Difference-Form Persuasion Contests

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ABSTRACT: We explore the equilibrium properties of "difference-form" contest functions derived in Skaperdas and Vaidya (2012) for cases - such as in litigation, lobbying, or political campaigning - in which contestants use resources to persuade an audience. We find that these difference-form contests can also support interior pure strategy Nash equilibria in which both parties contribute positive resources in equilibrium, in addition to corner pure strategy equilibria typically associated with difference-form contests as identified by Baik (1998) and Che and Gale (2000). Further, the reaction functions of the two contestants have very different characteristics than those identified in Baik (1998). In one case, we find that the reaction function of each player is non-responsive to the level of resources devoted by the rival so that the equilibrium is invariant to the sequencing of moves and unlike Che and Gale (2000) there is no "preemption effect" on rent dissipation. In the other case, the reaction functions of the stronger and weaker players respond very differently to the expenditures of the rival with the weaker player having a greater marginal incentive to invest resources into the contest relative to the stronger player. In all cases the extent of rent dissipation is partial.

Keywords: Rent-seeking, lobbying, litigation, contest functions

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1. Introduction

A large variety of economic activities can be thought to be about persuasion where competing parties attempt to influence the opinions and hence the decisions of their relevant audiences through costly production of ‘information’ or evidence.[1] These include among many others, advertising (Schmalensee 1972), electoral campaigning (Snyder 1989; Baron 1994; Skaperdas and Groffman 1995), marketing (Bell et al. 1975), litigation (Farmer and Pecorino 1999; Bernardo et al. 2000; Hirshleifer and Osborne 2001; Robson and Skaperdas, 2008) and rent-seeking or lobbying (Tullock 1980). In each of these settings, contest functions have often been employed to translate the resources or costly efforts employed by the competing parties into probabilities of their view prevailing over the relevant audience. [2]

The most widely applied contest functions involve variations of the Tullock or ratio form in which a contestant’s winning probability depends on the ratio of his effort or resource devoted in the contest relative to that by all the competing contestants.[3] Another family of contest functions, called the “difference-form” has also been applied, albeit less so, where a contestant’s probability of winning depends upon the difference of efforts or resources expended. Hirshleifer (1989) was among the first to explore the equilibrium characteristics of a logistic difference-form contest and showed that they can be considerably different from a Tullock contest.[4] More general difference-form contests have been studied by Baik (1998) and Che and Gale (2000). Baik (1998) studies 2-player contests in which the win probability of player 1 takes the form:[5]

$$p_1(R_1,R_2) = f(d) \text{ where } d = \sigma R_1 - R_2, \sigma > 0,\ f' > 0, \quad f'' < 0 \text{ for } d > 0 \text{ and } f(-d) = 1 - f(d) \tag{1}$$

Che and Gale (2000) examine 2-player contests involving the piece-wise linear difference-form contest function as given by:[6]

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[1] For a recent survey on the quantitative impact of such persuasion activities in voting, marketing and financial markets, see DellaVigna and Gentzkow (2010).

[2] For a recent review on the theoretical foundations of contest functions see Jia et al. (2013).

[3] Tullock (1980) originally applied this functional form in the context of rent-seeking.

[4] For applications of such logistic form contest to rent-seeking, see Munster and Staal (2011, 2012).

[5] The win probability of player 2 is always $1 - p_1(R_1, R_2)$. Hirshleifer (1989) can be understood as a special case of (1).

[6] Polishchuk and Tonis (2013) illustrate specific conditions under which a difference-form contest function of the type in (2) can arise as an optimal allocation mechanism for the prize allocator in a rent-seeking context where the contestants’ valuations of the prize are private information. Corchon and Dahm (2010, 2011) provide alternative positive and normative foundations for this contest function and also extend it to a 3-player setting. Their formulation allows for non-linearity of the type R_i^σ where $\sigma > 0$. Pelosse (2011) provides an interpretation of this type of contest function as equilibrium conjectures of players in a conflict setting involving correlated equilibria with endogenous group formation. Grossman and Helpman (1996) apply this function to determine the effect of campaign contributions on voting behavior of “uninformed” voters.

$$p_1(R_1, R_2) = \max\{\min\{\tfrac{1}{2} + s(R_1 - R_2),\ 1\}, 0\} \text{ where } s > 0 \qquad (2)$$

In all such applications until recently, the persuasion process by which resources expended by the contestants translate into win-probabilities has not been clarified. In the lobbying context, resources expended by competing sides to influence a decision maker are often considered venal – where they are interpreted as transfers or bribes.[7] However, such an interpretation does not encompass lobbying activities that can be naturally thought of as persuasion even when no bribes are exchanged. Skaperdas and Vaidya (2012) explicitly derive the contest functions as win probabilities in a game of persuasion where competing parties invest resources to produce evidence from which an audience updates its priors using Bayesian inference. They show that both the ratio-form and a difference-form contest function can be derived as an outcome of such a process. The equilibrium characteristics of the ratio-form contest are well known and have already been extensively examined in the contest literature.[8]

In contrast, the equilibrium characteristics of the difference-form contest function derived in Skaperdas and Vaidya (2012) are hitherto unexplored. As discussed in the next section, under symmetry, it takes the form:

$$p_1(R_1, R_2) = \frac{1}{2} + \alpha\frac{1}{2}\left[h(R_1) - h(R_2)\right] \qquad (3)$$

Under certain asymmetries between players, it takes the form:

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\left\{\left[(\tfrac{\Gamma-1}{\Gamma})h(R_1) - (\tfrac{1-\delta}{\delta})h(R_2)\right] + \left[\left(\tfrac{1-\delta}{\delta}\right) - \left(\tfrac{\Gamma-1}{\Gamma}\right)\right]h(R_1)h(R_2)\right\} \qquad (4)$$

(3) is different from the piece-wise linear difference-form contest examined by Che and Gale (2000) as specified in (2) due to the non-linearity induced by the evidence realization probability function $h(.)$ which by assumption is bounded between 0 and 1, strictly increasing and strictly concave in the resources expended by each party, $R_i, i = 1, 2$. (3) is also different from (1) as examined by Baik (1998). A major difference is in the arguments of the two functions. In contrast to (1), the argument of (3) involves differences in evidence realization probabilities of the two contestants which respond non-linearly to the level of resources due to the strict concavity of $h(.)$. The form in (4) is further apart from (1) and (2) due to the presence of a cross-product term $h(R_1)h(R_2)$ which alters the marginal incentive of the two parties to invest resources in favor of the weaker side as examined in detail in the subsequent sections.

