“Oscillatory dynamics of city-size distributions
in world historical systems” [1]

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Abstract. Oscillatory patterns of expansion/contraction have long characterized the dynamics of demographic, economic, and political processes of human societies, including those of exchange economies and globalization. Major perturbations in city-size distributions are shown to exist for major regions in Eurasia in the last millennium and to exhibit some of the characteristics of cyclical oscillations on the scale of 100s of years as well as longer fluctuations, from 400 up to 800 years, between periods of major collapse, often punctuated by lesser collapse. Variations in timing, irregularities in amplitudes, and ups and downs in our measures appear to correlate with some of the peaks and troughs in urban population growth and show long-cycle correlations with J.S. Lee's (1931) sociopolitical instability (SPI) data on the durations of internecine wars for China.

We focus here on central civilization within the world cities database, including China and Europe, and the Mid-Asian region between. These data are likely to reflect changes in the macro regions connected by trade networks, where we would expect synchronization. Our interpretation of city-size distribution oscillations is that they follow, with generational time lags, rises and falls in the expansion/contraction of multi-connected trade network macro zones, with Zipfian city-size hierarchies tending to rise with trade network expansions and fall with contractions. City system rise and fall also tend to couple with oscillations of population relative to resources interacting with SPI in total cycles that average about 220 years. Time-lagged synchronies in the dating of phases for city distributions in different regions that are connected by multiple routes of trade, as noted tentatively by Chase-Dunn and Manning (2002:21), at least in the rising and more Zipfian phase, support the existence of city-system rise and fall cycling. We find evidence that rise and fall in Silk Road connectivities between China and Europe had time lagged effects on the growth of power law tails in European urban hierarchies; that changes in Mid-Asia city distributions led weakly those in China while those in China led strongly those in Europe, at different time lags.

Maximal likelihood of two different measures of city size distributions proved to be of central importance to this paper, as they provide unbiased estimates of statistical parameters and improve confidence limits significantly, the more so for those smaller sample sizes in city data available in many historical periods. Analyses of these estimates supports six major sets of hypotheses about urban system evolution and fail to contradict two more speculative hypotheses about evolutionary learning in global systems.

INTRODUCTION

Globalization, world-system, and historical dynamic theory offer complementary perspectives for the study of city systems as the politicoeconomic engine of interstate networks. Here we combine these perspectives to examine a dynamical perspective on systems of cities. Globalization theory applied to Eurasia in the last millennium (e.g., Modelski and Thompson 1996) focuses on centers of economic innovation and political power and their successive periods of rise and fall in dominance. Units of larger scale, as for example polities, are shown to operate at successively longer time scales in rise and fall than the economic innovation centers within those polities. World-system theory for similar regions and processes (e.g., Chase-Dunn and Hall 1997) differs in focusing as well on innovation at the peripheries of states and empires, that is, on the marcher or boundary polities that resist the encroachment of expanding empires. Marcher states that amalgamate to defeat the spread of empire often defeat polities formally organized on a much larger scale through superior cohesive or decentralized organization able to forge a superior combative skill or technology. World-systems theory often limits itself to the more prominent types of relations, such as trade in bulk goods and interstate conflict, that form distinct macroregional networks. The structural demographic approach to the political economics of agrarian empires (e.g., Turchin 2003, 2006) is capable of yielding a more dynamical historical account of how central polities rise and fall as their internal cohesion disintegrates with population growth into factional conflict and of how once dominant polities and economies contend with marcher states that coalesce into formidable opponents on their frontiers.

Several of the problems in extending these kinds of complementary approaches to globalization, world-systems, and historical dynamics relate to how networks – social, political, and economic – fit into the processes of change and dynamical patterns that are observed historically. One aspect of major problems that might engage network research involves how changing network fluctuations of long distance trade influence inter- and intra-regional dynamics. The Silk Road trade so important in the connections through the marcher states and later empires of Mongol Central Asia between China and the Middle East, for example, also facilitated the diffusion of economic inventions from East to West that were crucial in the rise of the European city system. These included paper money, institutions of credit, and vast new knowledge, weaponry, and technologies. Spufford (2002), for example, shows how important were the transmissions of innovations from China from a European perspective, while Temple (1987) summarizes the work of Needham (1954-2004) to show the debt of the West to China. A related problem, among many other open network problems in historical research is how regional and long-distance trade networks are coupled, along with conflicts and wars, to the rise and fall of cities and city systems. That is the problem we take up here.

We approach the problems of the rise and fall of commercial trade networks, regional city systems, regional conflicts, and the historical dynamics of globalization and world-system interactions in Eurasia, during the last millennium, with a concern for valid comparative measurement of large scale phenomena. Tertius Chandler (1987) and other students of historical city sizes (Pasciuti 2006, Modelski 2003, Bairoch 1958, Braudel 1992, and others) have made it possible to compare the shapes of city-size distribution curves. These are the data we examine here in order to gauge and compare the dynamics of city system rise and fall, both within distinct regions and as changes in one region (such as China) affect changes in others (such as Europe, to give but one example).

Our approach here is to divide up Chandler's (1987) Eurasian largest city-sizes data into three large regions – China, Europe, and the Mid-Asian region in between – and measure variations over time that depart from the Zipfian rank-size distribution. Zipfian rank-size is the tendency for cities ranked 1 to n in size to approximate a size of M/r where r is a city's rank compared to the largest city and M is a maximum city size M that gives the best fit for the entire distribution (this formulation allows the rank 1 largest city size S_1 to differ from its expected value under a Zipfian fitted to an extensive set of the larger cities). The Zipfian distribution has been taken to be a recurrent and possibly universal pattern for city sizes as well as many other complex system phenomena. What we find for Eurasia and regions within Eurasia is that there are systematic historical periods that show significant deviations from the Zipfian. Some of these deviations show the characteristics of a regional collapse of city systems from which there is eventual recovery (unlike cataclysmic collapse exemplified by the Mayan cities system).

The periods of rise and fall of city systems for each Eurasian region, however, are different. This allows us to test the hypothesis that the rise and fall measure for China anticipates with a time lag the rise and fall measure for Europe, which is a prediction for the period starting in 900 CE consistent with Modelski and Thompson (1996), Temple (1987) and Needham (1954-2004). Finally, for the region of China we have sufficient time-series data to test the predictions from the historical dynamics model of Turchin (2005). This allows some limited results on whether some of the same processes are operative for the rise and fall of city systems as for the historical dynamics of state and empire rise and fall.

