Preface

The article by R. Duncan Luce published in this issue has the sad distinction of being his last single-authored publication. He died peacefully at his home on August 11, 2012, at the age of 87. With him died an era in the history of psychology.

Duncan Luce, more than anyone else, was responsible for creating and shaping modern mathematical psychology, and more than anyone else he labored to bring closer the day when all theoretical psychology becomes mathematical. He saw mathematics not as an optional method but as an integral part of any sufficiently deep reasoning and understanding. To him, a psychological theory that is not mathematical was merely an unclear, preliminary one. His working philosophy can be characterized by the following scheme: (1) observe behavior and note regularities, (2) create a set of simple principles (axioms, primarily in the language of abstract algebra) from which the observed regularities can be shown to follow, and (3) test experimentally either the axioms themselves, if this is possible, or at least some of their consequences derived as theorems. The article in the present issue is a good illustration of this philosophy, even though it is narrower in scope than much of his earlier work. It contains a highly unusual personal note in which Duncan Luce explains why, feeling that the end was not far away, he decided to publish an article in which experimental tests were proposed but not yet conducted.

Duncan Luce did not think axioms of an area needed any justification beyond experimental corroboration or feasibility in view of experimental corroboration of their consequences. His usual first response to criticism was to ask, "Do you think I made a mistake in derivations? Do you think my axioms contradict known facts?" If his critic could not say "yes" to either of these questions, he viewed the criticism as less than serious, even if he enjoyed discussing it as an intellectual pastime. And he often did. Perhaps because he felt secure behind his mathematical fortifications and because he was always curious about other people's thoughts, he was remarkably open to all kinds of challenges, especially if his opponent used the language of mathematics in raising them. He sometimes got exasperated with what he thought was his opponent's intellectual intransigence and declared further discussion a waste of time, but the next day, more often than not, he would renew the discussion with enthusiasm and vigor.

Because the journal plans to publish an obituary for R. Duncan Luce in a later issue, I will skip an enumeration of his extraordinary accomplishments and honors. The fact that his work was officially recognized (Luce was a member of the National Academy of Sciences, a recipient of the Congressional Medal of Science, and a founder and director of the Institute for Mathematical Behavioral Sciences at the University of California at Irvine) is a tribute to his remarkable ability to impress and evoke respect in a community that is not traditionally receptive to complex mathematical theories. It would not be an exaggeration or slight to say that most psychologists do not understand the mathematics of *Foundations*

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of Measurement, the three-volume treatise that Duncan Luce viewed as one of his (in collaboration with his colleagues) main achievements. However, most psychologists consider measurement theory an important and integral part of their science.

The language of the representational theory of measurement is abstract algebra, Duncan Luce's absolute favorite if one includes in it functional equations. It may not be common knowledge that Duncan Luce defended his PhD thesis in abstract algebra (semigroups) and that he is the author of very important algebraic notions, such as graph theoretic cliques and semiorders. His devotion to algebra is manifest not only in measurement theory but also in abstract psychophysics, his second main area of research interest. Thus, semiorders were introduced as a language formalizing the nature of psychophysical matching. Early in his career he came up with the choice theory and the low-threshold two-state theory of detection, both entirely algebraic even if applied to probabilities. Both have prominent place in the first, psychophysical volume of the *Handbook of Mathematical Psychology,* published in 1963, in my opinion one of Duncan Luce's most lasting contributions to science. His 1959 book *Individual Choice Behavior* contained the audacious attempt to derive Stevens's power function as the only possible psychophysical law on theoretical grounds alone, based on the admissible transformations of scales involved. Shortly afterwards, in an admirable act of scientific honesty, he recanted this attempt under what he recognized as valid criticism, but he later returned to a version of this derivation on a more sophisticated level.

In the last 10 years Duncan Luce worked primarily on what he called a global psychophysical theory. He based it on two basic algebraic operations: combining two stimuli in an overall sensation and determining the ratio of the subjective widths of two intervals of stimuli sharing a boundary. The article in the present issue is an offshoot of the theory for the latter operation. It is fitting that Luce's last article appears in *The American Journal of Psychology,* a journal that has published many influential articles on measurement and psychophysics in its storied past.