Due to these differences, equilibrium characteristics of contests that involve (3) and (4) are different and therefore worth examining. In this paper we uncover these characteristics and compare them with those of (1) and (2). Further, since the functional forms in (3) and (4) are explicitly grounded in the persuasion context, their parameters have natural inferential interpretations in terms of biases and decision thresholds and this allows us to establish the impact such parameters may have on equilibrium level of

[7] See for example, Grossman and Helpman (1994).
[8] See Perez-Castrillo and Verdier (1992) and Nitzan (1994).

resources, win-probabilities and rent dissipation. As will become apparent from the subsequent analysis, we find that contests involving (3) or (4) can yield interior pure strategy Nash equilibria where both parties contribute positive resources in equilibrium unlike Che and Gale (2000) and Baik (1998). The reaction functions of the two contestants also have very different characteristics than those identified in Baik (1998). In the case of (3) we find that the reaction function of each player is independent of the level of resources devoted by the rival. This result continues to hold even under certain specific types of asymmetries. This implies that the Nash equilibria of the simultaneous move game are identical to those of the sequential game where one of the players chooses resources earlier than the other. Further, unlike Che and Gale (2000), there is no "preemption" effect so that an increase in the valuation of prize by the player who values it more does not reduce the extent of rent dissipation. In case of (4), the reaction functions of the stronger and weaker players are analytically different. While the stronger player reduces his resource investment if the rival expends more, the opposite is true for the weaker player. This is due to the weaker player having a greater incentive to invest resources at the margin. These results are established and discussed in detail in the subsequent sections.

The paper is organized as follows. Section 2 provides a brief introduction to the difference-form contest functions of the type (3) and (4) as derived in Skaperdas and Vaidya (2012) hereafter referred to as "persuasion functions". Section 3 examines pure-strategy Nash equilibria of the symmetric persuasion function as given by (3). Section 4 examines equilibrium behavior involving asymmetric versions of (3) as well as those involving (4). In all such cases, to isolate the effect of the specific asymmetry introduced, all other aspects of the game are left symmetric for the two contestants. Section 5 concludes.

2. An introduction to difference-form persuasion functions

In this section, we briefly review the difference-form persuasion function as derived by Skaperdas and Vaidya (2012) as an outcome of a stochastic evidence production process. In their setting, two players (denoted by subscript $i = 1, 2$), compete to gather and present evidence in order to influence the verdict of a third party audience in their favor. Each player *i* can either produce a discrete piece of evidence in her favor denoted by e_i, or offer no evidence, denoted by e_ϕ. The production of such evidence is stochastic so that the amount of resources devoted by player *i* as denoted by R_i enhances her probability of finding favorable evidence $h_i(R_i)$.[9] It is assumed that $0 < h_i(R_i) < 1$,

[9] In a variety of contexts involving persuasion such as marketing and political campaigns, competing parties invest large resources into learning about the target audience and then delivering messages that are likely to be the most effective. As an example, consider the Obama campaign's investment of resources into collecting and studying information about potential supporters as gleaned from Facebook, voter logs, and telephone and in-person conversations with an objective to deliver personalized messages that are most likely to be effective in mobilizing potential voters. The effort involved operating a centralized office staffing over 300 workers including marketing experts with a payroll of over $3 million. For details see "Obama Mines for Voters with High-Tech tools", New York Times, March 8, 2012. Alternatively, one could also view the "evidence" as winning an endorsement from an entity considered as credible by the

$h_i'(R_i) > 0$ and $h_i''(R_i) < 0$. Thus depending on evidence realization, there are four possible states of the world that can be observed by the third party audience: (e_1, e_2), (e_1, e_ϕ), (e_ϕ, e_2) and (e_ϕ, e_ϕ) occurring with the following probabilities:
$h_1(R_1)h_2(R_2)$, $h_1(R_1)[1-h_2(R_2)]$, $[1-h_1(R_1)]h_2(R_2)$, and $[1-h_1(R_1)][1-h_2(R_2)]$ respectively. Each of these alternative states of the world can induce the audience to revise its prior probability of player 1 being the "correct" side denoted as π ($0 < \pi < 1$) with a posterior $\pi^*(e_i, e_j)$ where $i = 1, \phi$ and $j = 2, \phi$. Skaperdas and Vaidya (2012) employ the following parameterization:[10]

$$\pi^*(e_\phi, e_\phi) = \pi^*(e_1, e_2) = \pi \;;$$

$$\pi^*(e_\phi, e_2) = \delta\pi \; \textit{for some}\, \delta \in (0,1)\;;$$

$$\pi^*(e_1, e_\phi) = \begin{cases} \Gamma\pi \; \textit{if}\; \Gamma \le 1/\pi \\ 1 \; \textit{if}\; \Gamma > 1/\pi \end{cases} \; \textit{where}\; \Gamma > 1$$

Assuming that (i) the audience employs a threshold rule and decides in favor of player 1 iff $\pi^*(e_i, e_j) > \gamma$ ($0 < \gamma < 1$) (ii) γ is common knowledge and (iii) the two players don't observe π but have a common uniform prior over it, it is shown that when $\delta > \gamma$ the win probability of player 1 takes the following difference-form:[11]

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\left\{\left[\left(\tfrac{\Gamma-1}{\Gamma}\right)h_1(R_1) - \left(\tfrac{1-\delta}{\delta}\right)h_2(R_2)\right] + \left[\left(\tfrac{1-\delta}{\delta}\right) - \left(\tfrac{\Gamma-1}{\Gamma}\right)\right]h_1(R_1)h_2(R_2)\right\} \qquad (5)$$

Player 2's win probability is naturally given by $p_2(R_1, R_2) = 1 - p_1(R_1, R_2)$. The parameters determining $p_1(R_1, R_2)$ have natural inferential interpretations. γ captures a potential bias in decision threshold level used by the audience. When $\gamma < \frac{1}{2}$, the bar for posterior probability is lowered in favor of player 1 as the audience is more easily convinced about her superiority and vice versa. $\left(\frac{\Gamma-1}{\Gamma}\right)$ represents the inferential power of player 1's evidence (e_1). Notice that intuitively, it increases in Γ (the factor by which the

decision maker. Hence when only one player gains an endorsement, they have an edge in convincing the decision maker. On could also view this process as shopping for the right delegates to represent their case where the quality of the delegate has a bearing on how the case is represented and therefore on how the decision maker rules.