In Part I we pose the problem of instabilities in city sizes and systems drawing on Chandler's data for 26 historical periods from 900 CE to 1970. Part II examines ways of measuring departure from Zipfian distributions of city sizes and introduces the data used for city sizes and possible correlates of city system change. Part III gives the results of the scaling of city sizes for different regions so as to measure city system changes. In Part IV we examine the time-lagged interregional cross-correlations for these measures, and summarize results for crossregion synchrony. Part V examines correlations and time-lags between our three Eurasian regions and for other variables related to known historical oscillations where we have adequate data for hypothesis tests. The variables tested include such variables as trade connectivity, internecine warfare within China and development of credit and currency systems that facilitate international exchange as well as innovative national markets. VI concludes with a summary and implications of the findings.

PART I: CITY SYSTEM INSTABILITIES

Jen (2005:8-9) defines *stability* in terms of dynamical recoveries from small perturbations that return to an original state. Seriousness of the question of city system instability and of major departures from the Zipfian derive from the assumption that city economies are organized as networks that involve trade and war, and depend on innovation to join the leading economic or political sectors of more global networks. The two main factors that make for instability are competition and population growth.

Economic competition, aided by power politics, tends to make for oscillations that may return to what might be called structural stability. That is, they make for economic and political limit cycles rather than conservative stationarity. Populations of polities, empires, regions, and global world systems, also exhibit limit cycles if we average out trends of population growth, e.g., over the last several millennia. Jen defines *structural stability* as the ability to return from instability through other dynamics than the original (e.g., by varying external parameters) that are qualitatively similar to the original dynamic, as for example the Lotka-Volterra type of oscillatory limit cycle. While economic and political systems are not stable in the strict sense they may have the resilience to return to structural stabilities as they pass through oscillatory limit cycles with differing but qualitatively similar dynamics. Major population growth trends, however, as they interact with dynamical oscillations or limit cycles, may lead to *structural instability*, an inability to return to stability even through other dynamics than the original but that recover qualitative similarity to the original. The imperative of incessant competitive innovation for successful cities and city systems forms part of what leads to overgrowth of population relative to resources and to subsequent system crashes. Historically, these instabilities lead eventually to industrial revolutions that, rather than conserve materials and energies, may push extravagant degradation of resources into dynamically irreversible crises such as global warming and problems of structural instabilities. Unless innovation turns toward conservation the problems created will not be solved in the next century or possibly not in next millennium. The issues here are ones of scale, expansions of scale (size of cities, size of polities and empires, size of economies), the dynamic interactions that operate at different scales, and how these couple spatially and temporally (as described, for example, in Modelski and Thompson 1996).

The first questions of this study, then, are whether city systems as central economic actors and sites for multitudes of agents are stable or unstable, and if unstable, what kinds of models are appropriate for consistency with their dynamics. The thesis here is that it is not just individual cities that grow and decline but entire regional (and global) city systems. Here, drawing on our earlier work (White, Kejžar, Tsallis and Rozenblat 2005), Michael Batty (2006:592) states our case for us. "It is now clear that the evident macro-stability in such distributions" as urban rank-size or Zipfian hierarchies at different times "can mask a volatile and often turbulent micro-dynamics, in which objects can change their position or rank-order rapidly while their aggregate distribution appears quite stable…." Further, "Our results destroy any notion that rank-size scaling is universal… [they] show cities and civilizations rising and falling in size at many times and on many scales." What Batty shows, using the same data as do we for historical cities (Chandler 1987), is legions of cities in the top echelons of city rank being swept away as they are replaced by competitors, largely from other regions.[5]

PART II: DATA

City Size Data for Historical Eurasia

Chandler's (1987) database on historical city sizes is complemented by overlapping UN population data from 1950 to the present (in the interest of brevity we do not present these results here). Chandler reconstructed urban populations from many data sources. These included areas within city walls times number per unit area (see Appendix A), connected house-to-house suburbs lying outside the municipal area, data from city histories provided by city librarians, estimates from numbers of houses times numbers per house, and the cross-checking of different estimates (see Pasciuti and Chase-Dunn 2002). From 900 CE to 1970 his size estimates cover

[5] Noting from the shared database that the top echelon of cities in a single region may be swept away in a short period by interregional competition, Batty refers to our work on instabilities at the level of city systems.

over 26 historical periods, usually spaced at 50 year intervals, always comprise a set of largest cities suitable for scaling in a single period. These data include 80 Chinese, 91 European, and in between a much larger number of Mid-Asian cities.

Figure 1 shows numbers of cities in the dataset for in each period when they fall below 21. European cities in the top 75 world cities rise from 8 to 21 from 1100-1575, while those of China drop from 19 to 11 between 1200 and 1650 CE. Mid-Asia has more than 20 cities in the top 75 up to 1875. One concern is whether there are too few cities in some periods to differentiate characteristics of the tail of the size distribution (largest cities) from that part of the size distribution that reaches down to smaller cities.

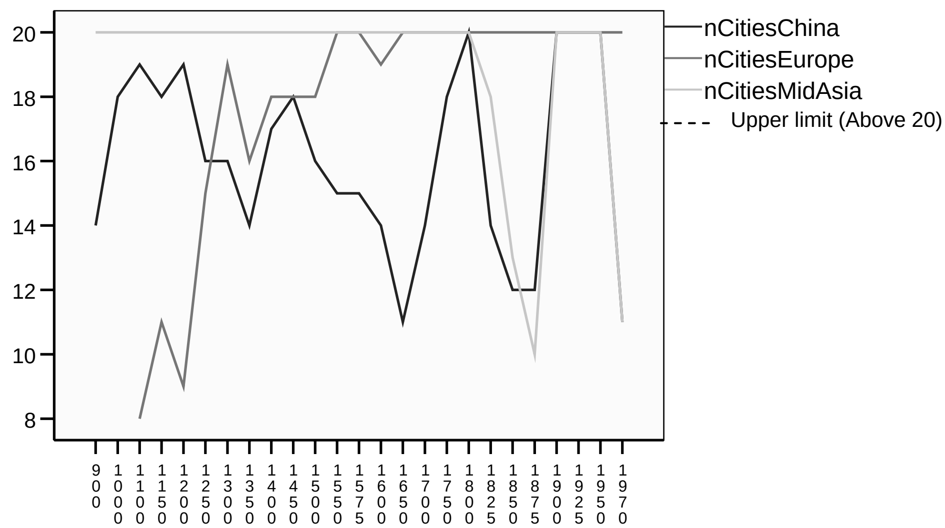

Figure 1: Number of Cities in Top 75 World Cities in each region when they fall below 21

Trade Routes (Eurasia). The total length of the Eurasian long-distance trade routes between 3500 BCE and 1500 CE at 50 year intervals have been calculated by Ciolek (2005) from trade-route maps drawn by Sherratt (2003). World-system pulsations in expansion and contraction of trade routes are shown by Turchin (2007). Turchin calculated a connectivity index for the Silk Routes between China and England. His data also show how instances of epidemics are concentrated near the high points of trade before periods of collapse.

