F(s) = E[\mathbf{X}(s)], \tag{2}$$

$$F^*(x) = E[1/\mathbf{X}^*(s)], \tag{3}$$

and

$$E[F(\mathbf{S}_n)\,F^*(\mathbf{S}_n)]$$

exist. Then,

$$E\left(\frac{\mathbf{R}_n}{\mathbf{R}_{n-1}}\,\middle|\,\mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s'\right) = CF(s)\,F^*(s'), \tag{4}$$

$$E(\mathbf{R}_n \mid \mathbf{S}_n = s) = A_nF(s) \tag{5}$$

where

$$A_n = A_{n-1}CE[F(\mathbf{S}_{n-1})\,F^*(\mathbf{S}_{n-1})] \tag{6}$$

and A_1 is a constant.

COROLLARY 1. *Suppose that the distribution of* $\mathbf{S}_n$ *is independent of n and let*

$$B = E[F(\mathbf{S})F^*(\mathbf{S})], \tag{7}$$

then

$$A_n = A_1(CB)^{n-1}, \tag{8}$$

$$\frac{E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s')}{E(\mathbf{R}_n \mid \mathbf{S}_n = s)} = \frac{F(s')F^*(s')}{B}, \tag{9}$$

$$\frac{E\left(\dfrac{\mathbf{R}_n}{\mathbf{R}_{n-1}} \,\middle|\, \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s'\right) E(\mathbf{R}_n \mid \mathbf{S}_n = s)}{E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s')} = CB\frac{F(s)}{F(s')}. \tag{10}$$

COROLLARY 2. *Suppose that for each s,* $\mathbf{X}(s)$ *and* $\mathbf{X}^*(s)$ *are identically distributed, then*

$$F(s)F^*(s) \geqslant 1. \tag{11}$$

Proof. By Eqs. (1), (2), and (3), and the independence of $\mathbf{X}$ and $\mathbf{X}^*$,

$$\begin{aligned} E\left(\frac{\mathbf{R}_n}{\mathbf{R}_{n-1}} \,\middle|\, \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s'\right) &= CE\left(\frac{\mathbf{X}(s)}{\mathbf{X}^*(s')}\right) \\ &= CE(\mathbf{X}(s))\, E[1/\mathbf{X}^*(s')] \\ &= CF(s)F^*(s'), \end{aligned}$$

which is Eq. (4).

To establish Eq. (5), observe that since $\mathbf{R}_{n-1}$ and $\mathbf{R}_n/\mathbf{R}_{n-1}$ are independent *RVs*,

$$E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s') = E\left(\frac{\mathbf{R}_n}{\mathbf{R}_{n-1}} \,\middle|\, \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s'\right) E(\mathbf{R}_{n-1} \mid \mathbf{S}_{n-1} = s').$$

Substitute Eq. (4) and take the expectation over $\mathbf{S}_{n-1}$,

$$\begin{aligned} E(\mathbf{R}_n \mid \mathbf{S}_n = s) &= F(s)\, CE[F^*(\mathbf{S}_{n-1})\, E(\mathbf{R}_{n-1} \mid \mathbf{S}_{n-1})] \\ &= A_n F(s), \end{aligned}$$

where

$$\begin{aligned} A_n &= CE[F^*(\mathbf{S}_{n-1})\, E(\mathbf{R}_{n-1} \mid \mathbf{S}_{n-1})] \\ &= CE[F^*(\mathbf{S}_{n-1})\, A_{n-1}F(\mathbf{S}_{n-1})] \\ &= A_{n-1}CE[F^*(\mathbf{S}_{n-1})F(\mathbf{S}_{n-1})]. \end{aligned}$$

In Corollary 1, Eq. (8) follows directly from Eqs. (6) and (7). To prove Eq. (9), we again use the independence of $\mathbf{R}_n/\mathbf{R}_{n-1}$ and $\mathbf{R}_{n-1}$ and Eqs. (4), (5), and (6),

$$\begin{aligned} E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s') &= E\left(\frac{\mathbf{R}_n}{\mathbf{R}_{n-1}} \,\middle|\, \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s'\right) E(\mathbf{R}_{n-1} \mid \mathbf{S}_{n-1} = s') \\ &= CF(s)\,F^*(s')\,A_{n-1}F(s') \\ &= \frac{A_n}{B} F(s)\,F^*(s')\,F(s') \\ &= E(\mathbf{R}_n \mid \mathbf{S}_n = s)\,F(s')\,F^*(s')/B. \end{aligned}$$

Equation (10) follows directly from Eqs. (4) and (9).

