

## Tests of Assumptions About the Joint Receipt of Gambles in Rank- and Sign-Dependent Utility Theory

Younghée Cho, R. Duncan Luce, and Detlof von Winterfeldt

A rank- and sign-dependent utility theory is based on an operation of joint receipt of 2 independent gambles and 3 assumptions regarding the operation (R. D. Luce, 1992). The authors tested these 3 assumptions—segregation, duplex decomposition, and the additivity of certainty equivalents over joint receipt—using both judged and choice certainty equivalents (CEs). Median choice CEs provided support for both segregation and duplex decomposition but little support for additivity in gains or in losses, whereas median judged CEs also failed to support segregation. The latter failure appears to result from some subjects misunderstanding the instructions.

The most dominant normative (or rational) models for decision making under risk or uncertainty are those leading to some form of the subjective expected utility (SEU) representation. Von Neumann and Morgenstern (1947) gave the first fully developed theory for the case of expected utility in which the probabilities are prescribed, and Savage (1954) extended it to the case involving events for which the probabilities are not externally given. (For modern surveys of the literature on normative theories, see Fishburn, 1982, 1988; and Wakker, 1989.)

As is well known, many experiments have raised questions about the descriptive accuracy of such theories (for reviews, see Camerer & Weber, 1992; Schoemaker, 1982; Segal, 1987; von Winterfeldt & Edwards, 1986). Much of this work focuses not on the ability of the normative representation to provide an overall fit to data, which it does pretty well, but on specific properties that are both implied by the representation and are often assumed as behavioral axioms from which the representation is derived.

The natural question is, what substitute properties can be invoked that preserve the rational properties when they appear to be (approximately) sustained, and what substitutes can be offered for those that are badly violated by human subjects. The first, and so far the most important, alternative model is Kahneman and Tversky's (1979) prospect theory. They reviewed several kinds of violations of SEU and proposed an alternative, more descriptive representation. Prospect theory encompasses a limited, but important, set of alternatives: two outcome (binary) gambles with a prescribed probability distribution and trinary gambles in

which one outcome is no change in the status quo. The status quo is interpreted, conceptually, as the neutral point that partitions outcomes into gains and losses. In practice, it is often assumed to mean that no money has been exchanged, but in many situations it may be defined in terms of the available alternatives, in which case it is usually called something else, such as a reference level.

The importance of the status quo has been acknowledged for a long time. Von Winterfeldt and Edwards (1986) speculated that people evaluate gambles on the basis of a reference point rather than on the basis of final assets, and they provided examples from everyday experience. Although the importance of the status quo had been discussed, it had not been systematically used in developing a model prior to prospect theory and its generalizations. An explicit, if indirect, study of the role of reference levels is Luce, Mellers, and Chang (1993).

Like SEU, prospect theory expresses the utility of a gamble as a weighted sum of the utilities of the outcomes. Unlike SEU, however, the weights depend on more than the probability (or event) underlying the outcome. First, the model is rank-dependent in the sense that for binary gambles of both gains or of both losses, the weight used depends both on the probability of the event and on whether the outcome associated to that event is more or less preferred than the other outcomes. Second, a feature of prospect theory is that when a gamble includes both gains and losses, the weights generally do not add to 1. As a result, the consequence of no change has utility 0 and thus, the utility scale is a ratio, not an interval, scale. Third, a single weighting function is used both for gains and losses, and various asymmetries between gains and losses—e.g., risk aversion for gains and risk seeking for losses—are accommodated by requiring the shape of the utility function to differ for gains and losses.

The representation, although stated in detail and tested in various ways, was not given a very useful axiomatization. Moreover, as was noted, it was limited to risky alternatives (prescribed probabilities) and to binary and a special class of trinary gambles. Thus, generalizations were required. Luce (1991) axiomatized a binary version with uncertain

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events and, using a similar approach, Luce and Fishburn (1991) generalized the representation to arbitrary finite uncertain alternatives, and Luce (1992) developed a similar formulation for a theory of certainty equivalents of monetary gambles. They spoke of the representation as rank- and sign-dependent utility (RSDU). Independently, Tversky and Kahneman (1992) generalized prospect theory to essentially the same representation of finite uncertain alternatives and fit that representation to data. They spoke of the representation as weighted utility with cumulative weights.<sup>1</sup> Wakker and Tversky (1993) proposed an axiomatization that follows the same general approach of Wakker (1989) to rank-dependent utilities. The axiomatization of Luce and Fishburn (1991) differs in a very significant and controversial way from previous axiomatizations. We describe this difference and empirical tests of the primary axioms of the theory.

### Joint Receipt Operation

Any of these weighted-average utility representations, beginning with SEU, involve both multiplication and addition. In brief, the multiplication of utility by weights turns out to correspond to admissible (i.e., positive similarity) transformations of the utility function that are effected by changes in the events. The addition, however, has a less clear-cut meaning. One way it has been treated is through an application of additive conjoint measurement (see chapter 8 of Krantz, Luce, Suppes, & Tversky, 1971, and the appendix to Kahneman & Tversky, 1979). An alternative is to suppose that it corresponds to an empirical operation, just as is the case with the additive measurement of mass, length, and time durations. This is the tack pursued in Luce (1991, 1992) and Luce and Fishburn (1991), who suggested that we should augment the usual structure of outcomes and uncertain outcomes with an operation, called joint receipt, which simply formalizes the possibility of receiving two things at once; either pure outcomes, such as books, cameras, or money; or more complex gambles, such as a portfolio of stocks. There were relevant papers on joint receipt, two of which they were then aware (Slovic, 1967; Slovic & Lichtenstein, 1968) and three of which they were then unaware (Linville & Fischer, 1991; Thaler, 1985; Thaler & Johnson, 1990). Aspects of this work are discussed later. Kahneman and Tversky (1979) and Tversky and Kahneman (1981, 1986) implicitly invoked such an operation when they described editing of gambles.

In describing joint receipt and its properties, we restrict the present discussion to money gambles with prescribed probabilities, but the ideas apply to general outcomes and to uncertain rather than risky alternatives.

Joint receipt of two gambles means that they are both played independently and, possibly, successively, with no other significant event intervening between the two plays. In practice, this is most easily achieved by either spinning two independent, similar roulette wheels or spinning a single wheel twice in rapid succession. The results from both plays are received. Joint receipt of a gamble and a pure sum of money means that the gamble is played, and the result from that play together with the pure sum of money is received.

### Commutativity and Monotonicity

Let us denote the operation of joint receipt by  $\oplus$ . If  $g$  and  $h$  are either gambles or pure sums of money, the  $g \oplus h$  denotes receiving both  $g$  and  $h$ .

We assume the  $\oplus$  is commutative, that is, for all gambles  $g$  and  $h$ ,

$$g \oplus h \sim h \oplus g. \quad (1)$$

We also assume that joint receipt and preference interact monotonically, that is, for all gambles  $g$ ,  $h$ , and  $k$ ,

$$g \geq h \text{ if and only if } g \oplus k \geq h \oplus k, \quad (2)$$

where  $\geq$  denotes the binary preference relation.

### Segregation

We must consider a type of editing of gambles called *segregation* by Tversky and Kahneman (1986). Suppose  $g$  is a gamble of all gains with outcomes, denoted  $g_i > 0$ , where  $i$  runs over the event partition, and indexed so that  $g_n$  is the least preferred gain. Consider editing  $g$  by subtracting  $g_n$  from each of the outcomes, thereby yielding  $e$  as the outcome associated to event  $E_n$ . Call the resulting gamble  $g'$ . The segregation assumption states

$$g \sim g' \oplus g_n, \quad (3)$$

where  $\sim$  denotes indifference in preference. The parallel property is assumed to hold when the  $g_i$  are losses but with the smallest loss subtracted. It should be noted that Pfanzagl (1959) studied this property without any restriction on the arguments; he called it *consistency*.