Sociopolitical Instability: Internecine Wars (China only)

Turchin (2003:164) transcribed J. S. Lee's (1931) coded 5-year interval data on internecine wars in China, ranging from regional uprisings to wide-spread rebellions and civil wars, from 221

BCE (unification by the first Ch'in Dynasty Emperor) to 1929 to create a ten-year sequential intensity index to 1710. D.R. White converted Turchin's codes into a 25-year index running from CE to 1700 and coded 1725-1925 directly from Lee (1931). Lee coded the period to the end of the Ming Dynasty from a remarkably systematic inventory of conflicts by Chih Shao-nan from the Tih Wang Nien Piao and checked for accuracy by Tung-Kien (Lee 1931:114). The Tabula Annua (Seikainenkan), cross-checked and supplemented by the Tai Ping Tien Kuo Chan Ssi proved a reliable record of Ch'ing Dynasty wars, with those following from Lee's memory. The systematic pattern discussed by Lee for his graphs is one of two 800-year periods (200 BCE-600CE, then to 1385 CE) in which many more of the intra-territorial China conflicts occur in the last 400-500 than in the early 400-300 years, and a partial repetition up to 1927 of that same pattern. The other evident pattern is for conflicts to become much more likely at the transitions between dynastic periods.

Total Population and Population Change Data (Eurasia)

The early data on Chinese population from 900 to 1300 are controversial. White had data from Chao and Hsieh (1988), provided by Turchin (2003:164-165), he consulted Ho (1956, 1959), Steurmer (1980), Mi Hong (1992), Durand (1960), Heilig (1997, 1999, 2002), and the radical revision proposed by Heijdra (1995) and Mote (1999) that was critiqued by Marks (2002), and others. Given the uncertainty in absolute figures, we coded a binary variable for each 25-year period where 1 is given for a date at which there is a population peak before collapse, with 0 otherwise. The different total population estimates available to us for China over our full time frame agreed very closely as to where these population peaks occurred. In some cases two adjacent peaks were indicated.

Turchin (2006, 2007) provided population, carrying capacity, detrended population, and a misery index (inverse wages) for England that could be useful in comparisons with our European city data.

Monetary Liquidity (China only)

We coded for 900-1700 an index of monetization (liquidity) in China using Temple's (1986:117-119) discussions (drawing on Needham 1954-) on the development of credit, paper money, banking, and inflation (indexing lower liquidity) into a qualitative judgmental scale from 0-10.

PART III: THE Q/β SCALING AND HYPOTHESES

Measuring Departures from Zipf's Law for City Size Distributions

We begin by examining the city size distributions of macroregions in the Eurasian continent, over roughly the last millennium, which is the millennium of Eurasian and then planetary globalization. We examine the extent of instability of city systems in ways that are visually evident by inspection of changes of the shapes of Eurasian city size distributions, and as measured by shape indices. Figure 2 shows a semilog graph of the cumulative rank-size distribution for divisions of most of Eurasia (excluding Japan/Korea) into three regions: China (c900-c1970), Europe (e900-1970) and the Mid-Asian (m900-c1970) remainder. The curve to which power-law distributions should correspond is shown by the top (ZipfCum) power-law curve. Cumulative population size is logged on the y axis and the x axis is city size rank. The Zipfian curve forms a straight line when rank is also logged, with a Pareto log-log slope of 2. As can be seen visually, there is some departure from perfect parallelism in the empirical curves: some lines are more curved or less curved for the top cities than the Zipfian, most lines are flatter that the Zipfian for the smaller cities and many of the curves bend at different city ranks.

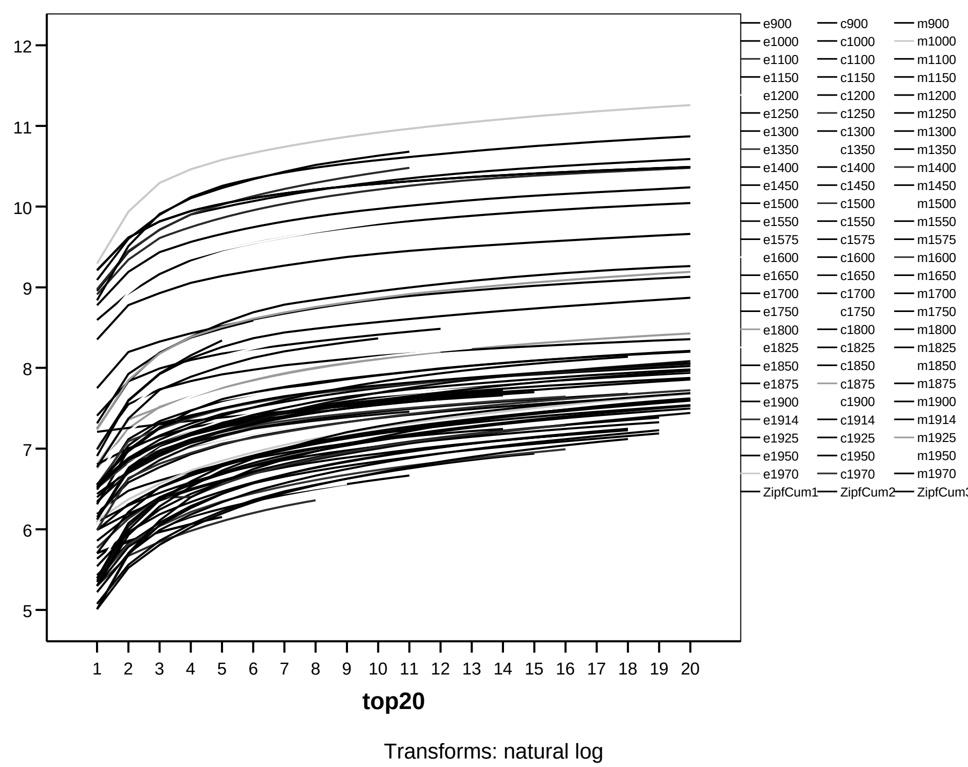

Figure 2: The Chandler Rank-Size City Data (semilog) for Eurasia (Europe, China, Mid-Asia)