In Corollary 2, let f_s denote the common distribution function of $\mathbf{X}(s)$ and $\mathbf{X}^*(s)$, then using Schwartz' inequality,

$$\begin{aligned} 1 &= \left(\int_0^1 df_s(x)\right)^2 \\ &= \left(\int_0^1 [x\, df_s(x)]^{1/2} \left[\frac{1}{x}\, df_s(x)\right]^{1/2}\right)^2 \\ &\leqslant \int_0^1 x\, df_s(x) \int_0^1 \frac{1}{x}\, df_s(x) \\ &= F(s)\,F^*(s). \end{aligned}$$

4. Discussion of Result

It will simplify matters slightly, without seriously limiting their applicability, to assume that the distribution of signal presentations is independent of trials. Moreover, it is convenient to take up the equations out of order.

Equation (5) says that the ordinary expected magnitude function is given by $A_nF(s)$, where by Eq. (8) $A_n = A_1(CB)^{n-1}$. Thus, there is a drift in this function over trials unless the observer selects $C = 1/B$. One might anticipate that a well-practiced observer could do this, but not a naive one. For one thing, B is an expectation over the entire set of stimuli and often responses are recorded before the observer is aware of the entire stimulus distribution. We might also observe that because magnitude functions seem to be well approximated by power functions of the signal intensity, $I(s)$, it follows from Eq. (5) that FI^{-1} is approximately a power function of $I(s)$. We use this shortly.

Equation (9) is one way to formulate sequential dependencies. They exist to the extent that $F(s)\,F^*(s)$ is not a constant. If $F(s)\,F^*(s)$ is constant, i.e., independent of s, then, by Eq. (7), the expectation of this constant is B and Eq. (9) has the value of unity. Assimilation occurs if and only if $F(s)F^*(s)$ is an increasing function of signal intensity, because then

$$E(R_n \mid S_n = s \text{ and } S_{n-1} = s') \gtrless E(R_n \mid S_n = s)$$

according as

$$F(s)F^*(s') \gtrless B$$

which, in turn, holds according as s' is large or small.

Given that the response ratio hypothesis is true, the most obvious function to plot is the estimated expected response ratio as a function of signal intensity ratio. The trouble with this is that it equals $CF(s)F^*(s')$, by Eq. (4), which does not have any simple relation to signal intensity ratio, even when FI^{-1} is a power function, unless

$$F^*(s) = E[1/\mathbf{X}^*(s)] = D/E[\mathbf{X}(s)] = D/F(s)$$

is approximately true. And this, according to Eq. (9), is equivalent to there being no sequential effects. So if there are sequential effects, we need to be able to plot an expression proportional to $F(s)/F(s')$; this is provided by Eq. (10).

To summarize: If there are no sequential effects, one can plot either the expected magnitude function or the expected response ratio. The former has the advantage of getting at F directly and creates no confusion if F is not a power function; it has the disadvantage of being subject to drift, which the expected response ratio is not. If, however, there are sequential effects, which can be tested through Eq. (9), then Eq. (10) is a much better way of getting at F than is Eq. (4).

5. Special Cases

Stemming in part from physiological results, a number of authors have recently postulated that signal intensity is represented internally as independent Poisson processes on parallel channels, and that decisions are based on statistics collected from these processes. For simplicity, we shall suppose that all of the processes are identical with intensity parameter $\mu(s)$ when signal s is presented.

Two quite different statistics have been proposed. McGill (1963, 1967) and Siebert (1965, 1968, 1970) have studied counting models in which the test statistic is the *number* or weighted combination of Poisson events that occur in a fixed time δ. Luce and Green (1972) have proposed that the test statistic be the *time* taken for a fixed sample size $k = J\kappa$ to occur, where κ is the number per channel and J is the number of channels. The reciprocal of the average time for k interarrival times is used as a measure of signal intensity. Since the observed RV is a time, these are called timing models. Green and Luce (1973) have obtained data in a speed-accuracy trade-off study that is consistent with both counting and timing models and have argued that both modes appear to be available to observers in, at least, auditory intensity detection.