Consider, for example, a binary gamble. Using the notation  $(x,p;y)$  to denote the gamble of receiving the money outcome  $x$  with probability  $p$  and  $y$  with probability  $1 - p$ , then for  $x$  and  $y$  both gains,  $x,y > 0$ , segregation implies

$$(x \oplus y, p; y) \sim (x, p; 0) \oplus y. \quad (4)$$

Although joint receipt is not a part of the classical SEU theory, segregation in fact has a rather strong normative flavor. The two situations in Equation 3, and likewise in Equation 4, are formally identical, and so it is normatively compelling that a rational decision maker be indifferent between the two expressions in each case.

The major role of segregation in the theory is a recursive one. A gamble of all gains (or of all losses) is systematically simplified by subtracting the least gain (or least loss) to get a gamble with one consequence 0, then the property next described is used to simplify that gamble to one with only  $n - 1$  consequences, and so on. The net result of the recursion is to introduce the rank dependence for gains and for losses separately.

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<sup>1</sup> More accurately, the weight associated to an event is the difference of weights of cumulative events that differ by the event in question.

### Duplex Decomposition

The next hypothesis concerns gambles having both gains and losses. Unlike segregation, it does not have a normative basis. Yet it is behaviorally plausible. It says that one should evaluate the gains and losses separately and combine the evaluations as a joint receipt. This is what one does in carrying out a cost-benefit analysis. Within the present context of binary money gambles it amounts to the following: if  $x, y > 0$ ,

$$(x, p; -y) \sim (x, p; 0) \oplus (0, p; -y). \quad (5)$$

It is to be kept in mind that the two gambles on the right are played independently. Thus, on the right one can receive either  $0 \oplus 0 = 0$  or  $x \oplus -y$ , neither of which are possibilities on the left, as well as  $x \oplus 0 = x$  or  $0 \oplus -y = -y$ , which are the two possibilities on the left.

This property was called duplex decomposition by Slovic (1967) and Slovic and Lichtenstein (1968), who studied it empirically. They found support for it within the noise level of their data, but there have been no subsequent tests of it, probably because it played no theoretical role until a few years ago.

Within prospect theory and all of its generalization, the utility representation takes the following form: for  $x, y > 0$ ,

$$U(x, p; -y) = U(x)W(p) + U(-y)W(1-p). \quad (6)$$

Recall that  $U(0) = 0$ , so

$$U(x, p; 0) = U(x)W(p)$$

and

$$U(0, p; -y) = U(-y)W(1-p). \quad (7)$$

Thus,

$$U(x, p; -y) = U(x, p; 0) + U(0, p; -y). \quad (8)$$

Thus, if  $U$  is additive over  $\oplus$  (see below), we see that prospect theory implies duplex decomposition (Equation 8). Moreover, duplex decomposition goes a long way toward implying prospect theory.

### Additivity of Utility and Certainty Equivalents (CEs) Over Joint Receipt

In arriving at RSDU, Luce (1991, 1992) and Luce and Fishburn (1991) postulated that utility is additive over joint receipt, that is, if  $g$  and  $h$  are gambles, then

$$U(g \oplus h) = U(g) + U(h). \quad (9)$$

This is a controversial, not directly testable assumption. To study it, we make the additional assumption (below) that for pure sums of money,  $x$  and  $y$ ,

$$x \oplus y \sim x + y. \quad (10)$$

If  $CE(g)$  denotes the certainty equivalent of  $g$ , that is, the sum of money such that  $CE(g) \sim g$ , then one can show<sup>2</sup> that

$$CE(g \oplus h) = CE(g) + CE(h). \quad (11)$$

We naturally assume that  $CE(x) = x$  when  $x$  is a pure sum of money.

It should be noted that if the range of the CEs is all real numbers, then together Equations 9 and 11 imply (using the well-known uniqueness of additive representations) that  $U$  is proportional to  $CE$ . Later we provide evidence against Equation 11 and hence against this proportionality.

### Thaler's Hedonic Rule

Thaler (1985) discussed implications of a rule—he called it *hedonic*—that he argued may summarize some observations about how people respond to joint receipts of monetary gains and losses. Although the rule is not stated explicitly, judging by Linville and Fischer (1991) and Thaler and Johnson (1990), for pure sums the utility function  $U$  is assumed to satisfy the condition:

$$U(x \oplus y) = \max[U(x + y), U(x) + U(y)]. \quad (12)$$

Thaler described implications of this on the assumption that  $U$  is concave for gains and convex for losses, but when subjects contemplate the receipt of two outcomes separated by a week, Thaler and Johnson (1990) concluded that the evidence supports concavity in both cases separately, but not in general for mixed gains and losses. Fishburn and Luce (in press) investigated in some detail the implications of Equation 12 together with various assumptions about the form of  $U$ .

The assumption of Equation 9, made by Luce (1991, 1992) and Luce and Fishburn (1991), coupled with Equation 12 is tantamount to postulating

$$U(x+y) \leq U(x) + U(y), \quad (13)$$

which is equivalent to saying that  $U$  is everywhere concave, which is suspect.<sup>3</sup>

### Hypotheses to Be Tested

From segregation, Equation 4, and Equation 10 we see that

$$\begin{aligned} CE(x+y, p; y) \\ = CE[(x, p; 0) \oplus y], \quad x, y > 0 \quad \text{or} \quad x, y < 0. \end{aligned} \quad (14)$$

<sup>2</sup> By Equations 9 and 10 and the fact that  $CE(g) \sim g$ ,  $U[CE(g \oplus h)] = U(g \oplus h) = U(g) + U(h) = U[CE(g)] + U[CE(h)] = U[CE(g) \oplus CE(h)] = U[CE(g) + CE(h)]$ , from which Equation 11 follows.

<sup>3</sup> A note on terminology: When Equation 9 holds for sums of money, Thaler (1985) refers to the joint receipt  $x \oplus y$  as being “segregated,” which of course is different from our and Tversky and Kahneman’s (1986) use of the same term. In the other case,  $U(x \oplus y) = U(x + y)$ , he refers to  $\oplus$  as being “integrated.” This inconsistency in usage will undoubtedly cause confusion.

And from duplex decomposition, Equation 5, we see that

$$\text{CE}(x,p;-y) = \text{CE}[(x,p;0) \oplus (0,p;-y)], x,y > 0. \quad (15)$$

The third property is the additivity of CEs, Equation 11. Thus, we have a test of the additive property of joint receipt separately from its relations to the properties of gambles, which we call a test of the additivity of joint receipt.

Specifically, we will test the additivity of joint receipt, Equation 11, for the stimuli that we use in studying segregation and duplex decomposition. To be explicit, Equations 11 and 14 yield

$$\text{CE}[(x,p;0) \oplus y] \quad (16)$$

$$= \text{CE}(x,p;0) + \text{CE}(y) = \text{CE}(x,p;0) + y,$$

and Equations 11 and 15 yield

$$\text{CE}[(x,p;0) \oplus (0,p;-y)] = \text{CE}(x,p;0) + \text{CE}(0,p;-y). \quad (17)$$

If we accept the conclusions of Thaler and Johnson (1990), then we should anticipate that the data will support Equation 16 and fail to support Equation 17. We found that to the extent there is a difference, it goes the other way.

### *Judgment and Choice Tasks*

There is one major problem in all this—how does one determine CEs? Certainly, the natural interpretation is that in a choice experiment,  $\text{CE}(g)$  is the amount of money that makes the decision maker indifferent between  $\text{CE}(g)$  and  $g$  when choosing between them. Because estimating that value requires collecting a great deal of choice data, experimenters have been much more inclined to ask subjects to give judgments of their CE values.