Figure 3 shows the same data (all three regions) in a log-log plot where power-law city distributions would all be straight lines and Zipfian distributions would all have the same slope. The lines are neither parallel nor of the same slope, nor do they have curvatures in the same places. Our measurements of the properties of these distributions will be aimed at the hypothesis that these variations provide indicators useful to showing how city system fluctuations fit into economic and historical dynamics.

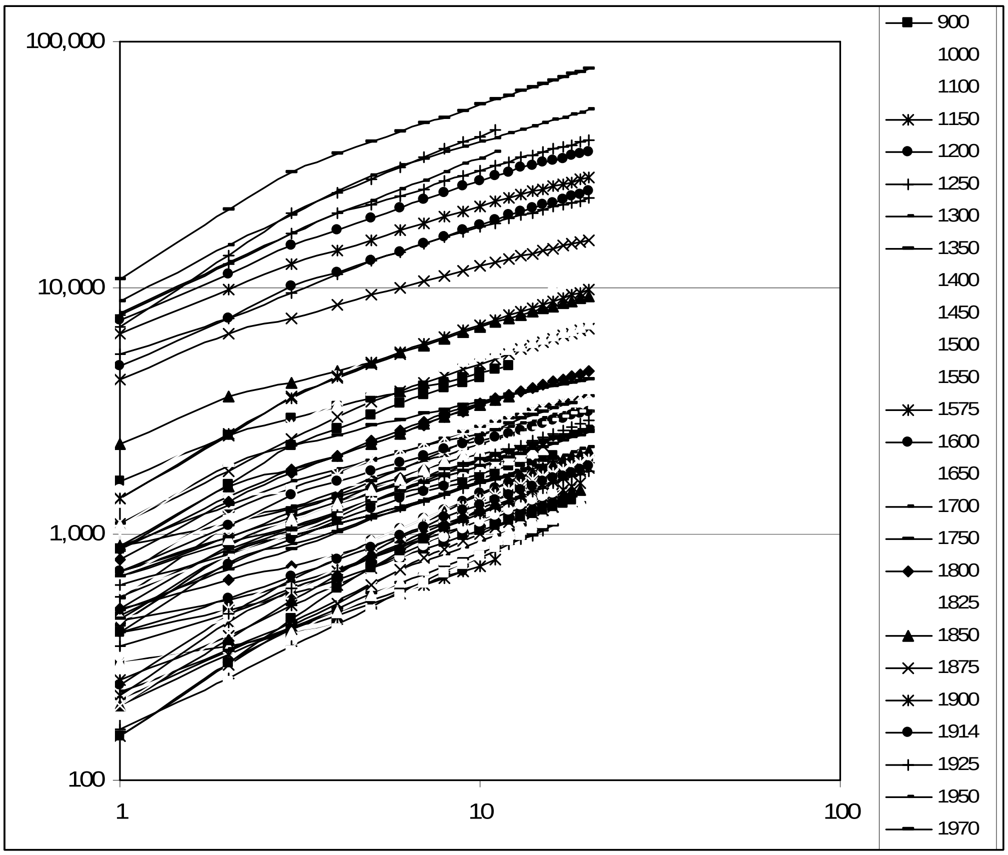

Figure 3: The Chandler Rank-Size City Data (log-log) for Eurasia (Europe, China, Mid-Asia)

We use two measures of how these curves vary. Each measurement model is based on a standard continuous function with parameter fitting based on recasting the data as a cumulative probability in its complementary form. $P(X \geq x)$ is the probability that an urbanite will reside in a city of at least size x. Each model is aimed at capturing the complete (Pareto II) shape of the empirical $P(X \geq x)$ or part of that shape (Pareto I for the tail) as a cumulative complementary distribution function (CCDF). It is important to use maximal likelihood estimates in fitting the parameters of distributional models, and especially so with small samples. MLE estimates are unbiased, meaning that with *n* independent samples of the same data, the expected values of the estimated parameters for each sample would converge to the true parameter values.

The first measure is the standard (type I) Pareto distribution with a single parameter β where the Zipfian is the special case of $\beta = 2$:

$$P_\beta(X \geq x) = x^{-\beta} \quad (1)$$

Fit to this distribution captures the extent to which the lines are straight in Figure 3, as is typical for the tails of city size distributions.

Fit to a second function, a type II generalized Pareto distribution with a cut-off σ (Arnold 1983), captures the extent to which a given log-log distribution is curved. This function allows recognition that Zipf's and power laws for city sizes almost always have one or more cut-offs of a lower size below which the power-law changes or ceases to apply.[6] This function fits to the urban distribution a shape parameter Θ, analogous to β, and a scale parameter σ.

$$P_{\Theta,\sigma}(X \geq x) = (1 + x/\sigma)^{-\Theta} \quad (2)$$

The type II Pareto has an extra cut-off parameter σ, but given unbiased maximal likelihood estimation (MLE) of parameters, there are many advantages of a parameter that specifies the crossover where the curve breaks for a power-law or Zipfian tail. The Pareto II function:

a) can fit an exponential function or a collapsed tail in cases where a power-law or Zipfian tail is inapplicable.
b) with MLE, yields parameter estimates that are unbiased by sample size. MLE runs can be batched for multiple datasets, and the MLE commands are easy to copy and paste into R.[7] Standard errors are typically very small for city size data and the true values of the parameters are likely to be within these limits.

MLE provides unbiased parameter estimates that allow 1-to-1 functional transformations of the parameters into equivalent models (Brown, 1986). The q-exponential $Y_q(S \geq x)$ = Y0 (1-(1-

[6] For example, excluding two primate city outliers, the next largest 16 cities for 1998 in the U.S. (over 11 million) show a steep log-log slope, those ranking down to .5 million show a shallower slope, those to 1 million a much shallower slope, and then the power-law disappears altogether (Malacarne et al. 2001:2).