Here we examine the response ratio hypothesis for these two models. First, the timing model. Let $\mathbf{T}(s)$ and $\mathbf{T}^*(s)$ be, respectively, the sum of $k(s)$ and $k^*(s)$ interarrival times of a Poisson process. These are distributed as gamma of intensity $\mu(s)$ and order $k(s)$ or $k^*(s)$. (We do not attempt to say how the two samples are divided between channels and *IAT*s per channel.) Because

$$E[\mathbf{T}(s)] = k(s)/\mu(s),$$

plausible candidates for $\mathbf{X}(s)$ and $\mathbf{X}^*(s)$ are

$$\mathbf{X}(s) = k(s)/\mathbf{T}(s) \qquad \text{and} \qquad \mathbf{X}^*(s) = k^*(s)/\mathbf{T}^*(s).$$

It follows (see Luce & Green, 1972, p. 28) that

$$F(s) = E[\mathbf{X}(s)] = k(s)\, E[1/\mathbf{T}(s)] = \mu(s)\, k(s)/[k(s) - 1]$$
$$F^*(s) = E[1/\mathbf{X}^*(s)] = E[\mathbf{T}^*(s)]/k^*(s) = \mu(s).$$

Thus,

$$F(s)\, F^*(s) = k(s)/[k(s) - 1].$$

From the discussion of Eq. (9), we conclude that there are sequential effects in the timing model if and only if $k(s)$ is not a constant. Moreover, since $\hat{E}(\mathbf{R}_n \mid \mathbf{S}_n = s)$ is approximately a power function of signal intensity, Eq. (5) together with the above form for $F(s)$ suggests that $\mu(s)$ is also approximately a power function of intensity unless $k(s)$ depends strongly on s.

In analyzing a considerable body of magnitude estimation data, we have been led to the view that there may be an attention factor that affects the sample size (Green & Luce, 1974). The evidence points to an attention mechanism, about 20 dB wide and centered about the preceding signal, that results in sample sizes an order of magnitude greater when the following signal is in that region than when it is outside it. Since the expected response ratio, Eq. (4), is of the form

$$C[\mu(s)/\mu(s')]\, k(s)/[k(s) - 1],$$

the attention phenomenon means that the factor $k(s)/[k(s) - 1]$ tends to be larger when the difference between s and s' is large than when it is small. This affects the apparent exponent of the response ratio function because the multiplier is also an increasing function of the intensity ratio. A similar argument applies to Eq. (10). One may, nonetheless, elect to study these functions because they are not subject to drift. This is especially important if one wishes to study the variability of the responses.

For the counting model, the random variable $\mathbf{X}(s)$ is the count of the number of Poisson events with intensity $\mu(s)$ that occur in some total time $\Delta = J\delta$ divided by that time. There is only one difficulty: the probability of getting no count whatsoever is non-zero, which would make the response ratio infinite. To avoid that, we take $\mathbf{X}^*(s)$ to be 1 plus the observed count divided by Δ^*. It is not difficult to show that

$$F(s) = E[\mathbf{X}(s)] = \Delta\mu(s)$$
$$F^*(s) = E[1/\mathbf{X}^*(s)] = [1 - e^{-\Delta^*\mu(s)}]/\mu(s).$$

Thus,

$$F(s)\,F^*(s) = \Delta[1 - e^{-\Delta^*\mu(s)}],$$

which is an increasing function of $\mu(s)$. Note, however, that $e^{-\Delta^*\mu(s)}$ is the probability of a zero count. On a priori grounds, one suspects this is negligible, and plausible choices for Δ^* and μ make it so. Therefore, for all practical purposes, the counting model also predicts sequential effects if and only if Δ depends on s. We also conclude, as in the timing model, that the intensity parameter μ is approximately a power function of signal intensity provided Δ is not a strong function of s.