Sometimes these judgments have been formulated as buying prices, sometimes as selling prices, sometimes as attractiveness ratings, and sometimes explicitly as estimates of

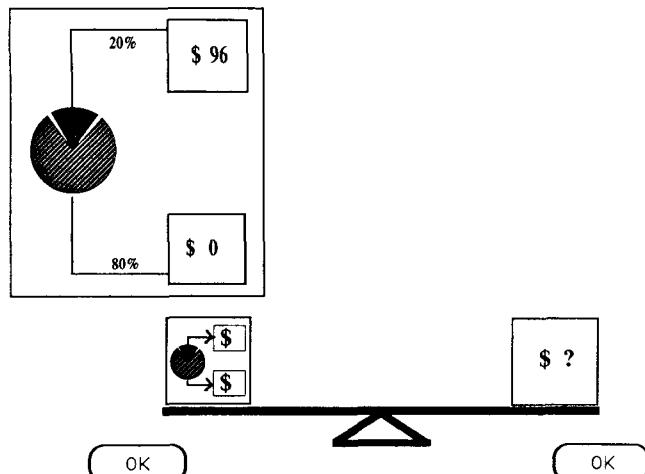


Figure 1. A typical display of a binary gamble used in the judgment task.

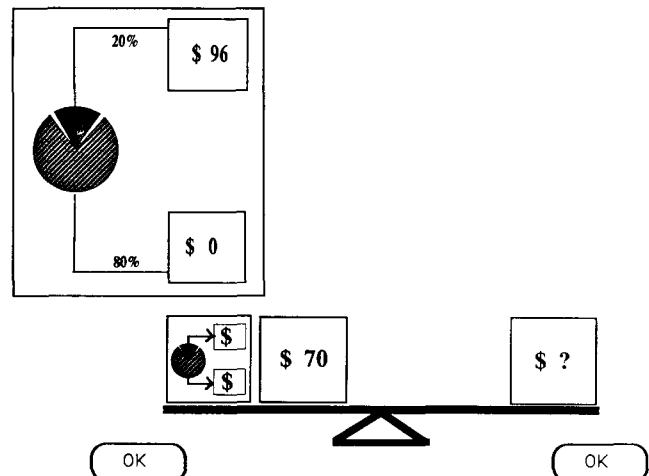


Figure 2. A typical display of a joint receipt of a simple gamble and a sure amount of money used in the judgment task.

the choice CEs. It is fairly obvious that buying and selling prices differ, and so at most one (and probably neither) agrees with the choice indifference value. What is worse is that even when one describes what one means by a choice indifference point and asks subjects to report it, they fail to do so. This was demonstrated explicitly by Bostic, Herrnstein, and Luce (1990) in a study of preference reversals. They compared judged and choice CEs obtained using two closely related psychophysical methods, up-down and Parameter Estimation by Sequential Testing (PEST), and found considerable discrepancies between choice and judged CEs. Using a quite different experimental approach, Tversky, Slovic, and Kahneman (1990) also inferred that preference reversals are caused by a failure of procedural invariance in the sense that the high outcome, low probability gamble is overpriced in the judgment task. Tversky, Sattath, and Slovic (1988) drew the same conclusion. These studies imply that the estimates obtained using choice and judgment tasks are not the same.

Because of these differences, we felt it was important to test the four properties of Equations 14, 15, 16, and 17 using both judged and estimated choice indifference points.

### *Method*

#### *Subjects*

Two hundred thirty-five undergraduate students at the University of California at Irvine participated in the experiment for course credits. Ninety-one took part in the judgment task, and 144 took part in the choice task.

#### *Stimuli*

The same 36 test gambles were used in both the judgment and choice tasks and an additional 144 filler gambles were also available for the choice task. (The construction of these gambles is described in the Design and Procedures Section.) The stimuli were

presented on a computer screen where the indifference was represented as the leveling of a balance beam. Typical displays for a binary gamble, for the joint receipt of a pure sum of money and a simple gamble, and for the joint receipt of two simple gambles are shown in Figures 1, 2, and 3, respectively. On the right side of the balance beam, a green square was presented with a question mark in the judgment task and with a monetary value in the choice task. On the left side of the balance beam, the gamble was displayed by two green squares for outcomes, a schematic pie chart where red or blue areas represented the complementary probability for one or the other of two outcomes, and lines connecting the probability to corresponding outcome. The enlarged version of this gamble was presented above the left side of the balance beam so that the outcomes and probabilities were visible.

Some results of the judgment task, to be described below, led us to modify the display of Figure 2 for the choice test so that the pure sum of money was shown in a separate box of the same size as the gamble. That display is shown in Figure 4.

### Design and Procedures

The experiment consisted of two sets of stimuli chosen to test the segregation and duplex decomposition hypotheses, and two closely related sets of stimuli to test the additivity of joint receipt. All tests were of a  $2 \times 3 \times 2$  within subject factorial design: 2 types of gambles by 3 probabilities ( $p = .2, .5, .9$ ) by 2 outcomes.

*Segregation.* The types of gambles were either  $(x+y,p;y)$ , denoted BG for "binary gamble," or  $(x,p;0) \oplus y$ , denoted JRS for "joint receipt segregation," where  $x,y > 0$  or  $x,y < 0$ . The value of  $x$  was either  $+$96$  or  $-$96$  and that of  $y$  was either  $+$70$  or  $-$70$ . Thus, with the three probability levels, there were a total of 12 gambles, 6 BGs and 6 JRSs.

*Duplex decomposition.* The types of gambles were either  $(x,p;-y)$ , denoted BG as above, or  $(x,p;0) \oplus (0,p;-y)$ , denoted JRD for "joint receipt decomposition," where  $x,y > 0$ . The value of  $x$  was fixed at  $+$96$  and  $-y$  was either  $-$40$  or  $-$160$ . Again, there were 6 BGs and 6 JRDs.

*Additivity of joint receipt.* This was tested relative to the JR gambles used in the segregation and decomposition tests, Equations 16 and 17. In the segregation case, it was only necessary to obtain the CEs for  $(x,p;0)$ , denoted SG for "simple gamble," and

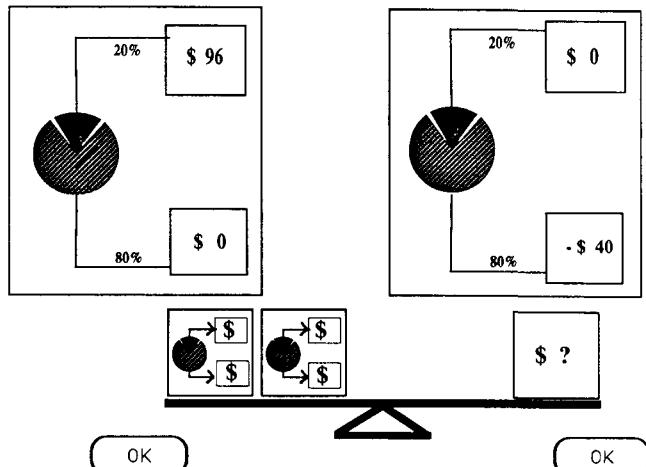


Figure 3. A typical display of the joint receipt of two simple gambles used in the judgment task.