[10] Notice that the posterior probability of the audience responds purely to the evidence in front of it and does not take into account the strategies of the competing parties in terms of the resources they put into the contest. This follows from the "limited-world" Bayesian assumption about the audience in Skaperdas and Vaidya (2012). There is some empirical literature on persuasion that provides evidence supporting such non-strategic behavior by specific audiences. DellaVigna and Gentzkow (2010) survey empirical evidence on the impact of financial advice by analysts on small investors. The financial analysts face strong incentives to bias their recommendations in favor of companies they are affiliated with. Despite this, Malmendier and Shanthikumar (2007) find that individual investors take such recommendations at face value and do not distinguish between affiliated and non-affiliated analysts. These findings are corroborated by De Franco et al. (2007) and an experimental study by Cain et al. (2005). Eyster and Rabin (2010) examine a model where the receivers of information adjust too little for sender's credibility.

[11] Details of this derivation can be found in Skaperdas and Vaidya (2012).

audience's prior is revised in favor of player 1). Similarly, $\left(\frac{1-\delta}{\delta}\right)$ represents the inferential power of player 2's evidence (e_2) and intuitively, it declines with δ. Notice that these parameters allow for various asymmetries so that $p_1(R_1,R_2) \neq p_2(R_1,R_2)$ when $R_1 = R_2$. The sources of such asymmetries could be a bias in the threshold ($\gamma \neq \frac{1}{2}$), differences in inferential power of evidence put forward as captured via $\left(\frac{\Gamma-1}{\Gamma}\right)$ and $\left(\frac{1-\delta}{\delta}\right)$ as well as differences in the evidence production functions as embodied in $h_i(R_i)$. The implications of these asymmetries for equilibrium behavior of the contestants are explored in detail in section 4.

When $\gamma = \frac{1}{2}$, $h_i(.) = h_j(.) = h(.)$ and $\left(\frac{\Gamma-1}{\Gamma}\right) = \left(\frac{1-\delta}{\delta}\right) = \alpha$, the asymmetries vanish and (5) reduces to the symmetric form in (3).

In the subsequent sections we examine the equilibrium characteristics of simultaneous move contests involving both the symmetric and the asymmetric versions of the difference-form persuasion functions as in (3) and (5). While examining the asymmetric cases, we consider the effect each type of asymmetry can have on the equilibrium characteristics.

3. Equilibrium behavior under symmetric difference-form persuasion function

In this section, we examine persuasion contests involving the symmetric difference-form persuasion function, where the win probability of player 1 is given by (3) and that of player 2 is simply $p_2(R_1,R_2) = 1 - p_1(R_1,R_2)$. We assume that $0 < \alpha < 1$ and the probability function $h(R_i)$ satisfies assumption 1:

Assumption 1: $h(R_i)$ is differentiable and strictly concave with $0 \leq h(R_i) < 1,\ h''(R_i) < 0, h'(R_i) > 0$ for every $R_i \geq 0$ and $h'(R_i) \to 0$ as $R_i \to \infty$.

Suppose that player 1's valuation of the prize is v_1 while that of player 2 is v_2 and without loss of generality, $v_1 \geq v_2$. Accordingly, the expected payoff of player *i*, is given by:

$$U^i(R_i,R_j) = \left\{\frac{1}{2} + \frac{\alpha}{2}\left[h(R_i) - h(R_j)\right]\right\} v_i - R_i \text{ for } i, j = 1, 2 \text{ and } i \neq j \tag{6}$$

where the cost per unit of resource expended is normalized to unity. Each player's decision problem involves maximizing her expected payoff U^i with respect to R_i taking R_j as given and under the constraint $R_i \geq 0$. Given the assumptions on $h(.)$, as long as $h'(0) > \dfrac{2}{\alpha v_i}$ the reaction functions of the two players are given by:

$$h'(R_i^*) = \frac{2}{\alpha v_i} \quad \text{for } i = 1, 2 \tag{7}$$

Accordingly, the optimal resource expenditure of each player is given by Proposition 1 below:[12]

Proposition 1: *Under a symmetric difference-form persuasion function as in (3):*

(i) *The reaction function of each player is independent of the resources devoted by her rival*

(ii) *If* $h'(0) > \frac{2}{\alpha v_i}, i = 1, 2$ *then the pure-strategy Nash equilibrium involves both players investing in positive but potentially different level of resources with* $R_i^* = (h')^{-1}\left(\frac{2}{\alpha v_i}\right)$

(iii) *If* $v_1 > v_2$ *and* $\frac{2}{\alpha v_2} \geq h'(0) > \frac{2}{\alpha v_1}$ *then the pure-strategy Nash equilibrium involves* $R_1^* = (h')^{-1}\left(\frac{2}{\alpha v_1}\right)$ *and* $R_2^* = 0$

(iv) *If* $h'(0) \leq \frac{2}{\alpha v_i}, i = 1, 2$ *then in the pure-strategy Nash equilibrium neither player invests any resources towards the contest*

(v) *When* $v_1 = v_2$, *the Nash equilibrium is always symmetric with either both players investing the same positive level of resources into the contest or both investing zero resources to it*

(vi) *There is partial dissipation of rents in any pure strategy Nash equilibrium*

From proposition 1, it follows that there exists a unique pure strategy Nash equilibrium where the resource expenditures by players depend on their valuations of the prize and the marginal productivity of resource expenditure. As long as each player's valuation of the prize is sufficiently high and the probability function $h(R_i)$ is responsive to resources expended, both players expend positive but potentially different level of resources in equilibrium. The agent who values the prize more expends more so that $R_1^* > R_2^*$ iff $v_1 > v_2$. This represents different equilibrium behavior than that in Che and Gale (2000) and Baik (1998) where any pure strategy Nash equilibrium always involves one of the players investing zero resources into the contest. When the equilibrium involves only one of parties investing positive resources to the contest, this player is always player 1 with a higher valuation of the prize. Proposition 1 thus implies that persuasion function (3) can support all feasible types of pure strategy Nash equilibrium

[12] See the appendix for a proof.

depending on the valuations of the prize and the sensitivity of $h(R_i)$ to resources. Further, since each player is assured of a positive payoff even if it were to not expend any resources towards the contest regardless of the rival player's choice, it follows that in any pure strategy Nash equilibrium, the extent of rent dissipation is partial.

Also notice that in contrast to Baik (1998), the reaction function of each player is independent of the rival's effort. This is because, unlike Baik (1998) where the win probability depends on a linear difference in resources, in our framework, persuasion function (3) depends on the difference between non-linear transformations of resources via the evidence probabilities. This leads to payoff functions that are additively separable in the resources resulting in strategic independence in the reaction functions. There are two interesting implications from this.