The model proposed by Cross (1973) is, with some slight changes in notation,

$$E(\mathbf{R}_n \mid \mathbf{S}_n = s) = aI(s)^{\beta-b}$$
$$E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s') = cI(s)^{\beta-b}\,I(s')^b.$$

It is easily seen that this agrees with our results if we set

$$F(s) = I(s)^{\beta-b}$$
$$F^*(s) = I(s)^{2b-\beta}$$
$$a = A_n$$
$$c = Ca.$$

A literature closely related to the response ratio hypothesis concerns expected ratio judgments. Denote by $\mathbf{R}(s, s')$ the response random variable when the observer is asked to report the subjective ratio of signal s to signal s'. Following the discussion for magnitude estimation, it is plausible to postulate

$$\mathbf{R}(s, s') = C\mathbf{X}(s)/\mathbf{X}(s'). \qquad (12)$$

Assuming independence of the representations, we obtain

$$E[\mathbf{R}(s, s')] = Cf(s)/f^*(s'). \qquad (13)$$

Comrey (1950) (see Torgerson, 1958, pp. 104–112) introduced the special case of Eq. (13) in which $C = 1$ and $f^* = f$, and he suggested a method for estimating f

(see also Ekman (1958)). Sjöberg (1962) compared this with $C \neq 1$ and $f^* = f$ and with the general form of Eq. (13), using the data of three authors. He found the two special cases inadequate, but Eq. (13) fit the data with $Cf(s)/f^*(s) > 1$. As we showed in Corollary 2, $f(s)/f^*(s) > 1$ and so if $C \geqslant 1$, this is to be expected. These ideas, along with statistical procedures and further data, are reported in Sjöberg (1971). Svenson and Åkesson (1966a, b; 1967) explored the hypothesis

$$E[\mathbf{R}(s, s')]\, E[\mathbf{R}(s', s)] = 1,$$

which holds if $C = 1$ and $f = f^*$, but not otherwise. It fails, as we would expect if there are sequential effects. In 1966b they ask if the nature of this failure is, for individuals, systematic over modalities, and they conclude it is not. In 1967, they show that $Cf(s)/f^*(s)$ tends to be >1 when estimated for pairs for which $I(s) < I(s')$ and to be <1 when estimated for pairs for which $I(s) > I(s')$.

6. Data

The most striking evidence for the existence of sequential effects is suggested by Eq. (9),

$$E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s')/E(\mathbf{R}_n \mid \mathbf{S}_n = s) = F(s')\,F^*(s')/B.$$

If we average over the various values of s the empirical estimates of the left side, we obtain an estimate of $F(s')\,F^*(s')$, which it will be recalled is constant if and only if there are no sequential effects. This has been done for the data, averaged over observers, reported by Cross (1973) and Ward (1973). It has also been done separately for each of the six observers in Green & Luce (1974). These curves are shown in Fig. 1. The pattern is extremely similar in all cases. The effect is assimilative. It is of the same general magnitude, from about .85 at 42.5 dB to about 1.15 at 90 dB in Cross' and in Green's and Luce's data and somewhat more extreme in Ward's. A power function fit, with exponents from .019 to .030 for the former and of .088 for Ward, is a reasonable, although by no means unique, summary of the trend. It is, therefore, clear that $F(s')\,F^*(s')$ is not a constant, and so the timing model can hold only if $k(s)$ depends on s and the counting model only if Δ depends on s.

Next consider Eq. (10),

$$\frac{E(\mathbf{R}_n/\mathbf{R}_{n-1} \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s')\, E(\mathbf{R}_n \mid \mathbf{S}_n = s)}{E(\mathbf{R}_n \mid \mathbf{S}_n = s \text{ and } \mathbf{S}_{n-1} = s')} = CBF(s)/F(s').$$

Denote the expression on the left by $G(s, s')$. Observe that if F is actually a power function of signal intensity, then $G(s, s')$ should depend only on $I(s)/I(s')$ and should