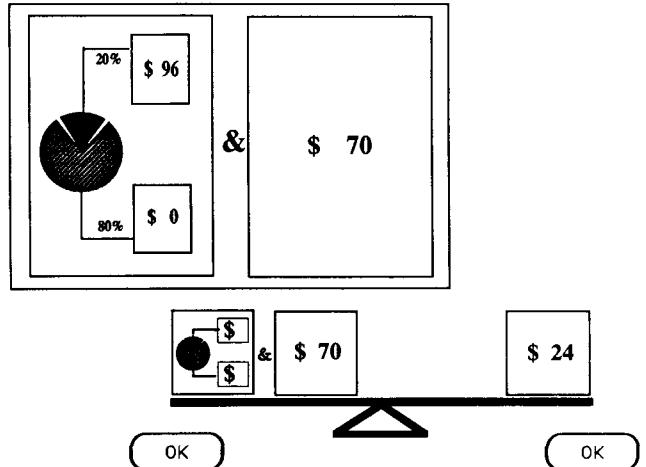


Figure 4. A typical display of the joint receipt of a simple gamble and a sure amount of money used in the choice task.

simply add  $CE(y) = y$  to these values. Thus, the types of gambles were the above JRS plus SG of the form  $(x,p;0)$ . With  $x$  being either  $+$96$  or  $-$96$ , 6 additional SGs were added to the stimulus set. In the decomposition case, the types of gambles were the JRD from above together with the two types of SG:  $(x,p;0)$  and  $(0,p;-y)$ , where  $x$  was  $+$96$  and  $-y$  was either  $-$40$  or  $-$160$ . Because the SG of type  $(x,p;0)$  was already included in the segregation case, only the 6 SGs of type  $(0,p;-y)$  were added to the stimulus set.

Thus, in total, there were  $12 + 12 + 6 + 6 = 36$  test gambles. In addition, filler gambles for the choice task, whose role is described below, were constructed by multiplying the outcomes of the test gambles by  $\frac{1}{3}, \frac{1}{2}, 2$ , or 3, resulting in a total of  $36 \times 4 = 144$  filler gambles.

*Judgment task.* The 36 gambles described above were presented<sup>4</sup> randomly, one at a time, and the subjects were asked to state the monetary value for each gamble for which they were indifferent between receiving the judged monetary value and playing the gamble. They typed this amount into the computer. It was emphasized that their judged value for the gamble should be such that they felt no preference in choosing between the gamble and the judged value. They were cautioned not to switch the complementary probabilities with each other and to maintain the judged value between the minimum and maximum outcomes. Following each response, they were asked to confirm that it was the intended value; if not, they were asked to correct the value. The next stimulus was presented only when the subject responded YES. For practice trials, 11 typical gambles, one corresponding to each type of gamble, were presented. This part of the experiment lasted for 30–45 min.

*Choice task.* Because we used a version of the sequential PEST procedure to determine choice CEs, the full experiment was too lengthy to be run in its entirety on each subject. Rather, each subject confronted only half of the stimuli, and each stimulus was judged by only one half (i.e., 72) of the subjects. This was carried out as follows. The stimuli were partitioned into four sets: those having to do with segregation and its associated additivity, with  $y = \$70$  for one subset and  $y = -\$70$  for the second, and those having to do with decomposition and its associated additivity, with

<sup>4</sup> The program for carrying out the judgment task was developed by Bradley Hutchings under our direction.

$y = -\$40$  for the third set and  $y = -\$160$  for the fourth. Experimental sets of stimuli were then formed by taking all possible pairs of these four subsets, resulting in a total of six sets. Twenty-four subjects were assigned at random to each experimental set, and each stimulus was presented to 72 subjects. Depending on which pairing occurred, subjects encountered either 18 test and  $4 \times 18 = 72$  filler stimuli or 21 test and  $4 \times 21 = 84$  filler stimuli.

Subjects chose between playing a gamble and taking a sure amount of money by clicking the OK button corresponding to their choice.<sup>5</sup> When each gamble was presented for the first time, the money amount that was presented on the right side of the balance beam was chosen according to a uniform distribution over the interval between the minimum and maximum outcomes of the gamble. After that, whenever that gamble appeared next, an interactive computer program kept track of the subject's choices and modified the money amount in the following systematic fashion. For a given gamble, if the subject chose to play the gamble rather than receive the sure amount of money, then on the next presentation of this gamble the amount of money was increased. Conversely, if the subject originally chose to receive the money rather than play the gamble, then on the next presentation of this gamble the money amount was decreased. If at any point the subject selected the gamble when the money exceeded the maximum outcome of the gamble or selected the money when it was less than the minimum outcome, a warning message appeared requesting the subject to reconsider the choice. The money amounts were changed in step size on successive presentations of the gamble according to the following rules:

1. The initial step size was  $\frac{1}{5}$  of the difference of the maximum and minimum outcomes.
2. Whenever the choice reversed (i.e., either from the gamble to the money or vice versa), the step size was halved.
3. When there was no reversal, the step size either remained the same or changed according to the following subrules:
  - 3-1. After a sequence of three choices in the same direction, the step size was doubled, and it was repeatedly doubled until a reversal occurred.
  - 3-2. If a reversal followed a doubling of the step size, the step size was decreased by one half (Rule 2), and two more steps in the same direction were required in order to double the step size.
4. When the step size reached  $\frac{1}{50}$  of the difference of the minimum and maximum outcomes, which was either \$2 or \$4, the test gamble was no longer presented. The choice CE was estimated to be the average of the lowest accepted sure amount and the highest rejected sure amount.

At the onset of the experimental run, there was a total of either 18 or 21 (depending on the pairing) test gambles and no filler gambles. Each of these gambles was presented in random order and after a subject's response, the sure money amount was modified according to the above rules and stored in the computer for later presentation. When the subject responded to all of the 18 or 21 test gambles, the entire set was re-randomized and presented one by one using the modified sure amounts, and once again following the subject's response, the sure amounts were modified according to the rules and stored. This procedure continued, and when, eventually, a gamble reached the termination criterion of a step size  $\frac{1}{50}$  of its outcome range, it was removed and replaced by a filler gamble. By introducing filler gambles, we masked the removal of test gambles and kept the average reappearance interval of each test gamble constant throughout the experiment.

The experiment was concluded when all test gambles were terminated and replaced by filler gambles. The experiment took from  $1\frac{1}{2}$  to 2 hr.

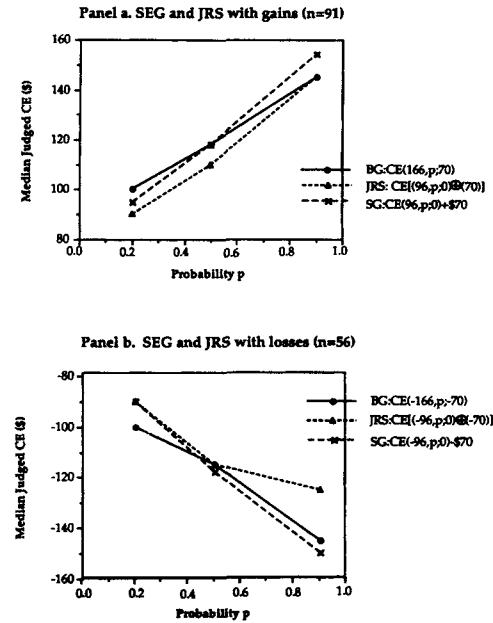


Figure 5. Panel a: The median judged certainty equivalents (CEs) as a function of the probability  $p$  of the various gambles used in testing for segregation of gains and its additivity of joint receipt. Panel b: The median judged CEs as a function of the probability  $p$  of the various gambles used in testing for segregation of losses and its additivity of joint receipt. SEG = segregation; BG = binary gamble; JRS = joint receipt segregation; SG = simple gamble.

## Results and Discussion

Because the individual data were irregular and in some cases asymmetric (see below), we believe that median rather than mean CEs are better estimates to use. The estimated CEs for BG, JRS or JRD, and SG are presented together in each figure as a function of the probability to win (or lose) the specified outcome.

The main concern of this study was the equality of the judged and of the choice CEs of two different types of gambles at fixed levels of  $p$  and  $y$ . To compare the statistical difference of each pair of gamble types at fixed levels of  $p$  and  $y$ , nonparametric median tests are reported. In doing median tests, the assignment of ties to the grand median is an issue. We chose to partition the ties as equally as possible to the two categories, but in case of an odd number of ties we opted to be conservative and assigned the extra one to the larger category.