Firstly, notice that the rival player's valuation of the prize has no impact on a player's equilibrium choice of resource spending. This implies that player 2's choice of resources is not affected by the level of v_1 even if $v_1 > v_2$. Accordingly an increase in v_1 will not discourage or "intimidate" player 2 into putting lesser resources into the contest. As stated in part (ii) of the above proposition, R_i^* is non-decreasing in v_i and non-responsive to v_j where $i, j = 1, 2$ and $i \neq j$. Hence unlike Che and Gale (2000), there is no preemption effect as the level of rent dissipation is not decreasing in v_1.

Secondly, the order in which the players choose resources is irrelevant and hence the equilibria of the simultaneous move game are identical to that of a sequential move game where either player 1 or player 2 chooses resource level first. These findings are summarized in corollary 1.

Corollary 1: *Under a symmetric difference-form persuasion function as in (3):*

(i) There is no preemption effect as the level of rent dissipation $\sum_i R_i^*$ *is non-decreasing in* v_1.

(ii) The equilibria of a simultaneous game are identical to those of a sequential game where one of the two players chooses resource level first relative to the rival.

4. Equilibrium characteristics under asymmetric persuasion functions

As discussed briefly in section 2, various factors can give rise to asymmetry in the persuasion function described in (5). In this section we will explore the implication of each of those sources of asymmetry for equilibrium behavior of players. We begin with the case of asymmetry in the evidence production process.

4.1 Asymmetric evidence production and its impact on equilibrium spending

To isolate the effect of asymmetric evidence production in this subsection, the following assumption will apply:

<u>Assumption 2:</u> *Let* $\gamma = \frac{1}{2}$, $0 < \left(\frac{\Gamma - 1}{\Gamma}\right) = \left(\frac{1-\delta}{\delta}\right) = \alpha < 1$ *and* $v_1 = v_2 = v$. *However,* $h_1(R_1) \neq h_2(R_2)$ *when* $R_1 = R_2$.

Assumption 2 ensures that the only source of asymmetry that arises in the persuasion contest is via the difference in the evidence production probabilities. To explore potential implications for this asymmetry, we postulate the following truth-enabled production probabilities as stated in assumption 3:

<u>Assumption 3:</u> *Let* $h_1(R_1) = \theta\, h(R_1)$ *and* $h_2(R_2) = (1-\theta)\, h(R_2)$ *where* $h(.)$ *has the same characteristics as in the symmetric persuasion function case, and* $\theta \in (0,1)$.

Following Hirshleifer and Osborne (2001) and Robson and Skaperdas (2008), θ can be interpreted as the *degree of truth* or, in the case of litigation, as the *level of property rights protection*. For example, if the truth (or property rights) is with player 1, then $\theta \in (\frac{1}{2}, 1)$, so that when $R_1 = R_2 = R$, $h_1(R) > h_2(R)$. This implies that the side arguing for the truth (player 1 in this instance) will have a higher chance of getting favorable evidence when both players invest the same level of resources. The closer is θ to 1, the easier it is to argue for player 1 who is on the side of the truth (or, the better defined property rights are). Analogously, if the truth were with player 2, then $\theta \in (0, \frac{1}{2})$. For the sake of brevity, we will assume that $\theta \in (\frac{1}{2}, 1)$.

Given assumptions 2 and 3, the persuasion function in equation (5) reduces to:

$$p_1(R_1, R_2) = \frac{1}{2} + \frac{\alpha}{2}\left[\theta\, h(R_1) - (1-\theta)\, h(R_2)\right] \tag{8}$$

The expected payoffs to players 1 and 2 may then be written as:

$$U^1(R_1, R_2) = \left\{\frac{1}{2} + \frac{\alpha}{2}\left[\theta\, h(R_1) - (1-\theta)\, h(R_2)\right]\right\} v - R_1$$
$$U^2(R_1, R_2) = \left\{\frac{1}{2} + \frac{\alpha}{2}\left[(1-\theta) h(R_2) - \theta h(R_1)\right]\right\} v - R_2$$

Each player *i*'s decision problem is to maximize her expected payoff U^i taking R_j as given and under the constraint $R_i \geq 0$ for $i, j = 1, 2, i \neq j$. Given our assumptions regarding $h(.)$, the reaction functions of the two players are:

$$R_1^* = \begin{cases} (h')^{-1}\left(\dfrac{2}{\alpha\theta v}\right) \text{when } h'(0) > \dfrac{2}{\alpha\theta v} \\ 0 \text{ otherwise} \end{cases} \tag{9}$$

$$R_2^* = \begin{cases} (h')^{-1}\left(\dfrac{2}{\alpha(1-\theta)v}\right) \text{when } h'(0) > \dfrac{2}{\alpha(1-\theta)v} \\ 0 \text{ otherwise} \end{cases} \tag{10}$$

By inspection of (9) and (10), it is clear that the reaction functions of both players are analogous to the case of asymmetric prize valuations where $v_1 > v_2$ as examined in section 3. Hence it is clear that proposition 1 continues to apply qualitatively. Since $\theta \in (\frac{1}{2}, 1)$, we have $\dfrac{2}{\alpha\theta v} < \dfrac{2}{\alpha(1-\theta)v}$. This implies that when the equilibrium consists of only one of the two players actively engaging resources in the contest, it is player 1 who is the active player. Further, when the Nash equilibrium involves both players investing resources into the contest, it is always the case that $R_1^* > R_2^*$.[13] This implies that the side arguing for the truth puts in more resources in equilibrium and has a higher probability of winning. The closer is θ to 1, the greater is this effect.

This discussion suggests that when the only source of asymmetry is a tilt in the evidence production towards the player arguing for the truth, this natural advantage to that player gets reinforced through the equilibrium choice of resources. However, the previous section reminds us that when the valuations of the prize are asymmetric, this effect can be potentially dominated when the player who values the prize higher is not the one arguing for the truth.

4.2 Bias in the decision threshold and its impact on equilibrium behavior

To study the effect of a bias in the decision threshold, in this sub-section, we allow for $\gamma \neq \frac{1}{2}$ while suppressing other sources of asymmetry as stated in assumption 4.

<u>Assumption 4:</u> *Let* $\frac{1}{1+\alpha} \geq \gamma \neq \frac{1}{2}$, $0 < \left(\frac{\Gamma-1}{\Gamma}\right) = \left(\frac{1-\delta}{\delta}\right) = \alpha < 1$, $v_1 = v_2 = v$ *and* $h_1(.) = h_2(.) = h(.)$.[14]

With assumption 4, the persuasion function in (5) reduces to:

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\alpha\left[h(R_1) - h(R_2)\right] \tag{11}$$

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[13] This follows due to strict concavity of $h(.)$ and $\theta \in (\frac{1}{2}, 1)$.