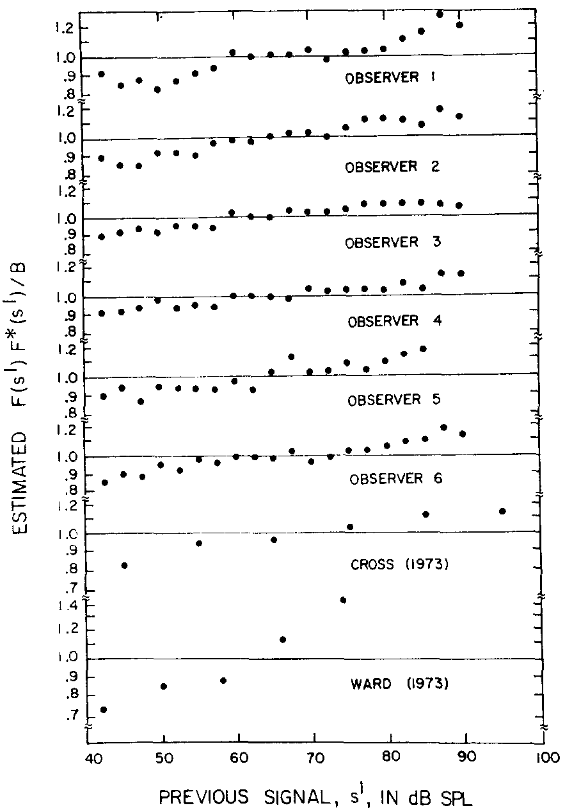

FIG. 1. Estimates of $F(s')F^*(s')/B$ using Eq. (9) from group data reported by Cross (1973) and Ward (1973) and from individual data reported by Green and Luce (1974). Reading from top to bottom, the exponents of power functions to fit these data are: .030, .028, .019, .019, .027, .022, .028, and .088.

be independent of $I(s)$. To test this prediction, we calculated for each observer the following statistic. For s and s' such that $d = I(s)/I(s')$, define $H(s; d) = G(s, s')$ and let $H(.; d)$ denote the mean of it over all s. If the hypothesis is correct, $H(s; d)$ should be approximately equal to $H(.; d)$ for all s and for all d. So a plot of

$$\frac{1}{m} \sum_{d} H(s; d)/H(\cdot; d),$$

where m is the number of different signal ratios, versus $I(s)$ should fluctuate about 1, but not increase systematically with $I(s)$. This function is shown in Fig. 2, and we see only a very slight growth with $I(s)$ except for the largest signal level. Deviations of F

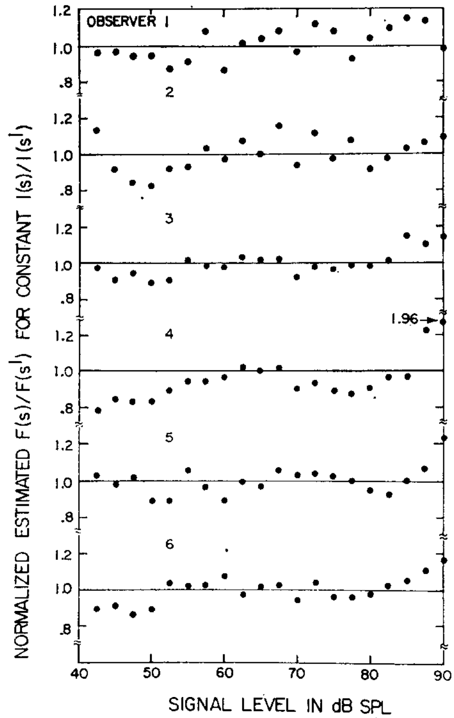

FIG. 2. Estimates of $(1/m) \sum_d H(s; d)/H(.; d)$ versus $I(s)$. According to Eq. (10), this function should be 1 if $F(s)$ is a power function of $I(s)$.

from a power function are the most likely explanation for the slight trend. In particular, we have various reasons to suspect that the 90 dB signal was often absolutely identified and assigned a value independent of the value given to the preceding signal. The problem of absolute identification intruding in magnitude estimations is not to be underestimated.