[7] A typical batch of instructions, for example, might be:

```
china.900 <- c(500,150,90,81,75,75,70,65,60,58,49,47,40,40)
china.900.tsal.fit <- tsal.fit(china.900,xmin=40) # Assigns the results of the fit to the object
china.900.tsal.fit # Displays the estimated parameters and information about the fit
(these estimates run in seconds, and those following run in minutes)
china.900.tsal.errors <- tsal.bootstrap.errors(china.900.tsal.fit,reps=100)
china.900.tsal.errors # Displays the bootstrapped error estimates.
```

$q)x/\kappa)^{1/(1-q)}$ and its shape (q) and scale (κ—kappa) parameters is a 1-to-1 functional transformation of equation (2) where $q=1+1/\Theta$ and $\kappa = \sigma/\Theta$. Y_q has been previously used (Malacarne, Mendes, and Lenzi 2001) for city-size distribution scaling. Bootstrap estimates of the standard error and confidence limits of the q,κ parameters derived from Θ,σ are provided by Shalizi's (2007) R program for MLE. There are many other advantages of Y_q and Pareto II that we do not exploit here.[8]

A continuity theorem often used with the Y_q model (Tsallis 1988) gives the relation in the Pareto II and Y_q functions, when $q>1$, between a Y_q tail that asymptotes to a power-law with slope 1/(q-1) and a Pareto I tail with slope β. The Y_q curvature, however, captures that of the smaller cities in the distribution. The theorem is important to the study of city sizes where smaller size cities do not follow a power law and we often lack data on the actual distribution for smaller size cities, a problem that we take up next. More generally, although that is beyond the scope of this paper, the q-exponential is interpretable as one of the theories developed in "behavioral statistical physics" (Farmer 2007) and q=1 (the limiting case of an infinite Θ) has a special meaning as an entropic system "at rest," lacking complex interactions and power-law behaviors. For cities, $q \leq 1$ is an indicator of system collapse!

Measurement Error for Pareto II Departures from Zipf's Law for City Size Distributions

There are two potential sources of error for Pareto II parameters that are not soluble without MLE. They are sufficiently important to merit discussion here because they affect accuracy and interpretation of results, and provide crucial information needed for others to replicate results using MLE. One is the boundary specification of the region studied. China and Europe, for example, are more naturally bounded areas that our Mid-Asia, which runs from Cordoba to India and Southeast Asia and is partly defined by the spread of Islam. The second is

[8] These include:

a) Y_q estimates an expected "largest city size" M consistent with the body of the size distribution. This requires simultaneous estimation of M and Y(0) to solve $Y(0)\, P_{\Theta,\sigma}\,(X \geq M)=M$.

b) The total urban population can be estimated from Y_q without having data on all smaller cities, although this feature is not utilized here.

c) Equation (1) and Y_q may be fitted *without* the largest city so as to derive an expected size for the largest city given our model.

d) This gives our model a ratio measure of the largest city size to its expected size from Y_q.

e) Y_q has a known derivative $Y_q'(x)=Y0/\kappa[1-(1-q)x/\kappa]^{(q/(1-q))}$ giving the slope of the curve $Y_q(x)$ for any city size x.

f) Solving for $Y_q(M)=M$ for the estimated largest city size M consistent with Y_q gives $Y_q'(M)$ as the slope of the Y_q at M and converges with $\beta=1/(q-1)$ for the Pareto power-law slope.

the size of the sample: if there is a small number of largest cities for which there are data, estimation of the scale parameter $\sigma = \kappa/(q\text{-}1)$ of the Pareto II distribution may err because the sample does not provide the relevant evidence for curvature away from a power-law tail. In this case σ will be close to zero to insure an approximation to the ordinary Pareto because the 1 in $1+x/\sigma$ from equation 2 becomes negligible when the values of x are divided by a diminishingly small σ (i.e., are multiplied by a large constant). In China and Europe we find a strong coefficient of determination (significant at $p <.005$) between $\sigma < .04$ and having fewer than 18 cities in the sample. In this case σ is commonly in the range $.00001 > \sigma > .0000001$, as contrasted to $\sigma > 5$ (ranging up into the thousands) for the larger city samples. *Must we conclude when σ is small, in the cases of small samples of the largest cities, that the appropriate distribution function is Pareto I and not Pareto II (q-exponential) when Pareto I fit may be due to missing data?* These small values of σ(«1) are nonarbitrary, however, and the fitted Pareto I parameter β ($\approx\Theta$ in this case) must converge to $1/(q\text{-}1)$ when $q>1$.

We are thankful to Shalizi (2007) for writing an MLE program in R for our use in estimating parameters Θ,σ (Arnold 1983) in equation (2).[9] He also gave us his R program for standard MLE fits to the Pareto distribution.

The left graph of Figure 4 shows some of the early, middle and late period q-exponential curves for Chinese cities fitted by MLE using a normalized cumulative probability distribution. These are for periods where $\sigma > 1$ ($\kappa>1/\Theta=q\text{-}1$). These cases also fit the q-exponential better than a power-law MLE for the same data. What this graph shows is a very regular pattern for both the actual data and the fitted curves. First, all the empirical curve tend to asymptote toward a power-law (Pareto I) tail for the larger cities. Second, there is considerable variation in the slopes of these tails. Third, there is a second horizontal asymptote convergent on $P(X \geq x)=1$ where “cities proper” cease to occur at smaller settlement sizes. This allows an estimate and reconstruction of total urban population at different city sizes. Fourth, there are distinct historical periods, ones that come in three clumps, within which there are ups and downs in slopes. Sixth, each period

[9] Commentary on the White et al. (2004) paper on the present topic elicited the suggestion from Cosma Shalizi that we should consider MLE estimates for fitting q-exponentials to city size data. Given our small sample sizes from the Chandler data as we moved from a world sample (75 data points) to regional samples, we called on Shalizi in late 2006 to derive the MLE equation for us, which he did,. He then found earlier derivations such as those of Arnold (1983) and others. We are greatly indebted to Shalizi for writing functions in R to make MLE (unbiased) estimates of the q-exponential parameters and bootstrap estimates of the standard errors of these estimates.

tends to have something of a fixed pivot (shown by the arrow) below the crossover from the power-law tail to smaller sizes. The center graph in Figure 4 shows the same Pareto II curves but with additional fits for random samplings of the same data points. Additional fittings like these allow bootstrap estimates of the standard errors of estimates, which in almost all our empirical cases are very small.