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Analogs in Luce's Global Psychophysical Theory of Stevens's Psychophysical Regression Effect

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This article is based on the magnitude production representation in Luce's (2004, 2011) global psychophysical theories for subjective intensity of both binary and unary continua. It is shown that the slopes of certain predicted linear relationships for magnitude production and estimation differ when, in the two procedures, the respondent-generated reference signals differ. This contrasts with Stevens's (1975) regression effect, which arises because the exponents of the psychophysical scale differs with procedural changes. Four experiments are suggested to evaluate these predictions, which, importantly, should be analyzed for individual respondents, not data averaged over respondents.

Background

PSYCHOPHYSICAL

Stevens (1975, pp. 32, 102–109, 271–272), drawing on data of Stevens and Greenbaum (1966), pointed out that plots of average magnitude estimates (ME) (with all magnitudes represented in dB) and of average magnitude productions (MP) (again with magnitudes in dB) are approximately straight lines, with the ME slope somewhat shallower than that of MP. He called this a regression effect because of a similar but not necessarily related phenomenon known as least-squares statistical regression that occurs when variable y is regressed against variable x and compared with the plot of y versus x when x is regressed against y. It is well known that these two slopes do not quite agree.

In ME the experimenter selects signals x and y, and the respondent states a response $\mathbf{p} = \mathbf{p}(x, y)$, so that the perceived ratio of the subjective intensity of y to the subjective intensity of x is $\mathbf{p}$. I use boldface for any magnitude that is selected by the respondent. In practice some measure of central tendency, $\bar{\mathbf{p}}$, such as mean, median, or geometric mean, is usually reported. In MP the experimenter selects signal x and a number $p > 0$, and the respondent states the signal $\mathbf{y} = \mathbf{y}(x, p)$ such that the perceived ratio of the subjective intensity of $\mathbf{y}$ to the subjective intensity of x is p. More specifically, Stevens argued from data that

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psychophysical functions are power functions, so that in ME the relation is

$$\mathbf{p} = \left(\frac{y}{x_0}\right)^{\beta_{ME}}, \tag{1}$$

and in MP the relation is

$$p = \left(\frac{\mathbf{y}}{x_0}\right)^{\beta_{MP}}. \tag{2}$$

Converting these to dB coordinates yields

$$\mathbf{p}_{dB} = \beta_{ME}\left(\frac{y}{x_0}\right)_{dB}, \tag{3}$$

$$p_{dB} = \beta_{MP}\left(\frac{\mathbf{y}}{x_0}\right)_{dB}. \tag{4}$$

Thus, we see the psychophysical regression effect is due to the typical empirical finding that

$$\beta_{ME} < \beta_{MP}. \tag{5}$$

Despite Stevens's name for it, this is not really a case of a statistical regression effect.

Ehtibar Dzhafarov pointed out in his referee reports that even if the power function assumption is wrong, any strictly monotonic increasing transformations of the coordinates produce the following relations. Select the modulus of x_0 to be 1 so the curves intersect $(x_0,1)$ and

(R1) to the right from $(x_0,1)$ the MP curve exceeds the ME curve;

(R2) to the left from $(x_0,1)$ the ME curve exceeds the MP curve.

This is indeed true, but in the literature the regression effect is usually described in terms of linear relations (3) and (4) with (5) typically satisfied.

This article reports comparable but different linear relations and my rationale for why the slopes differ for the ME and MP linear equations that follow from my well-sustained global theory of psychophysics, described next. It is not an explanation or prediction of Stevens's regression but, rather, analogous properties exhibited by my theory.

MODELING

The present results arise from a somewhat more general mathematical representation of Stevens's (1975) method of magnitude production. The representation follows mathematically from testable and tested behavioral properties (Luce, 2004, 2008, 2012). Experimental tests of my theory were reported for audition by Steingrimsson and Luce (2005a, 2005b, 2006, 2007), for brightness by Steingrimsson (2009, 2011, in preparation-a), and for perceived contrast by Steingrimsson (2012, in preparation-b).