### Judged CEs

Figure 5 presents the median judged CEs for the segregation and the corresponding additivity tests with  $y = +\$70$  and  $y = -\$70$ . Figure 6 presents the median judged CEs for the decomposition and the corresponding additivity tests with  $y = -\$40$  and  $y = -\$160$ .

<sup>5</sup> The program for carrying out the choice task was developed by Kiho Jeon.

*Segregation test.* Comparing the median judged  $\text{CE}(x + y, p; y)$  and  $\text{CE}[(x, p; 0) \oplus y]$ , for both  $y = +\$70$  and  $y = -\$70$ , the pairs at  $p = .2$  were significantly different ( $P < .05$  and  $P < .01$ , respectively).<sup>6</sup>

*Duplex decomposition test.* The median judged  $\text{CE}(x, p; y)$  and  $\text{CE}[(x, p; 0) \oplus (0, p; y)]$  were compared; no pair was significantly different at either value of  $y$ .

*Additivity of joint receipt test.* Within the segregation context, additivity was tested by comparing  $\text{CE}[(x, p; 0) \oplus y]$  and  $\text{CE}(x, p; 0) + y$ . For  $y = +\$70$ , all of the pairs at  $p = .2$ ,  $p = .5$ , and  $p = .9$  were significantly different ( $P < .05$ ,  $P < .05$ , and  $P < .01$ , respectively) and for  $y = -\$70$ , those at  $p = .2$  and  $p = .9$  were significantly different ( $P < .01$  and  $P < .05$ ). For the decomposition context, comparing  $\text{CE}[(x, p; 0) \oplus (0, p; y)]$  and  $\text{CE}(x, p; 0) + \text{CE}(0, p; y)$  yielded no significant difference. Observe that this is exactly contrary to what we would anticipate from Thaler and Johnson (1990); however, the latter concerned the contemplation of a joint receipt that was to be realized with a 1-week separation.

As an additional analysis, we simply counted the number of subjects who gave identical CEs for both test gambles and those whose CEs indicated preferences in either direction. Symmetry of these distributions indicates support for the hypotheses tested. Table 1 shows the numbers and the corresponding  $\chi^2$  values. Overall, a fairly small proportion

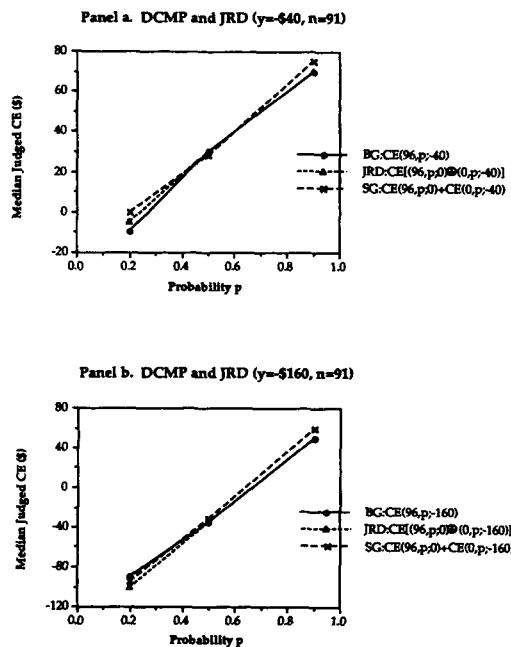


Figure 6. Panel a: The median judged certainty equivalents (CEs) as a function of the probability  $p$  of the various gambles used in testing for duplex decomposition and its additivity of joint receipt for a gain of \$96 and a loss of \$40. Panel b: The median judged CEs as a function of the probability  $p$  of the various gambles used in testing for duplex decomposition and its additivity of joint receipt for a gain of \$96 and a loss of \$160. DCMP = duplex decomposition; JRD = joint receipt decomposition; BG = binary gamble; SG = simple gamble.

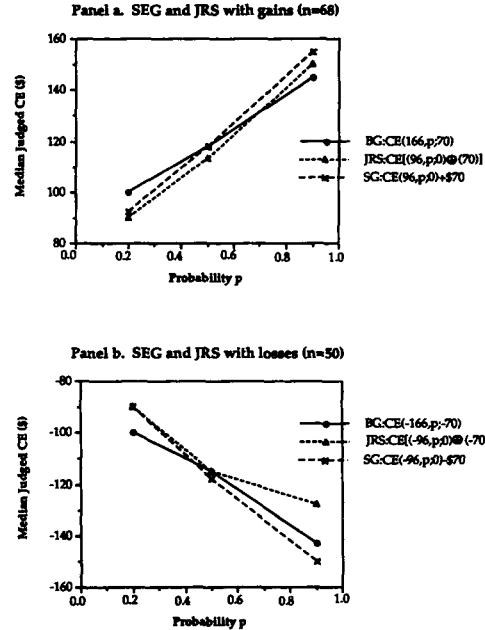


Figure 7. Panel a: The median judged certainty equivalents (CEs) as a function of the probability  $p$  of the various gambles used in testing for segregation and its additivity of joint receipt of gains for the 68 subjects who did not give any CE less than the certain amount, \$70. Panel b: The median judged CEs as a function of the probability  $p$  of the various gambles used in testing for segregation and its additivity of joint receipt of losses for the 68 subjects who did not give any CE more than the certain amount, -\$70. SEG = segregation; JRS = joint receipt segregation; BG = binary gamble; SG = simple gamble.

of subjects had identical CEs, which is not too surprising, given that the CEs were assessed at different times and that the gambles were fairly complex. To test the symmetry of two distributions, we applied a  $\chi^2$  test; the results are presented in the last column of Table 1. Again, the ties were partitioned between the two frequencies, but in case of an odd number of ties, the extra one was assigned to the larger frequency. This likely underestimates the true  $\chi^2$ , because, by definition, the contribution from the indifference frequencies was zero.

*Segregation test.* Consistent with the median data, there is some tendency for subjects to assign a higher certainty equivalent to the binary gamble in the gains condition, but to the joint receipt of the segregated gamble in the losses condition.

*Duplex decomposition.* Most distributions of preferences seem quite symmetrical, and none are significantly different.

*Additivity of joint receipt test.* There are quite a few asymmetries in the preference distributions. In the segregation context, there appears to be a tendency to subadditivity (i.e.,  $\text{CE}[\text{JRS}] \leq \text{CE}[\text{SG}] + \$70$ ) for gains, but for losses,

<sup>6</sup> To avoid confusion, we use (lowercase)  $p$  when referring to probability and (uppercase)  $P$  when referring to statistical significance.