[14] Since $0 < \alpha < 1$ it follows that $\frac{1}{2} < \frac{1}{1+\alpha} < 1$.

Since $0 < h(.) < 1$, as long as $\gamma \leq \frac{1}{1+\alpha}$, (11) is naturally bounded between 0 and 1 for any (R_1, R_2).

Notice that when $\gamma > \frac{1}{2}$ the decision threshold favors player 2 as $p_1(R_1, R_2) < p_2(R_1, R_2)$ when $R_1 = R_2$. The opposite is true when $\gamma < \frac{1}{2}$. To illustrate the implications of such threshold bias, we use (11) to construct each player's expected payoff in the game as shown below:

$$U^1(R_1, R_2) = \{(1-\gamma) + \alpha\gamma[h(R_1) - h(R_2)]\}v - R_1$$

$$U^2(R_1, R_2) = \{\gamma + \alpha\gamma[h(R_2) - h(R_1)]\}v - R_2$$

The maximization of the above expected payoffs simultaneously by the players, leads to the following reaction functions:

$$R_i^* = \begin{cases} (h')^{-1}\left(\dfrac{1}{\alpha\gamma v}\right) \text{when } h'(0) > \dfrac{1}{\alpha\gamma v} \\ 0 \text{ otherwise} \end{cases} \text{ for } i = 1, 2 \tag{12}$$

It is apparent by observing (12) that the equilibrium behavior of the players is qualitatively the same as in proposition 1 part (v). Hence unlike other sources of asymmetry examined so far, threshold bias does not invoke asymmetry in equilibrium spending as both players invest the same amount of resources in the contest. However, the equilibrium spending of each player increases with γ when they both actively participate in the contest. Hence the threshold bias alters the equilibrium intensity of the contest and thus the extent of rent dissipation.

4.3 Bias in the sensitivity to evidence and its impact on equilibrium behavior

We now allow for differences in receptivity of the audience towards evidence presented by the two contestants so that $\left(\frac{\Gamma-1}{\Gamma}\right) \neq \left(\frac{1-\delta}{\delta}\right)$ and examine the impact of such evidence bias on players' equilibrium behavior. As with previous cases, we suppress other sources of asymmetry as is stated in Assumption 5.

<u>**Assumption 5:**</u> *Let* $\alpha_1 = \left(\frac{\Gamma-1}{\Gamma}\right) \neq \left(\frac{1-\delta}{\delta}\right) = \alpha_2$, $\gamma = \frac{1}{2}$, $v_1 = v_2 = v$ *and* $h_1(.) = h_2(.) = h(.)$. *Further, without loss of generality,* $0 < \alpha_2 < \alpha_1 < 1$ *so that the evidence bias is in favor of player 1.*

When assumption 5 applies, the persuasion function in (5) becomes:

$$p_1(R_1, R_2) = \tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_1 h(R_1) - \alpha_2 h(R_2) - (\alpha_1 - \alpha_2)h(R_1)h(R_2)\right] \tag{13}$$

The win probability of player 2 is $p_2(R_1, R_2) = 1 - p_1(R_1, R_2)$. Notice that the evidence bias $(\alpha_1 - \alpha_2)$ works its way via a cross product term $h(R_1)h(R_2)$ with a negative co-efficient for player 1 which makes (13) distinct from other asymmetries examined so far. Lemma 1 identifies the basic characteristics of the persuasion function in (13):

Lemma 1:

(i) $\frac{\partial p_i}{\partial \alpha_i} = \frac{h(R_i)(1-h(R_j))}{2} > 0$ for $i, j = 1, 2, i \neq j$

(ii) $\frac{\partial p_i}{\partial \alpha_j} = -\frac{h(R_j)(1-h(R_i))}{2} < 0$ for $i, j = 1, 2, i \neq j$

(iii) $\frac{\partial p_1}{\partial R_1} = \frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]h'(R_1) > 0$ and $\frac{\partial p_2}{\partial R_2} = \frac{1}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)\right]h'(R_2) > 0$

(iv) $\frac{\partial p_1}{\partial R_2} = -\frac{1}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)\right]h'(R_2) < 0$ and $\frac{\partial p_2}{\partial R_1} = -\frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]h'(R_1) < 0$

(v) $\frac{\partial^2 p_1}{\partial R_1^2} = \frac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]h''(R_1) < 0$ and $\frac{\partial^2 p_2}{\partial R_2^2} = \frac{1}{2}\left[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)\right]h''(R_2) < 0$

(vi) $\frac{\partial^2 p_1}{\partial R_1 \partial R_2} = -\frac{1}{2}\left[(\alpha_1 - \alpha_2)h'(R_2)\right]h'(R_1) < 0$ and $\frac{\partial^2 p_2}{\partial R_1 \partial R_2} = \frac{1}{2}\left[(\alpha_1 - \alpha_2)h'(R_1)\right]h'(R_2) > 0$

(vii) $p_1(R_1, R_2) > p_2(R_1, R_2)$ when $R_1 = R_2$ and $\alpha_1 - \alpha_2 > 0$

Property (i) and (ii) imply that each player's win probability is increasing in the strength of its evidence and decreasing in that of its rival. Property (vii) implies that when both players put in the same level of resources the evidence bias (in favor of player 1) provides an advantage to player 1 in terms of a higher win probability. These properties suggest that despite the negative co-efficient in the cross-product term for player 1, the first order effects of the evidence bias are to enhance the win probability of player 1. However, the evidence bias does have the opposite effect on the marginal incentive to put in resources towards the contest for player 1. This can be first appreciated by observing the marginal impact of resources on the win probabilities in (iii). Notice that the evidence bias is reducing the marginal return to player 1's investment in the contest while it is adding to that of player 2. Further, notice that as per (vi), when $\alpha_1 - \alpha_2 > 0$, an increase in the rival's investment of resources towards the contest discourages player 1 while it encourages player 2 to increase its resources. These two properties imply that the evidence bias strengthens the weaker player's marginal incentive to put in resources relative to the stronger player. Intuitively, the weaker party is tempted to invest more at the margin in an attempt to compensate for the evidence bias against it. As we will observe subsequently, this property manifests itself also via differences in the shapes of the reaction functions for the two players. The sign of the partial derivatives described in properties (iii), (iv) and (v) follow straightforwardly from our assumption of monotonicity and strict concavity of $h(.)$.