Let X denote the set of signals described in terms of physical intensity presented to the respondent, less the corresponding threshold intensity. This is definitely not a dB measure. Suppose that the experimenter presents signal $x \in X$ and a number p to the respondent, who in turn is asked to produce the signal $\mathbf{y} = \mathbf{y}(x, p) \in X$ that stands in the subjective proportion p to the given x. The behavioral invariances of the theory lead to the following numerical representation: There exist a strictly increasing, psychophysical ratio scale ψ over X, a strictly increasing cognitive number distortion function W over positive numbers, and a parameter $\rho \in X$, called a reference signal, such that $\rho < x$ and $\rho < y$, and together they satisfy the following magnitude production constraint:

$$W(p) = \frac{\psi(\mathbf{y}) - \psi(\rho)}{\psi(x) - \psi(\rho)}. \tag{6}$$

In this theory, neither of the two functions ψ nor W is itself a direct function of the parameter ρ. Of course, respondent selected intensity $\mathbf{y}$ and in MP and $\mathbf{p}$ in ME presumably do depend on the magnitude of ρ. In some experiments the experimenter presents the reference signal ρ; in others it is not explicitly presented but rather it is treated as a respondent-generated signal. I do not have any theory that describes how the respondent generates ρ, quite probably dynamically from trial to trial (Steingrimsson & Luce, 2012). Presumably what is needed is some sort of a dynamic theory. There is no doubt that the respondent-generated reference signal is, at present, the weakest aspect of my theory. So from the perspective of the scientist ρ becomes a parameter that has to be estimated from data.

Stevens (1975) assumed the function W was the identity function, as did Narens (1996) in his attempt to account for Stevens's formulation in terms of deeper theoretical ideas. Empirical study of two of Narens's predictions, a type of commutativity and an implication of W being the identity functions, provided very vivid evidence in support of commutativity and against W being the identity function (Ellermeier

& Faulhammer, 2000; Steingrimsson & Luce, 2007; Zimmer, 2005). Indeed, Steingrimsson and Luce (2007) gave a theoretical condition equivalent to W being the following generalized Prelec (1998) function (see also Luce, 2001):

$$W(p) = W(1)\begin{cases} \exp[-\omega(-\ln p)^{\mu}] & (0 < p \leq 1) \\ \exp[\omega'(\ln p)^{\mu'}] & (1 < p) \end{cases}. \qquad (7)$$

They also offered empirical evidence for loudness that the condition was satisfied. And Steingrimsson (2011) did also for brightness.

Because in my theory W is independent of experimental procedure, in particular of MP or ME, these four parameters, ω, ω', μ, and μ', do not depend on the reference signals ρ_{MP} and ρ_{ME}.

A Regression-Like Effect

Luce (2012) summarized his earlier theory for binary or 2-dimensional (2-D) stimuli when pairs of related receptor organs such as the eyes, the ears, perhaps the nostrils, the arms, and possibly other, more artificial pairings are stimulated. However, motivated by earlier work on utility (Luce, 2010), I also proposed a theory for unary or 1-dimensional (1-D) stimuli such as the subjective intensity of vibration, force, preferences over money, and probably taste. Extensive empirical tests of both the underlying behavioral assumptions and predictions from the representations of the 2-D cases have been carried out with success. The work on 1-D cases is far less systematic and less clear except, possibly, for utility of money, but even there substantial gaps remain.

Of these two theories, we need here only two things: First is the prediction of the theory for magnitude production, (6), which does not differ between the 2-D and 1-D cases. Second, at least for the 2-D cases (see Steingrimsson, in preparation-a; Steingrimsson & Luce, 2006) but probably not for the 1-D ones, the possibility that the psychophysical scale can be approximated as a power function of x, which is the signal intensity less the threshold intensity. So $x = 0$ denotes the threshold.

Representations of Magnitude Productions and Estimations

Magnitude production was described earlier for the general case, (6), but in the present context it is appropriate both to use boldface to identify respondent chosen signals and to denote the magnitude production reference signal as ρ_{MP}:

$$W(p) = \frac{\psi(\mathbf{y}) - \psi(\rho_{MP})}{\psi(x) - \psi(\rho_{MP})}. \qquad (8)$$

The only real difference between magnitude estimation and production is that in ME the respondent is given signals x, y by the experimenter and is asked to report the number $\mathbf{p}$ "such that y is subjectively $\mathbf{p}$ times as intense as x." So I continue to assume the same constraint (6), where, of course, $\mathbf{p}$ is respondent generated and so is placed in boldface and the experimenter given y is not, yielding

$$W(\mathbf{p}) = \frac{\psi(y) - \psi(\rho_{ME})}{\psi(x) - \psi(\rho_{ME})}. \qquad (9)$$

So the linear plots we explore in my regression-like effect are $W(p)$ versus $\psi(\mathbf{y})$ and $W(\mathbf{p})$ versus $\psi(y)$, with x fixed at x_0. In Stevens (1975, p. 103) it was p_{dB} versus y_{dB}, as in (3) and (4).