Table 1

*Number of Subjects Whose Judged Certainty Equivalents (CEs) for One Type of Gamble Were Higher Than, Equal to, or Lower Than Those for the Other Type of Gamble and Chi-Square Values*

Test and gamble pair	Number of subjects			$\chi^2$
Segregation test				
BG>JRS	BG=JRS	BG<JRS		
(166,.2;70) vs. (96,.2;0) $\oplus$ (70)	52	7	32	4.8*
(166,.5;70) vs. (96,.5;0) $\oplus$ (70)	53	8	30	5.8*
(166,.9;70) vs. (96,.9;0) $\oplus$ (70)	37	29	34	0.1
Total	142	35	96	8.1**
(-166,.2; -70) vs. (-96,.2;0) $\oplus$ (-70)	11	9	36	12.07**
(-166,.5; -70) vs. (-96,.5;0) $\oplus$ (-70)	21	11	24	0.3
(-166,.9; -70) vs. (-96,.9;0) $\oplus$ (-70)	21	4	31	1.8
Total	53	24	91	8.6**
Duplex decomposition test				
BG>JRD	BG=JRD	BG<JRD		
(96,.2;-40) vs. (96,.2;0) $\oplus$ (0,.2;-40)	33	13	45	1.9
(96,.5;-40) vs. (96,.5;0) $\oplus$ (0,.5;-40)	36	19	36	0.01
(96,.9;-40) vs. (96,.9;0) $\oplus$ (0,.9;-40)	44	10	37	0.5
Total	113	42	118	0.1
(96,.2;-160) vs. (96,.2;0) $\oplus$ (0,.2;-160)	31	20	40	0.9
(96,.5;-160) vs. (96,.5;0) $\oplus$ (0,.5;-160)	35	20	36	0.01
(96,.9;-160) vs. (96,.9;0) $\oplus$ (0,.9;-160)	34	19	38	0.3
Total	100	59	114	0.8
Additivity of joint receipt test in segregation context				
JRS>SG	JRS=SG	JRS<SG		
(96,.2;0) $\oplus$ (70) vs. (96,.2;0) + (70)	26	17	48	5.8*
(96,.5;0) $\oplus$ (70) vs. (96,.5;0) + (70)	25	17	49	6.9**
(96,.9;0) $\oplus$ (70) vs. (96,.9;0) + (70)	15	17	59	22.3**
Total	66	51	156	30.3**
(-96,.2;0) $\oplus$ (-70) vs. (-96,.2;0) + (-70)	27	5	24	0.3
(-96,.5;0) $\oplus$ (-70) vs. (-96,.5;0) + (-70)	29	8	19	1.8
(-96,.9;0) $\oplus$ (-70) vs. (-96,.9;0) + (-70)	34	12	10	10.3**
Total	90	25	53	8.6**
Additivity of joint receipt test in duplex decomposition test				
JRD>SG	JRD=SG	JRD<SG		
(96,.2;0) $\oplus$ (0,.2;-40) vs. (96,.2;0) + (0,.2;-40)	36	11	44	0.9
(96,.5;0) $\oplus$ (0,.5;-40) vs. (96,.5;0) + (0,.5;-40)	43	21	27	3.2
(96,.9;0) $\oplus$ (0,.9;-40) vs. (96,.9;0) + (0,.9;-40)	33	10	48	2.5
Total	112	42	119	0.2
(96,.2;0) $\oplus$ (0,.2;-160) vs. (96,.2;0) + (0,.2;-160)	31	15	45	2.5
(96,.5;0) $\oplus$ (0,.5;-160) vs. (96,.5;0) + (0,.5;-160)	29	14	48	4.0*
(96,.9;0) $\oplus$ (0,.9;-160) vs. (96,.9;0) + (0,.9;-160)	46	5	40	0.5
Total	106	34	133	2.7

*Note.* BG = binary gamble; JRS = joint receipt segregation; JRD = joint receipt decomposition; SG = simple gamble. In this table, BG means CE(BG), JRS means CE(JRS), JRD means CE(JRD), SG in gains means CE(SG) + \$70, and SG in losses means CE(SG) - \$70. In the segregation test,  $n = 91$  for  $y = +\$70$  and  $n = 56$  for  $y = -\$70$ . In the duplex decomposition test,  $n = 91$ . For the additivity of joint receipt test in segregation context,  $n = 91$  for  $y = +70$  and  $n = 56$  for  $y = -70$ . For the additivity of joint receipt test in duplex decomposition test,  $n = 91$ .

\*  $P < .05$ . \*\*  $P < .01$ .

the tendency appears to be reversed. The distributions are fairly symmetrical in the duplex decomposition context.

*Discussion of judged CEs.* There is the possibility that the additivity results may be contaminated by some subjects who, when judging the joint receipt of a gamble and an amount of money, apparently failed to pay attention to both components and responded on the basis only of the gamble. This may have arisen because of the asymmetric stimulus presentation in which, on the left side of the balance beam,

the display for the sure amount of money was smaller than the one for the gamble (see Figure 2).

To test this operationally, let us assume that whenever the judged  $CE(g \oplus +\$70)$  was less than  $+\$70$  in the gains condition or  $CE(g \oplus -\$70)$  exceeded  $-\$70$  in the losses condition, the subject has ignored the  $+\$70$  or  $-\$70$  and responded solely to the gamble. According to this criterion, 23 of 91 subjects failed to respond to the joint receipt as was intended, and 68 are acceptable. The resulting median

judged CEs of these 68 subjects are presented in Figure 7 for  $y = +\$70$  and  $y = -\$70$ .

These data are slightly more consistent than are those for the entire set of subjects (Figure 5). In the median segregation test, only one pair for  $y = -\$70$  at  $p = .2$  was significantly different ( $P < .01$ ). In the median test of additivity of JRS, two pairs were significantly different:  $y = \$70$  at  $p = .9$  ( $P < .01$ ) and  $y = -\$70$  at  $p = .9$  ( $P < .05$ ). Of course, we are in danger of taking advantage of the larger variability of smaller samples in evaluating the null hypothesis.

Table 2 shows how the 68 acceptable subjects distributed among equal or two unequal CEs cases. Although the general tendency of the asymmetry for either subadditivity or superadditivity appears to be as in Table 1, the number of statistically different pairs is reduced by half in Table 2.

Thus, some subjects apparently failed to follow instructions, ignoring the sure money in a joint receipt and basing their judgment solely on the gamble. Possible ways to counter this problem in the future are to use symmetric stimulus displays, to alter the instructions, and to give feedback designed to keep them from ignoring the sum of money when providing their CEs. For the choice task, we adopted both the first and third methods.

### Choice CEs

Figure 8 presents the median choice CEs for the segregation and corresponding additivity tests with  $y = +\$70$  and  $y = -\$70$ . Figure 9 presents the median choice CEs for

the decomposition and corresponding additivity tests with  $y = -\$40$  and  $y = -\$160$ .

*Segregation test.* For both  $y = +\$70$  and  $y = -\$70$ , no pair was significantly different.

*Duplex decomposition test.* For both  $y = -\$40$  and  $y = -\$160$ , no pair was significantly different.

*Additivity of joint receipt test.* For the segregation stimuli, the pairs at  $p = .9$  for both  $y = +\$70$  and  $-\$70$  were significantly different ( $P < .01$ ). For the decomposition stimuli, one pair for  $y = -\$40$  at  $p = .9$  was significantly different ( $P < .05$ ).

Table 3 shows the number of subjects who gave identical CEs to both types of gambles and those whose CEs indicated preference in either direction. Not surprisingly, there are fewer cases of identical CEs, because the PEST method makes it fairly unlikely for a subject to home in on exactly the same CE for the two gambles, even if the gambles were considered indifferent.

*Segregation test.* The distributions appear fairly symmetric, indicating support for the segregation hypothesis. An exception is the case of  $(166,9;70)$  vs.  $(96,.9;0) + (70)$ , but even this distribution is not significantly different from the symmetrical one.

*Duplex decomposition.* All distributions (except one) are quite symmetrical.