Given the above characteristics of the persuasion function in (13), we now proceed to examine the equilibrium behavior of the two contestants. The expected payoffs to players 1 and 2 are as follows:

$$U^1(R_1, R_2) = \left\{\frac{1}{2} + \frac{1}{2}\left[\alpha_1 h(R_1) - \alpha_2 h(R_2) - (\alpha_1 - \alpha_2)h(R_1)h(R_2)\right]\right\}v - R_1$$

$$U^2(R_1, R_2) = \left\{\tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_2 h(R_2) - \alpha_1 h(R_1) + (\alpha_1 - \alpha_2)h(R_1)h(R_2)\right]\right\}v - R_2$$

Each player i aims to maximize U^i by her choice of $R_i \geq 0$, taking R_j as given where $i, j = 1, 2, i \neq j$. We first establish conditions under which we can expect a player to invest positive amount of resources into the contest. We begin with player 1. For player 1 to invest a positive amount of R_1 into the contest, it must be the case that the marginal benefit from investing exceeds the marginal cost at $R_1 = 0$, i.e.

$\tfrac{1}{2}\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]h'(R_1)v > 1$ at $R_1 = 0$ or,

$$h'(0) > \frac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)\right]v}$$

Notice that the level of R_2 affects the R.H.S. of the above inequality. An increase in R_2 increases $h(R_2)$ and therefore increases the R.H.S. via reducing the denominator. Given the properties of $h(.)$, we know that $h(.)$ attains its lowest value when $R_i = 0$ and we label it as $\underline{h} = h(0)$. Similarly, let $\overline{h} = \lim_{R \to \infty} h(R)$ where $0 < \overline{h} < 1$. Accordingly, from the above discussion, given the strict concavity of $h(.)$, we have lemma 2:

Lemma 2:

(i) If $h'(0) \leq \dfrac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)\underline{h}\right]v}$ *then it will never be optimal for player 1 to invest positively in the persuasion contest irrespective of the level of* R_2.

(ii) If $h'(0) > \dfrac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)\overline{h}\right]v}$, *then it will always be optimal for player 1 invest a positive level of* R_1 *regardless of the level of* R_2.

Following a similar logic, we arrive at Lemma 3 which pertains to player 2.

Lemma 3:

(i) If $h'(0) \leq \dfrac{2}{\left[\alpha_2 + (\alpha_1 - \alpha_2)\overline{h}\right]v}$ *then it will never be optimal for player 2 to invest positively in the persuasion contest irrespective of the level of* R_1.

(ii) If $h'(0) > \dfrac{2}{\left[\alpha_2 + (\alpha_1 - \alpha_2)\underline{h}\right]v}$, *then it will always be optimal for player 2 to invest a positive level of* R_2 *regardless of the level of* R_1.

Using Lemmas 2 and 3, we can identify a condition that rules out a corner solution for either contestant as given in Lemma 4:

Lemma 4: When $h'(0) > \text{Max}\left[\frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)\bar{h}]v}, \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)\underline{h}]v}\right]$ *both players will always invest positive level of resources into the contest regardless of the level invested by the rival.*

When Lemma 4 holds, we can ignore the possibility of a corner solution and accordingly, the reaction functions of the two players are given by the first order conditions as follows:

$$h'(R_1) = \frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]v} \tag{14}$$

$$h'(R_2) = \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]v} \tag{15}$$

Equation (14) represents the reaction function for player 1 while (15) represents that of player 2. Notice that the evidence bias enters negatively in the reaction function of player 1. This coupled with strict concavity of $h(.)$ implies that as R_2 increases, the optimal level of R_1 falls. Hence the reaction function of player 1 is negatively sloped. However, for player 2, the evidence bias enters positively in the denominator which implies that as R_1 increases, the optimal level of R_2 increases giving a positive slope to the reaction function of player 2. These reactions functions suggest that the evidence bias makes player 2 aggressive at the margin relative to player 1. The equilibrium behavior is summarized in Proposition 2:[15]

Proposition 2: *Under Assumption 5 and Lemma 4, when the evidence bias is in favor of player 1, the equilibrium behavior induced by the persuasion function (13) is as follows*:

(i) *player 1's optimal resource expenditure is strictly positive as given by her reaction function (14) and is inversely related to the resource expenditure of player 2*

(ii) *player 2's optimal resource expenditure is strictly positive as given by her reaction function (15) and is positively related to the resource expenditure of player 1*

(iii) *A unique interior pure strategy Nash equilibrium exists and determined by the crossing point of the reaction functions of the two players*

[15] Proofs of proposition 2 parts (i) and (ii) follow immediately from inspection of (14) and (15) via the preceding discussion. For proofs of parts (iii)-(v) please see the appendix.

(iv) *In principle, we can have 3 alternative kinds of pure-strategy Nash equilibrium: (1) symmetric equilibrium with* $R_1^* = R_2^*$ *(2)* $R_1^* > R_2^*$ *so that the player favored by the evidence bias invests greater resources in equilibrium (3)* $R_1^* < R_2^*$ *so that the weaker player invests greater resources to counterbalance the evidence bias in equilibrium*

(v) *There is partial dissipation of rents in any Nash equilibrium*

The conditions under which the symmetric equilibrium $R_1^* = R_2^* = R^*$ holds are given by:

$$h(R^*) = \tfrac{1}{2} \tag{16}$$

$$h'(R^*) = \frac{4}{[(\alpha_1 + \alpha_2)]v} \tag{17}$$

It is apparent that the level of R^* implied by (17) depends on the values of α_1, α_2 and v and is very unlikely to be consistent with (16). Hence, in general, the symmetric equilibrium is unlikely and easily disturbed by small changes in either α_1, α_2 or v. Therefore in most instances, the Nash equilibrium will be asymmetric with either $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