Note that for some purposes it can be useful to rewrite (8) and (9) in the following equivalent forms

$$\psi(\mathbf{y}) = \psi(x)W(p) + \psi(\rho_{MP})[1 - W(p)]. \qquad (10)$$

and

$$\psi(y) = \psi(x)W(\mathbf{p}) + \psi(\rho_{ME})[1 - W(\mathbf{p})]. \qquad (11)$$

For those familiar with utility theories (e.g., Luce, 2000), these two expressions are very typical of the subjective utility representation for binary gambles.

Two observations follow readily from the generic representations (8) and (9), respectively, and the third concerns experimental design:

1. $\bar{\mathbf{y}} \equiv x$ iff $W(p) = 1$ and $y \equiv x$ iff $W(\mathbf{p}) = 1$.
2. Because W is strictly increasing, choose any p such that $W(p) \neq 1$ Then from (8), we have
   $x = \rho_{MP} \Leftrightarrow \bar{\mathbf{y}} = \rho_{MP}$.
3. In experimental practice we have attempted to use designs where we think ρ will be selected so that $\rho < x$ and $\rho < \mathbf{y}$ and will not vary much from trial to trial. This means avoiding choices of p that are so near 0 that the respondent cannot maintain a fixed ρ. Steingrimsson and Luce (2012) discuss aspects of such experimental design features.

An Expansive Approach to the Regression Phenomenon

The basic idea of (8) and (9) is very simple, but it is different from Stevens's (1975) functions because it embodies a 3-dimensional constraint on the three variables involved, x, y, and p, and measures signals differently: intensity rather than dB. So this family of curves exists in 3-dimensional space, but it can be plotted in 2-dimensional space either by fixing one of the variables or choosing to make one of the variables a parameter in a 2-dimensional plot of the other two. I explore two variants of the former.

The classic work studied the case where $x = x_0$. My theory implies that the two triples $(x_0, y, \bar{\mathbf{p}})$ and $(x_0, \bar{\mathbf{y}}, p)$ simply do not agree, which is much the same as Dzhafarov's conditions (R1) and (R2). In particular, the averaged individual slopes of MP data for the linear relation is somewhat larger than the equivalent slope of the linear ME expression, whereas in my view linearity is key to the regression phenomenon. As Steingrimsson pointed out to me, the empirical regression may be confounded by the fact that ordinary statistical regression was probably used to estimate such slopes.[1] Once one has collected new data on individual respondents, it should be possible to disentangle the separate sources of regression. Of course, the full 3-dimensional approach would be extremely expensive in terms of respondent hours, so we vary just two of the three variables. In Stevens's work, it was reduced to two variables by fixing $x = x_0$. In the next section, I offer an explanation for my version of the standard regression effect, and in a later section predictions are made for the case where p is fixed at a value p_0.

The W(p) *Versus* ψ(y) *Regression-Like Effect*

In much experimental practice, x is often set at a fixed value x_0, and measures of central tendency are used so (8) and (9) specialize to

$$W(p) = \frac{\psi(\bar{\mathbf{y}}) - \psi(\rho_{MP})}{\psi(x_0) - \psi(\rho_{MP})}, \tag{12}$$

and

$$W(\bar{\mathbf{p}}) = \frac{\psi(y) - \psi(\rho_{ME})}{\psi(x_0) - \psi(\rho_{ME})}. \tag{13}$$

Proposition 1: *Given (12) and (13), the MP slope exceeds the ME slope iff* $\rho_{ME} < \rho_{MP}$.

Proof: *The slopes of (12),*

$$\frac{1}{\psi(x_0) - \psi(\rho_{MP})},$$

and of (13),

$$\frac{1}{\psi(x_0) - \psi(\rho_{ME})},$$

satisfy

$$\frac{1}{\psi(x_0) - \psi(\rho_{MP})} \gtrless \frac{1}{\psi(x_0) - \psi(\rho_{ME})} \tag{14}$$

$$\Leftrightarrow \rho_{ME} \lesseqgtr \rho_{MP}. \tag{15}$$

Power Function Representations

The results thus far are for a general functional form for ψ, whereas now I will limit myself to the case of power functions. For more than 25 years of research, S. S. Stevens (1975 presents a summary) argued strongly that psychophysical functions are well described by power functions:

$$\psi(x) = \alpha x^{\beta}, \alpha > 0, \beta > 0. \tag{16}$$

Luce (2004, 2011) provided a qualitative condition for this to be sustained that has been verified for audition by Steingrimsson and Luce (2006), for brightness by Steingrimsson (in preparation-a), and for contrast by Steingrimsson (in preparation-b). Some of the calculations that follow are simplified considerably when this form is correct, which may very well be limited to the 2-D cases.