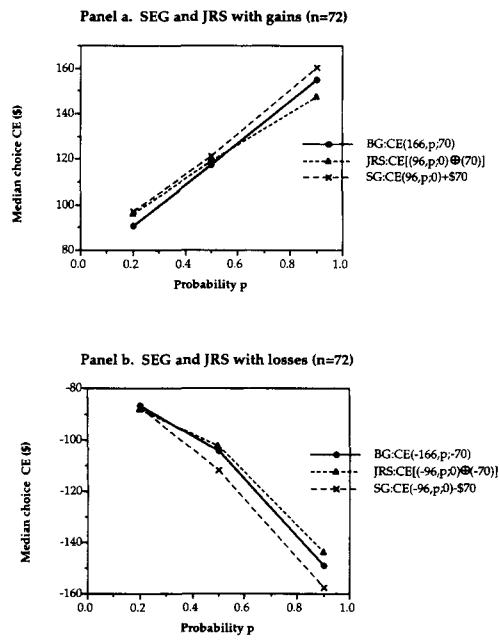
*Additivity of joint receipt test.* Almost half of the distributions are not symmetrical. In the segregation context, there again appears to be a tendency toward subadditivity in the gains case and superadditivity in the losses case. In the

Table 2  
Number of Subjects Whose Judged CEs for One Type of Gamble Were Higher Than,  
Equal to, or Lower Than Those for the Other Type of Gamble and Chi-Square Values

Gamble pair	Number of subjects			$\chi^2$
Segregation test				
	BG>JRS	BG=JRS	BG<JRS	
$(166,2;70)$ vs. $(96,2;0) \oplus (70)$	35	7	26	1.5
$(166,5;70)$ vs. $(96,5;0) \oplus (70)$	36	7	25	2.1
$(166,9;70)$ vs. $(96,9;0) \oplus (70)$	25	17	26	0.06
Total	96	31	77	2.0
$(-166,2;-70)$ vs. $(-96,2;0) \oplus (-70)$	10	9	31	9.7**
$(-166,5;-70)$ vs. $(-96,5;0) \oplus (-70)$	18	10	22	0.3
$(-166,9;-70)$ vs. $(-96,9;0) \oplus (-70)$	19	4	27	1.3
Total	47	23	80	7.7**
Additivity of joint receipt test in segregation context				
	JRS>SG	JRS=SG	JRS<SG	
$(96,.2;0) \oplus (70)$ vs. $(96,.2;0) + (70)$	23	14	31	0.9
$(96,5;0) \oplus (70)$ vs. $(96,5;0) + (70)$	22	14	32	1.5
$(96,9;0) \oplus (70)$ vs. $(96,9;0) + (70)$	12	15	41	13.2**
Total	57	43	104	11.3**
$(-96,2;0) \oplus (-70)$ vs. $(-96,2;0) + (-70)$	21	5	24	0.3
$(-96,5;0) \oplus (-70)$ vs. $(-96,5;0) + (-70)$	26	8	16	2.0
$(-96,9;0) \oplus (-70)$ vs. $(-96,9;0) + (-70)$	32	9	9	11.5**
Total	79	22	49	6.0*

Note. CEs = certainty equivalents; BG = binary gamble; JRS = joint receipt segregation; SG = simple gamble. In this table, BG means  $CE(BG)$ , JRS means  $CE(JRS)$ , SG in gains means  $CE(SG) + \$70$ , and SG in losses means  $CE(SG) - \$70$ . In the segregation test,  $n = 68$  for  $y = +\$70$  and  $n = 50$  for  $y = -\$70$ .

\*  $P < .05$ . \*\* $P < .01$ .



**Figure 8.** Panel a: The median choice certainty equivalents (CEs) as a function of the probability  $p$  of the various gambles used in testing for segregation of gains and its additivity of joint receipt. Panel b: The median choice CEs as a function of the probability  $p$  of the various gambles used in testing for segregation of losses and its additivity of joint receipt. SEG = segregation; JRS = joint receipt segregation; BG = binary gamble; SG = simple gamble.

duplex decomposition case (mixed gains and losses), the tendency is toward subadditivity.

### General Discussion

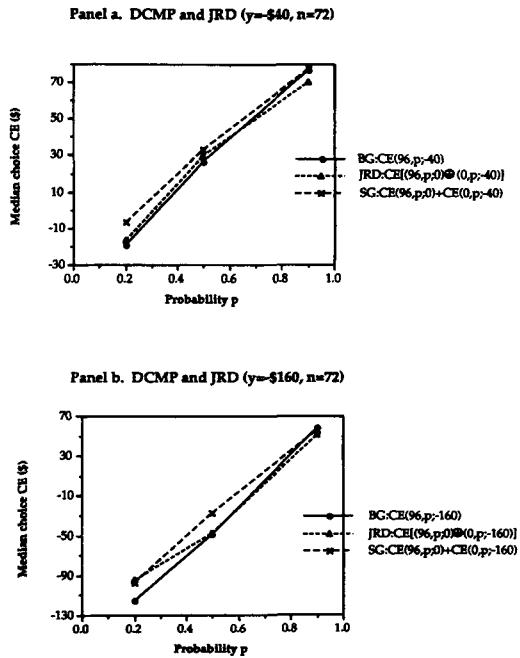
RSDU theory consists of three major parts: a rank-dependent, weighted utility for the subgamble of gains; a distinct rank-dependent weighted utility (RDU) for the subgamble of losses; and a weighted additive combination of the separate utilities for gains and losses (with the sum of two weights not adding to one). This is a generalization of Kahneman and Tversky's (1979) prospect theory, and it is closely related to Tversky and Kahneman's (1992) generalization.

One axiomatic approach (Luce & Fishburn, 1991) to this representation rests on the concept of joint receipt of two gambles. A basic axiom underlying the RDU parts, called segregation, asserts that a person is indifferent between a gamble of gains and the joint receipt of the smallest gain and a gamble formed by reducing each consequence by that smallest gain. This assumption has a strongly normative flavor. A similar statement holds for losses. A basic assumption underlying the weighted sums of utilities of gains and losses, called duplex decomposition, is that a gamble having a gain, a loss, and the status quo as consequences is indifferent to the joint receipt of two independent gambles,

one in which the loss is replaced by the status quo and the other in which the gain is replaced by the status quo. Duplex decomposition does not have a normative basis because the two independent gambles have a different distribution of consequences than does the original gamble. The third assumption is that the utility of the joint receipt is the sum of the utilities of the two components.

In the present experiment we studied each of these three hypotheses. The technique was to establish CEs for each side of an asserted indifference and then to test the null hypothesis that the two values are equal. Both median and mean tests were examined, but only the former were reported because the individual data are quite irregular and asymmetric. CEs were estimated in two ways: Simple judgments and choice-based ones using the PEST technique. In general, the median-choice data provided somewhat greater support for the hypotheses than did the median-judgment data. This is indirect evidence that the two types of CEs are not the same. This fact was demonstrated directly by Bostic et al. (1990), indirectly by Tversky et al. (1990), and parametrically in Mellers, Chang, Birnbaum, and Ordóñez (1992). The present data are not suited to studying the nature of the difference.

The segregation assumption was not rejected for two thirds of the median-judged CE data, whereas the choice



**Figure 9.** Panel a: The median choice certainty equivalents (CEs) as a function of the probability  $p$  of the various gambles used in testing for duplex decomposition and its additivity of joint receipt for a gain of \$96 and a loss of \$40. Panel b: The median choice CEs as a function of the probability  $p$  of the various gambles used in testing for duplex decomposition and its additivity of joint receipt for a gain of \$96 and a loss of \$160. DCMP = decomposition; JRD = joint receipt decomposition; BG = binary gamble; SG = simple gamble.