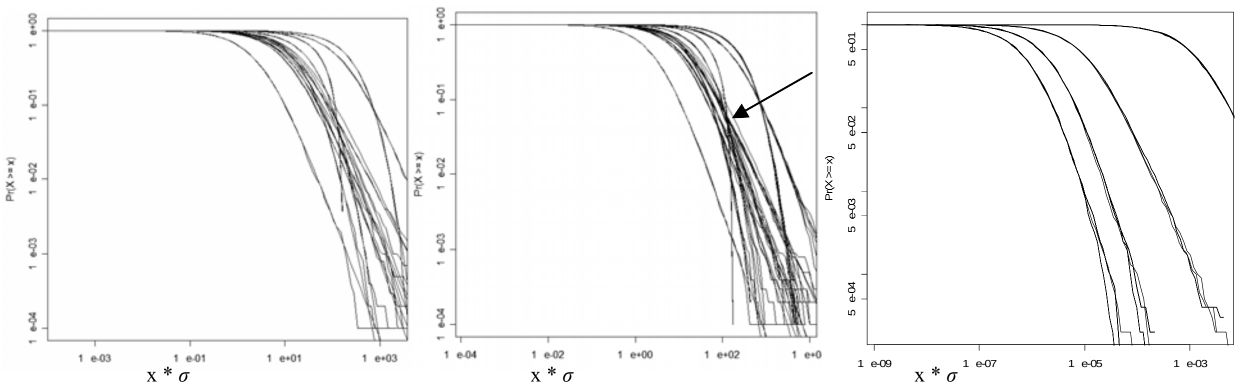

Figure 4: Probability distributions of Chandler Rank-Size City Data (log-log) for China

The right graph in Figure 4 shows the normalized probability MLE fits for China in historical periods where values of σ are small («1). Small σ may be caused by small samples, as seen in many periods in Figure 1 for China or Europe or by summation of distributions from different regions (as with Mid-Asia). The precision of ML parameter estimation solves the problem of comparability of q values given very different σ estimates («1 and >1) but relabels the x axis in the σ«1 case on a scale that allows focus on the small values of σ but also to see the same shapes of the Pareto II distribution as in the cases where $\sigma > 1$.[10] The bootstrap standard errors for σ and κ are small within each period (but not across periods). The σ«1 values are thus technically correct, along with q.

The Mid-Asian region has more cases where σ«1 but these are neither caused by nor correlated with small samples. Rather, the broad Mid-Asian power-law tails are likely to result from summing somewhat divergent regions each having shorter power-law segments in their tails, with summation making for a longer Pareto tail in the aggregate (Farmer 2006). What this indicates is that the magnitude of the Θ parameters might be underestimated (longer tails with

[10] If we remove the σ«1 cases for China, for example, the relationship expected of our estimates, namely, (-)β ~1/(1-q), is closely fulfilled but is contradicted where σ«1.

lower Θs than the true values for properly specified regions).[11] Since our comparisons depend on relative variations within or across regions, these might not affect our findings on Mid-Asia.

The MLE results, then, allow us to solve the potential measurement errors for Pareto II departures from Zipf's law. Remarkably, when only the power-law tail is present in the sample, without any overt indication of departures from the power-law, MLE still estimates q correctly in a way that converges with a power-law fit for the tail, but it retains the prediction that the fitted q and κ, in the normalized probability distribution, represent a correct prediction of both smaller city sizes and total population.

HYPOTHESES

Several linked hypotheses build on one another, each supposing the previous hypotheses to be supported, and each adding greater specificity in relation to the parameters of the two models, β for the Pareto I and Θ for Pareto II (thus q for the q-exponential):

H1. The Zipfian (β=2 and q=1.5 for our CDF) is posited as the most likely historical norm for the tails (β) and bodies (q) of city size distributions.[12] Over long historical periods these should be expected as average values around which q and β fluctuate.

H2. Variations in q and β are *conservative* as population measures affected by births and normal mortality but may change quickly when influenced by migration, and sociopolitical instability (SPI), that is, internecine wars or outbreaks of violence.

H3. Variations in q and β are thus likely to exhibit stability within historical periods of multiple generations, with Zipfian values on both measures correlated with periods of stability and normality, followed by instabilities that may occur suddenly.

H4. As such q and β are indicators of rise and fall of urban system size distributions in both the body and the tail of the distributions, which may vary with considerable independence. Tails may a) collapse (shorten, q dropping toward or beyond 1), or grow into (b) longer (thinner, lower β slope) tails, (c) Zipfian (β=2), or (d) shorter power-law (thicker, higher β slope) tails.

4a) Collapse of tails should co-occur with sociopolitical instability (SPI), severe economic/political crisis, or major wars.

4b) Longer tails should be enhanced by capitals of empires and by exceptional centers of international trade that serve as economic magnets for migration from impoverished rural areas.

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[11] When σ«1 values are not correlated with small sample sizes, it is a signal to investigate the possibility that the region of aggregation is not properly specified. Having a rule-of-thumb for regional boundary misspecification will be helpful in further research in this and other areas of study.

[12] Elsewhere we explore estimates of the total urban population asymptote, Y(0), from fitting the Pareto II and equivalent q-exponential distributions.

4c) Zipfian tails should be enhanced by intraregional trade with positive urban-hierarchy feedbacks.

4d) Shorter tails should correlate with external conquest of a capital or of hub cities.

H5. Intraregional and interregional trade is crucial for city system rise, and economic collapse may be involved in city system collapse. Fluctuations of interregional trade may act either or both to *synchronize* city rise and fall between regions, or to *predict* from city rise and fall in a more developed region that time-delayed rise and fall will occur in a less developed region that is strongly connected by trade. Currency, credit, banking and liquidity are leading indices of development in measuring the interregional impact of trade.

