Deriving meaningful scientific laws from abstract, "gedanken" type, axioms: three examples

Jean-Claude Falmagne

University of California, Irvine

Abstract

A scientific law can be a faithful representation of the physical world only if its form is invariant with respect to changes in the units. This is referred to as the 'meaningfulness condition.' This condition is powerful. Under this condition, some abstract constraint, possibly verifiable by a thought experiment, may considerably restrict the mathematical form of a law. We give three examples of such abstract constraints in this paper. The three examples are:

1. The *translation equation:* $F(F(x,y),z) = F(x,y+z)$;
2. The *associativity equation:* $F(F(x,y),z) = F(x,F(y,z))$;
3. The *quasi-permutability equation:* $F(G(x,y),z) = F(G(x,z),y))$.

In each case, only one or a couple of mathematical representations are possible. Our examples include Beer's Law, the formula for the volume of a cylinder, , and the concatenation of weights. In passing, we obtain two new proofs of the Pythagorean Theorem. We also formulate an open problem in the same vein.