Table 3

*Number of Subjects Whose Choice CEs for One Type of Gamble Were Higher Than, Equal to, or Lower Than Those for the Other Type of Gamble and Chi-Square Values*

Test and gamble pair	Number of subjects			$\chi^2$
Segregation test				
	BG>JRS	BG=JRS	BG<JRS	
(166,.2;70) vs. (96,.2;0) $\oplus$ (70)	29	3	40	2.0
(166,.5;70) vs. (96,.5;0) $\oplus$ (70)	31	5	36	0.5
(166,.9;70) vs. (96,.9;0) $\oplus$ (70)	43	3	26	4.5*
Total	103	11	102	0.0
(-166,.2;-70) vs. (-96,.2;0) $\oplus$ (-70)	38	2	32	0.5
(-166,.5;-70) vs. (-96,.5;0) $\oplus$ (-70)	32	1	39	0.9
(-166,.9;-70) vs. (-96,.9;0) $\oplus$ (-70)	34	1	37	0.2
Total	104	4	108	0.1
Duplex decomposition test				
	BG>JRD	BG=JRD	BG<JRD	
(96,.2;-40) vs. (96,.2;0) $\oplus$ (0,.2;-40)	16	1	55	22.0**
(96,.5;-40) vs. (96,.5;0) $\oplus$ (0,.5;-40)	29	3	40	2.0
(96,.9;-40) vs. (96,.9;0) $\oplus$ (0,.9;-40)	38	7	27	2.0
Total	83	11	122	7.4**
(96,.2;-160) vs. (96,.2;0) $\oplus$ (0,.2;-160)	32	0	40	0.9
(96,.5;-160) vs. (96,.5;0) $\oplus$ (0,.5;-160)	35	1	36	0.1
(96,.9;-160) vs. (96,.9;0) $\oplus$ (0,.9;-160)	37	1	34	0.2
Total	104	2	110	0.2
Additivity of joint receipt test in segregation context				
	JRS>SG	JRS=SG	JRS<SG	
(96,.2;0) $\oplus$ (70) vs. (96,.2;0) + (70)	36	1	35	0.1
(96,.5;0) $\oplus$ (70) vs. (96,.5;0) + (70)	30	3	39	1.4
(96,.9;0) $\oplus$ (70) vs. (96,.9;0) + (70)	22	1	49	10.9**
Total	88	5	123	6.0*
(-96,.2;0) $\oplus$ (-70) vs. (-96,.2;0) + (-70)	33	2	37	0.2
(-96,.5;0) $\oplus$ (-70) vs. (-96,.5;0) + (-70)	46	2	24	6.7**
(-96,.9;0) $\oplus$ (-70) vs. (-96,.9;0) + (-70)	50	3	19	14.2**
Total	129	7	80	11.6**
Additivity of joint receipt test in duplex decomposition test				
	JRD>SG	JRD=SG	JRD<SG	
(96,.2;0) $\oplus$ (0,.2;-40) vs. (96,.2;0) + (0,.2;-40)	26	1	45	5.6*
(96,.5;0) $\oplus$ (0,.5;-40) vs. (96,.5;0) + (0,.5;-40)	29	1	42	2.7
(96,.9;0) $\oplus$ (0,.9;-40) vs. (96,.9;0) + (0,.9;-40)	23	3	46	8.0**
Total	78	5	133	14.5**
(96,.2;0) $\oplus$ (0,.2;-160) vs. (96,.2;0) + (0,.2;-160)	34	0	38	0.2
(96,.5;0) $\oplus$ (0,.5;-160) vs. (96,.5;0) + (0,.5;-160)	21	1	50	12.5**
(96,.9;0) $\oplus$ (0,.9;-160) vs. (96,.9;0) + (0,.9;-160)	28	1	43	3.6
Total	83	2	131	10.7**

Note. CEs = certainty equivalents; BG = binary gamble; JRS = joint receipt segregation; JRD = joint receipt decomposition; SG = simple gamble. In this table, BG means CE(BG), JRS means CE(JRS), JRD means CE(JRD), SG in gains means CE(SG) + \$70, and SG in losses means CE(SG) - \$70. For all types of tests,  $n = 72$ .

\*  $P < .05$ . \*\*  $P < .01$ .

data were more supportive. The violations of the judged segregation assumption may be attributed in part to misinterpretations of the instructions: Roughly one quarter of the subjects seem not to have understood that when judging the joint receipt of a gamble and a sum of money, they were to respond to the entire complex, not just the gamble. This problem was discovered before the choice data were collected and so the procedure was modified in two ways. The same size of display was used for both the gamble and the sum of money and an error message (described in the Design and Procedure section) appeared whenever a re-

sponse was unrealistic. There was no evidence of such a mistake in the choice data, suggesting that the difference between the choice and judged data may, in this experiment, be an artifact.

In summary, we conclude that these data are consistent with the hypothesis that subjects satisfy segregation.

The duplex decomposition received support from the median data in both the judged and choice procedures. This result has some theoretical implications. First, it provides additional evidence for the importance of the status quo in evaluating binary gambles. Second, it provides a descriptive

foundation for the nonrational combination of gains and losses that is embodied in RSDU.

The median data for both judged and choice CEs of the segregation stimuli fail to provide support for the additivity of CEs over joint receipt, although those for duplex decomposition do. These data have several implications. The weighted sum of gains and losses embodied in RSDU is supported; however, the details of the RDU representation, which is based on additive utility plus segregation, are apparently wrong. There also are possible implications for Thaler's (1985) hedonic rule (Equation 12) for utility of joint receipt.

The plots of the data for segregation stimuli suggest that to the extent that additivity of CEs over joint receipt fails, it is subadditive for gains and superadditive for losses. Working with pure sums of money, Thaler (1985) and Thaler and Johnson (1990) argued for the hedonic rule, but it was less clear what held for gambles. To that end, Luce (in press) axiomatized the structure of joint receipt and certainty equivalents and suggested generalizing Equation 12 to one in which the gambles are replaced by their CEs, that is,

$$\begin{aligned} U(g \oplus h) \\ = \max \{U[CE(g) + CE(h)], U[CE(g)] + U[CE(h)]\}. \quad (18) \end{aligned}$$

A natural question, then, is: What properties of CE are necessary and sufficient for Equation 12 to imply Equation 18? One of these conditions is the additivity of CE over joint receipt, and so the data make us suspicious about the validity of this rule. Detailed empirical tests of these conditions are now in progress.

As was noted, we concentrated on medians rather than means because of irregularity and asymmetry in the individual data. If these irregularities were not caused by an experimental artifact such as some subjects' misunderstanding the task, it would be desirable to design the experiment to compare the conditions without being affected by the artifact. Brothers (1990) and von Winterfeldt, Chung, Luce, and Cho (1993) measured a test-retest reliability of the judged CEs of the gamble and found substantial unreliability. Slovic and Lichtenstein (1968) suggested an alternate approach in which each binary gamble is presented twice and the absolute difference of the two CE(BG)s is used to compare the significant difference either between CE(BG) and CE(JRD) or between CE(JRD) and the sum of two CE(SG)s. The difficulty in doing this is the amount of time it takes. To follow this route, one will probably have to hire subjects for a number of experimental sessions.

Our results provide some additional support for the descriptive accuracy of RSDU beyond that already adduced for prospect theory and by the global fit carried out by Tversky and Kahneman (1992). More interestingly, it also provides some support for the idea of formulating RSDU, at least the sign aspect, using the concept of joint receipt.

To the degree that RSDU has descriptive support, it becomes a potential candidate in bridging the gap between a purely normative theory, such as SEU, and prescriptive practice, in which gains and losses are taken seriously (Bell,

Raiffa, & Tversky, 1988; von Winterfeldt & Edwards, 1986). As Luce and von Winterfeldt (1994) argue, to the extent that RSDU is descriptive, then eliminating rank dependence by making the weights additive for gains and losses separately, as with probabilities, but retaining sign dependence and duplex decomposition, is a possible prescriptive compromise between the normative and descriptive.

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### New Editor Appointed

The Publications and Communications Board of the American Psychological Association announces the appointment of a new editor for the *Journal of Experimental Psychology: General* for a 6-year term beginning in 1996. As of January 1, 1995, manuscripts should be directed to Nora S. Newcombe, PhD, Department of Psychology, Temple University, 565 Weiss Hall, Philadelphia, PA 19122.

Manuscript submission patterns make the precise date of completion of the 1995 volume uncertain. The current editor, Earl Hunt, PhD, will receive and consider manuscripts until December 31, 1994. Should the 1995 volume be completed before that date, manuscripts will be redirected to the new editor for consideration in the 1996 volume.