H6. Historical phases that are clearly marked in the population/instability cycles of structural demographic historical dynamics for periods in which agrarian empires that are relatively self-contained, lacking external perturbations (Turchin 2003, 2005, 2006), should correlate with some but not necessarily all of the phases of city system rise and fall, especially those involving SPI fluctuation. In this context, population growth with low pressure on resources should lead to rises in q and β provided that trade and liquidity allows economic elites to congregate in cities and to provide employment for skilled workers. Peaks of population pressure on resources followed by high SPI levels should precipitate city system declines. In these terms, city system rises and falls:

6a) are likely to be loosely but not strictly coupled into structural demographic variations.

6b) are also interactive with inter-regional competition and global wars (Modelski and Thompson 1996).

6c) are likely to exhibit both longer and shorter (i.e., less predictable) historical periods of stability given this combination of regionally endogenous and regionally exogenous dynamics.

Those hypotheses are testable. The following are more speculative observations about evolutionary tendencies predicated on our findings.

H7. In spite of the growth of gross world product (GWP) rising at faster rates than population, there is no evolutionary historical tendency evident toward either greater stability, greater instability, or alteration in the periodicities of city system ups and downs, in spite of, and probably because of, the pressures of rising global and rising urban populations. Evolutionary stability in city systems, as recovery from small perturbations, apparently remains something to be learned rather than taken for granted.

H8. Stability as recovery from large perturbations through structural demographic oscillatory dynamics, however, has apparently been learned, but urban instabilities are only weakly coupled to oscillatory structural demographic recoveries, so evolutionary learning is only partial. The introduction of new dynamics complicates the question of whether city systems will acquire greater stability

SCALING RESULTS

Using visual and statistical evidence for changes in the shape parameter, White, Kejžar, Tsallis and Rozenblat (2005) were able in an earlier study to date six Q-periods in Eurasia over the last millennial period. These changes and periods were seen to be related to the framework for studying globalization developed by Modelski and Thompson (1996). In studying multiple regions we take a more detailed and dynamical view of this relationship.

Figure 5 shows the q and β slope parameters fitted by MLE for the regions of China, Europe and the region between (Mid-Asia, from the Middle East to India),. Here, a Zipfian tail would have q=1.5 and β=2. The horizontal line shows that this slope and shape is approximated more recently in the early modern and modern period. We also show a normalized minimum of q and β in which we divide q by 1.5 and β by 2.0 to normalize for the Zipfian.

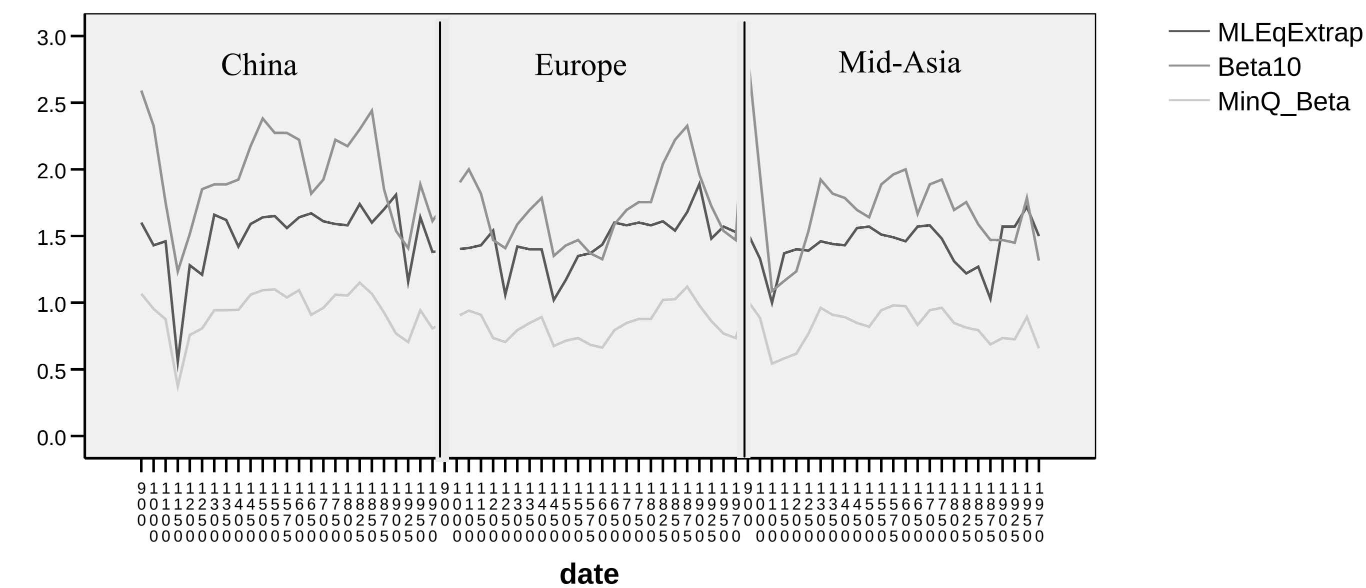


Figure 5: Values of q, β, and their normalized minimum

Table 1: Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Std.Dev/Mean
MLEqChinaExtrap	25	.56	1.81	1.5120	.25475	.16849
MLEqEuropeExtrap	23	1.02	1.89	1.4637	.19358	.13225
MLEqMidAsIndia	25	1.00	1.72	1.4300	.16763	.11722
BetaTop10China	23	1.23	2.59	1.9744	.35334	.17896
BetaTop10Eur	23	1.33	2.33	1.6971	.27679	.16310
BetaTop10MidAsia	25	1.09	2.86	1.7022	.35392	.20792
MinQ_BetaChina	25	.37	1.16	.9645	.16247	.16845
MinQ_BetaEurope	23	.68	1.26	.9049	.15178	.16773
MinQ_BetaMidAsia	25	.54	1.01	.8252	.13217	.16017

Valid N (listwise)	19						

These results support H1, that the Zipfian is the historical norm both for tails and bodies of city size distributions, approximating β=2 and q=1.5. Mean values for q in the three regions vary around q=1.5±.07, consistent with a Zipfian tail, and similarly for variations around β=2 ±.07 for the Pareto slope of the top 10 cities. Statistical runs tests of whether the variations around the means are random or patterned into larger temporal periods are shown in Table 2 and 3. The runs tests reject the null hypothesis ($p<.01$ for Europe, $p<.05$ for Mid-Asia, and $p<.06$ for China; $p<.00003$ overall), supporting H3.