When the Nash equilibrium is such that $R_1^* > R_2^*$, it immediately follows that $p_1(R_1^*, R_2^*) > p_2(R_1^*, R_2^*)$ (given Lemma 1, (vii)). Hence along such equilibrium, the player favored by the evidence bias also puts in more resources into the contest and therefore has a higher equilibrium probability of winning. In this instance, the equilibrium choices of resource expenditures by both players reinforce the advantage conferred to player 1 through the evidence bias. The necessary conditions for such equilibrium are:

$$1 > h(R_1^*) + h(R_2^*) \tag{18}$$

$$h(R_1^*) > h(R_2^*) \tag{19}$$

When $R_1^* < R_2^*$, the weaker player puts in greater effort in the equilibrium and at least partially offsets the disadvantage of the evidence bias favoring the rival. In this case, it is theoretically possible that $p_1(R_1^*, R_2^*) < p_2(R_1^*, R_2^*)$. If this were to happen, it represents a case where the advantage conferred to player 1 through evidence bias is overwhelmed by the greater marginal incentive to put in resources on player 2's part. It is useful to recall that while the evidence bias lowers the win probability of the weaker player for given levels of resources, it also provides a higher marginal incentive to her to compete in the contest. When $p_1(R_1^*, R_2^*) < p_2(R_1^*, R_2^*)$, the latter effect dominates. The necessary conditions for an equilibrium with $R_1^* < R_2^*$ are:

$$1 < h(R_1^*) + h(R_2^*) \tag{20}$$

$$h(R_1^*) < h(R_2^*) \tag{21}$$

By examining conditions (18) – (21), it is apparent that the nature of the $h(.)$ function will determine which one of the two types of asymmetric equilibria will eventuate. Further, since each player's expected payoff is strictly positive even if she does not put any resources into the contest, it follows that in any Nash equilibrium, it will always be the case that $v > R_1^* + R_2^*$ so that the rent dissipation will be partial.

The asymmetric equilibrium of the type $R_1^* < R_2^*$ is particularly intriguing as the disadvantaged player is associated with higher resource expenditure. To illustrate the plausibility of such equilibrium, we consider a specific functional form for $h(.)$ that satisfies assumption 1:

$$h(R_i) = \frac{R_i + \psi}{R_i + 1},\ 0 < \psi < 1 \tag{22}$$

By inspecting (22) it is apparent that the lowest value of $h(.)$ is $\underline{h} = h(0) = \psi$. Similarly, the highest value is $\overline{h} = \lim_{R\to\infty} h(R) = 1$. From these values, it is clear that for Lemma 4 to be satisfied it must be the case that:

$$h'(0) = 1 - \psi > \text{Max}\left[\frac{2}{\alpha_2 v}, \frac{2}{(\alpha_2 + (\alpha_1 - \alpha_2)\psi)v}\right]$$

Since $\alpha_1 > \alpha_2$, and $\psi > 0$, $\text{Max}\left[\frac{2}{\alpha_2 v}, \frac{2}{(\alpha_2 + (\alpha_1 - \alpha_2)\psi)v}\right] = \frac{2}{\alpha_2 v}$. Further since $0 < \psi < 1$, it follows that for Lemma 4 to be satisfied, the following two conditions must hold:

$$\frac{2}{\alpha_2 v} < 1 \tag{23}$$

$$\psi < 1 - \frac{2}{\alpha_2 v} \tag{24}$$

By inspection of conditions (23) and (24), it is clear that Lemma 4 will be satisfied and therefore an interior Nash equilibrium will exist when v is sufficiently large and ψ is not too high. Notice that the level of ψ directly affects the marginal incentive to expend resources in the contest. When $\psi \approx 1, h(R_i) \approx 1$ regardless of the level of R_i and hence there is little incentive to spend costly resources into the contest. Similarly, the higher is v, the greater is the incentive to invest in the contest.

Let us assume that v and ψ are such that both (23) and (24) are satisfied so that Lemma 4 holds. In this case, the Nash equilibrium is given by the simultaneous solution of the two reaction functions (14) and (15) taking into account (22) as given below:

$$\frac{1-\psi}{(R_1+1)^2} = \frac{2}{\left[\alpha_1 - (\alpha_1-\alpha_2)\frac{R_2+\psi}{R_2+1}\right]v} \tag{25}$$

$$\frac{1-\psi}{(R_2+1)^2} = \frac{2}{\left[\alpha_2 + (\alpha_1-\alpha_2)\frac{R_1+\psi}{R_1+1}\right]v} \tag{26}$$

By examining conditions (25) and (26), we arrive at proposition 3 below.

Proposition 3: *When (22), (23) and (24) are satisfied and $\psi > \frac{1}{2}$, the Nash equilibrium is strictly interior with $R_1^* < R_2^*$.*

Proposition 3 suggests that the counter-intuitive equilibrium $R_1^* < R_2^*$ is particularly likely when marginal incentives to expend resources are relatively weak for both contestants as ψ is moderately high with $\frac{1}{2} < \psi < 1 - \frac{2}{\alpha_2 v}$.[16]

5. Conclusion

We have examined generalized difference-form contests that are best thought of as "persuasion functions;" that is, as applying to instances, such as litigation, lobbying, or political campaigning, in which different parties expend resources in order to persuade an audience. Contrary to specific cases of difference-form contests, we have found that pure-strategy interior equilibria often exist; that the "preemption" property is not a general property of difference-form contests; and that in the case of asymmetric forms some counterintuitive outcomes are possible regarding the effects of resources on the probability of winning.

Contrary to contests that have been extensively studied under the Tullock form, with the exception of Besley and Persson (2009) we are unaware of any applied studies which have used a difference-form contest function. The presumed absence or difficulty in finding interior pure-strategy equilibria is perhaps the reason for this absence that, given our results, no longer holds. Since at least in the form in (3) it is easy to derive equilibria, it would be worthwhile to re-examine many applied settings that involve persuasion and determine whether they shed a difference light than those that come from existing studies.

[16] Proof of Proposition 3 is provided in the Appendix.

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Appendix

<u>Proof of Proposition 1:</u>

Let us examine the payoff function of player *i* in section 3 which is reproduced below:

$$U^i(R_i, R_j) = \left\{\frac{1}{2} + \frac{\alpha}{2}\left[h(R_i) - h(R_j)\right]\right\} v_i - R_i \text{ for } i, j = 1, 2 \text{ and } i \neq j \qquad \text{(A.1)}$$

(A.1) can be re-arranged as:

$$U^i = \frac{1}{2} v_i \{1 - \alpha h(R_j)\} + \frac{\alpha}{2} v_i\, h(R_i) - R_i \qquad \text{(A.2)}$$

Notice that the above payoff is additively separable in R_i and R_j. From this it follows that the optimal choice of R_i by player *i* will be independent of R_j. This proves part (i).

Given the assumption of strict concavity of $h(.)$ with $h'(R) \to 0$ as $R \to \infty$, if $h'(0) > \frac{2}{\alpha v_i}$ then $R_i^* > 0$ and it is given by the first order condition $h'(R_i^*) = \frac{2}{\alpha v_i}$ so that $R_i^* = (h')^{-1}\left(\frac{2}{\alpha v_i}\right)$. Otherwise, for $h'(0) \leq \frac{2}{\alpha v_i}$, $R_i^* = 0$. Parts (ii) – (v) follow straightforwardly from this result.