Finally, we show in Fig. 3 the several magnitude functions suggested by Eqs. (4), (5), and (10). The two ratio functions have been normalized to be 1 for a 0 dB difference, and the magnitude function, Eq. (5), has been normalized to be 1 for the smallest signal, 42.5 dB SPL. The most striking thing about the plot is how similar all three functions are. Note, however, this is only strictly true for Eqs. (4) and (10), which is to be expected since the sequential effects are really slight and FF^* is close to a constant. For observers 2 and 4, and to a lesser degree 5, the ratio functions differ by as much as 10 dB from the ordinary magnitude functions; moreover, the ratio functions appear to approximate power functions more closely.

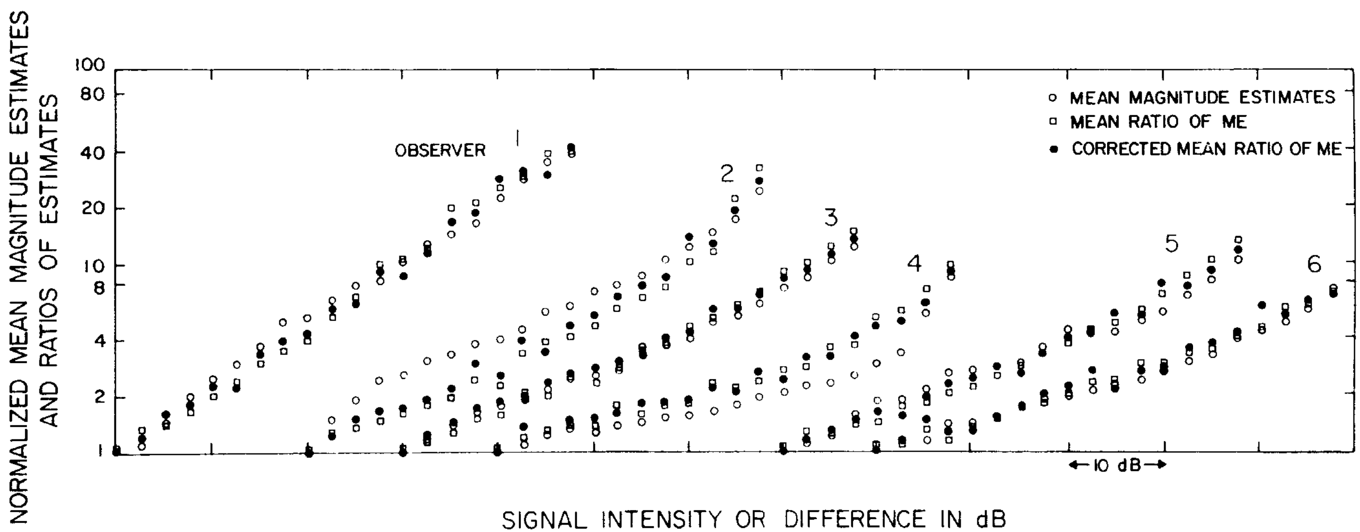

FIG. 3. Plots of mean magnitude estimates, mean ratio of successive magnitude estimates, corrected (Eq. (10)) mean ratio of successive magnitude estimates versus signal intensity in *dB*. The former is normalized to be 1 at the lowest signal, 42.6 *dB SPL*, at the latter two to be 1 at the smallest signal difference, 0 *dB*.

7. Conclusions

The supposition that observers have internal numerical representations of signals and that the ratio of these on successive trials determines the response ratio seems compatible with the instructions given in magnitude estimation procedures. It does not, however, predict sequential effects unless we assume that independent representations are used each time a given signal presentation enters into the response process. This leads to the hypothesis that a representation is destroyed when it is used to generate a response. An incidental effect is to increase response variability beyond that encountered in designs when only one signal representation is needed. From this hypothesis we derived expressions for drift, for sequential effects, for the mean magnitude function, and for two mean magnitude ratio functions. Cross' model is shown to be a special case.

The analysis of data from three experiments suggests that the sequential effects are probably slight for well-practiced observers, although perhaps not for naive ones, and so there is little need to use Eq. (10) to correct the more obvious mean ratio function, Eq. (4). On the other hand, Fig. 3 is fairly persuasive that the plot of successive response ratios is sometimes different from the ordinary magnitude function and better approximates a power function. Its use is strongly recommended in studying individual observers. In Green and Luce (1974), careful attention is given to the variability of response ratios.

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