Unlike Stevens (1975), my theory treats ψ as a fixed function, and so β may not vary with different tasks, such as ME and MP. My version of a regression-like effect is based on variations in the respondent-induced reference signals.

Of course, the existing empirical plots are p_{dB} versus y_{db}, whereas the present theory and earlier results would lead us to plot the power function version of ψ, (16):

$$W(p) = \frac{\bar{\mathbf{y}}^{\beta} - \rho_{MP}^{\beta}}{x_0^{\beta} - \rho_{MP}^{\beta}}, \tag{17}$$

$$W(\bar{\mathbf{p}}) = \frac{y^{\beta} - \rho_{ME}^{\beta}}{x_0^{\beta} - \rho_{ME}^{\beta}}. \tag{18}$$

The two slopes are thus

$$\frac{1}{x_0^{\beta} - \rho_{MP}^{\beta}} \text{ and } \frac{1}{x_0^{\beta} - \rho_{ME}^{\beta}}.$$

Estimation issues of such equations, especially β, are discussed in the second Appendix of Luce (2011).

Nonstandard 2-Dimensional Plots

The next approach to comparing MP and ME is to fix p at some value p_0 and plot $\psi(\bar{\mathbf{y}})$ versus $\psi(x)$ and $\psi(y)$ versus $\psi(\bar{\mathbf{x}})$. So the resulting equations from (10) and (11), with p_0 fixed and x and y varied, yield

$$\psi(\bar{\mathbf{y}}) = W(p_0)\psi(x) + \psi(\rho_{MP})[1 - W(p_0)] \quad (19)$$

$$\psi(y) = W(p_0)\psi(\bar{\mathbf{x}}) + \psi(\rho_{ME})[1 - W(p_0)]. \quad (20)$$

Both versions of $\psi(y)$ versus $\psi(x)$ are straight line functions with the same slope $W(p_0)$ but different intercepts when $\rho_{MP} \neq \rho_{ME}$, and so they are predicted to be parallel lines. These lines become identical when and only when $\rho_{MP} = \rho_{ME}$.

Four Experiments That Need to Be Done

In practice, the usual plots are p_{dB} versus $\bar{\mathbf{y}}_{dB}$ and $\bar{\mathbf{p}}_{dB}$ versus y_{dB}, which are approximately linear. Note that these are dB measures, not power functions.

Experiment 1 is aimed at discovering just how the usual plots are changed when (17) and (18) are used and each individual's data are analyzed separately.

Experiment 2 is exactly the same as Experiment 1 except that the experimenter gives the reference signal $\rho = \rho_{\mathrm{MP}} = \rho_{\mathrm{ME}}$ to the respondent, in which case according to Proposition 1 no regression effect should exist.

Experiment 3 is also based on (17) and (18). This entails fixing x_0 and varying y and p using standard MP and ME methods and estimating β. It should be done analyzing individual data, not averages over respondents.

Experiment 4 is based on (16), (19), and (20). This entails fixing p_0 and varying either x or y using standard MP and ME methods and estimating β. It too should be done for individuals separately.

Summary

This article concerned some linear relations involving magnitude productions and estimations whose underlying qualitative properties have been supported by published auditory and visual data. The usual method of specializing this 3-dimensional structure (6) to 2 dimensions results in my psychophysical theory of a simple analog of the so-called psychophysical regression effect. Another reduction from 3-D to 2-D, namely holding p fixed, leads to the prediction of parallel curves. Four simple experiments to test these predictions were outlined.

NOTES

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A personal note: The work pattern that Steingrimsson and I have established over many years of our collaboration is that I would work out, often at his instigation, a theoretical prediction, together we would design one or more relevant experiments, then he would prepare the stimuli and run the experiments, and we would jointly write up the resulting article. The pace has been such that the predictions reported here will take at least 2 years to test experimentally and put in publishable form. I concluded that it would be best to publish these theoretical results before the data are collected.

1. These confounds include asymmetries from the facts that slope estimates are not typically equal when x is regressed on y compared to when y is regressed on x, and different anchoring effects in ME than in MP.

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