By inspecting (A.2), it is clear that $U^i > 0$ when $R_i = 0$ regardless of the level of R_j since $0 < \alpha < 1$, and $0 \leq h(.) < 1$. Hence if $R_i^* > 0$, it must also be the case that $U^i > 0$.

Hence in any pure strategy equilibrium, it must be the case that:

$$U^{*1} = \left\{\frac{1}{2} + \frac{\alpha}{2}[h(R_1^*) - h(R_2^*)]\right\} v_1 - R_1^* > 0$$

$$U^{*2} = \left\{\frac{1}{2} + \frac{\alpha}{2}[h(R_2^*) - h(R_1^*)]\right\} v_2 - R_2^* > 0$$

Since by assumption, $v_1 \geq v_2$, it follows from the above two conditions that $R_1^* + R_2^* < v_1$ implying partial dissipation of rents. This proves part (vi).

Q.E.D.

Proof of Proposition 2 parts (iii) ,(iv) and (v):

Given the strict concavity of $h(.)$, when Lemma 4 holds, a corner solution is ruled out and both players always invest strictly positive level of resources as determined by the joint solution of the reaction functions given by (14) and (15) which are reproduced below.

$$h'(R_1) = \frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]v} \qquad \text{(A.3)}$$

$$h'(R_2) = \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]v} \qquad \text{(A.4)}$$

Since the reaction function of player 1 as given by (A.3) is continuous and strictly decreasing in R_2 while that defined by (A.4) for player 2 is continuous and strictly increasing in R_1, the existence of a crossing point representing the unique pure strategy Nash equilibrium is assured. Naturally, such pure strategy equilibrium will be either involve $R_1^* = R_2^* = R^*$ or $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

Along a symmetric equilibrium, equations (A.3) and (A.4) must hold simultaneously at $R_1^* = R_2^* = R^*$. This implies,

$$\frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R^*)]v} = \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R^*)]v}$$

The above equality implies that:

$$h(R^*) = \tfrac{1}{2} \qquad \text{(A.5)}$$

Further each of the equations (A.3) and (A.4) must also be satisfied along with (A.5) so that:

$$h'(R^*) = \frac{4}{[(\alpha_1 + \alpha_2)]v} \qquad \text{(A.6)}$$

When (A.5) and (A.6) are not simultaneously satisfied, the Nash equilibrium must be asymmetric with either $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

When $R_1^* > R_2^*$, strict concavity of $h(.)$, implies that $h'(R_1^*) < h'(R_2^*)$. Hence from (A.3) and (A.4), it must be the case that:

$$\frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2^*)]v} < \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1^*)]v} \quad \text{or}$$

$$1 > h(R_1^*) + h(R_2^*) \qquad \text{(A.7)}$$

Further, given the monotonicity of $h(.)$, it must also be the case that:

$$h(R_1^*) > h(R_2^*) \tag{A.8}$$

Hence along such a Nash equilibrium, both (A.7) and (A.8) must be satisfied.

Analogously, when $R_1^* < R_2^*$, the following conditions must hold:

$$1 < h(R_1^*) + h(R_2^*) \tag{A.9}$$

$$h(R_1^*) < h(R_2^*) \tag{A.10}$$

This proves parts (iii) and (iv).

To establish part (v), notice that player 1's win probability as given by (13) can be re-arranged as: $p_1(R_1, R_2) = \frac{1}{2} + \frac{1}{2}\left[\alpha_1 h(R_1)(1 - h(R_2)) - \alpha_2 h(R_2)(1 - h(R_1))\right]$. Notice that since $0 < \alpha_i < 1$ for $i = 1, 2$ and $0 \le h(.) < 1$, it follows that $\left|\alpha_1 h(R_1)(1 - h(R_2)) - \alpha_2 h(R_2)(1 - h(R_1))\right| < 1$ so that $0 < p_1(R_1, R_2) < 1$ for any level of resources invested by either player. Exactly the same holds for $p_2(R_1, R_2)$. From this it follows that each player is assured of a positive expected payoff from the contest even if she does not invest any resources to it. That is:

$U^1(0, R_2) > 0$

$U^2(R_1, 0) > 0$

Thus along any Nash equilibrium,

$$U^1(R_1^*, R_2^*) = \left\{\tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_1 h(R_1^*) - \alpha_2 h(R_2^*) - (\alpha_1 - \alpha_2) h(R_1^*) h(R_2^*)\right]\right\} v - R_1^* > 0$$

$$U^2(R_1^*, R_2^*) = \left\{\tfrac{1}{2} + \tfrac{1}{2}\left[\alpha_2 h(R_2^*) - \alpha_1 h(R_1^*) + (\alpha_1 - \alpha_2) h(R_1^*) h(R_2^*)\right]\right\} v - R_2^* > 0$$

From the above two inequalities, it follows that $v > R_1^* + R_2^*$ so that there is partial dissipation of rents.

Q.E.D.

<u>Proof of Proposition 3:</u>

Since by assumption Lemma 4 holds, the Nash equilibrium is given by the simultaneous solution of the two reaction functions (25) and (26) as reproduced below:

$$\frac{1 - \psi}{(R_1^* + 1)^2} = \frac{2}{\left[\alpha_1 - (\alpha_1 - \alpha_2)\frac{R_2^* + \psi}{R_2^* + 1}\right] v} \tag{A.11}$$

$$\frac{1 - \psi}{(R_2^* + 1)^2} = \frac{2}{\left[\alpha_2 + (\alpha_1 - \alpha_2)\frac{R_1^* + \psi}{R_1^* + 1}\right] v} \tag{A.12}$$

Now (A.11) and (A.12) can be re-arranged as:

$$\frac{(R_1^* + 1)^2}{(R_2^* + 1)^2} = \frac{\alpha_1 - (\alpha_1 - \alpha_2)\frac{R_2^* + \psi}{R_2^* + 1}}{\alpha_2 + (\alpha_1 - \alpha_2)\frac{R_1^* + \psi}{R_1^* + 1}} \tag{A.13}$$

Notice that the highest value of the numerator on the R.H.S. of (A.13) is $\alpha_1 - (\alpha_1 - \alpha_2)\psi$. The lowest value of the denominator on the R.H.S. of (A.13) is $\alpha_2 + (\alpha_1 - \alpha_2)\psi$. Hence when $\alpha_2 + (\alpha_1 - \alpha_2)\psi > \alpha_1 - (\alpha_1 - \alpha_2)\psi$, i.e. when $\psi > \frac{1}{2}$, R.H.S. of (A.13) is strictly less than 1. Since $R_i^* \geq 0$, it follows that $R_1^* < R_2^*$.

Q.E.D.