Table 2: Runs Tests at medians across all three regions

	MLE-*q*	Beta10	Min(*q*/1.5, Beta/2)
Test Value(a)	1.51	1.79	.88
Cases < Test Value	35	36	35
Cases >= Test Value	36	37	38
Total Cases	71	73	73
Number of Runs	20	22	22
Z	-3.944	-3.653	-3.645
Asymp. Sig. (2-tailed)	.0001	.0003	.0003

Table 3: Runs Test for temporal variations of *q* in the three regions

	mle_Europe	mle_MidAsia	mle_China
Test Value(a)	1.43	1.45	1.59
Cases < Test Value	9	11	10
Cases >= Test Value	9	11	12
Total Cases	18	22	22
Number of Runs	4	7	7
Z	-2.673	-1.966	-1.943
Asymp. Sig. (2-tailed)	.008	.049	.052

a Median

The time periods of successive values above and below the medians represent the rise and fall of q to Zipfian or steeper-than-Zipfian slopes alternating with low-q periods with truncated tails of the distributions. Relatively long city-slump periods occur in the medieval period for all three regions, a second slump occurs in Europe in 1450-1500, another in Mid-Asia in 1800-1850, and one in China in 1925 (not shown) when to q falls to 1.02.

As for H6, there are rough correlations for both secular cycles (Turchin 2003, 2005, 2006, 2007) and Modelski-Thompson (1996) globalization processes with dates of urban crashes, shown in Table 4 below.

Breaks	950	1150	1430	1640	1850
Cycle	1	2	3	4	5 (Modelski-Thompson 1996: Table 8.3)

Mid-Asia:	**1100,**		1450		**1825-75, 1914**	(**major**/minor **urban crashes)**
China:	**1150-1250,**			1650	**1925**	(**major**/minor **urban crashes)**
Europe:		**1250,**	**1450-1500,**		1950?	**(major**/minor **crashes)**

Table 4: Temporal breaks and urban crashes of β/q in the three regions

As shown by the lower dotted line in Figure 6, values of q below 1.3 might be considered as approach a city system crash or collapse/destruction of primate cities. China and Europe experience an abnormal rise in q in 1900 beyond 1.7 (upper dotted line). This results in a thin tailed distribution (extreme primate cities) that might be considered as a different kind of city system crisis. Some crashes have to do with wars, like the Song loss of their capital to the Jin in 1127. Global wars noted by stars on the lines in Figure 6, might have to do with the punctuations of these periods, but we are unable to evaluate that question statistically. Crashes in q often occur at long intervals (**bold** dates in Table 4), as in Figure 5, with β falling at shorter intervals.

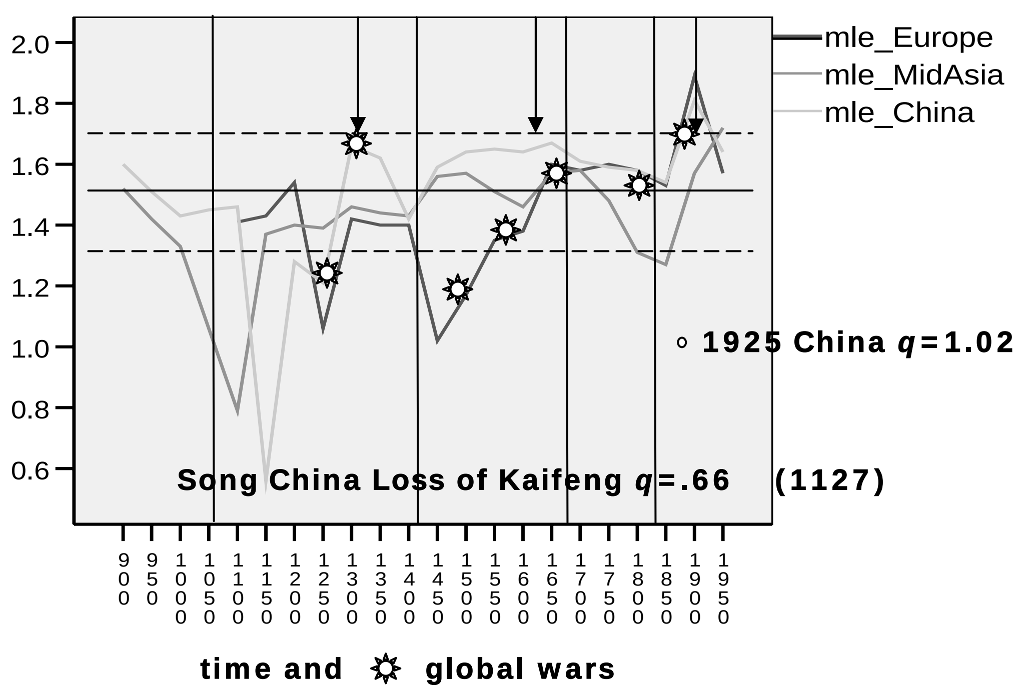


Figure 6: Fitted q parameters for Europe, Mid-Asia, China, 900-1970CE, 50 year lags. Vertical lines show approximate breaks between Turchin's secular cycles for China and Europe Downward arrow: Crises of the 14th, 17th, and 20th Centuries.

These results support H2, that variations in q and β are *conservative* (since births and normal mortality are slow to affect population measures) but may also *change quickly* in ways consistent with interurban migration, internecine wars and outbreaks of violence or general sociopolitical instability (SPI). They also support H3, that variations in q and β may have long periods of stability, with Zipfian values on both measures correlated with periods of stability and normality, and that stability may be followed by sudden instabilities, or drops in q, β, or both.

PART IV: CROSS-CORRELATION OF THE SCALING MEASURES

One of the major patterns of variability in city distributions is the primate city effect: the primate and top ranked cities often form a steeper urban hierarchy in periods of economic boom or empire, or when the primate city is a major international trade center. In periods of decline they may form a truncated tail compared to the body of the distribution. Further, the slope of the tail of the size distribution (β) tends to change faster than the shape body of the distributions. This is tested using data from all three regions using the autocorrelation function (AFC), where values of a variable in one time period are correlated its values for 1-16 time lags (each lag in this case of 50 years). The upper and lower confidence limits are at 95% for a two-tailed significance test ($p<.05$).The AFC of β compared to q shows a much higher short term continuity (1 lag of 50 years), a recovery period at 5-6 lags, and then autocorrelation largely disappears, while q varies more continuously with more stable long term autocorrelations (up to 16 lags or 800 years). The ratio of q/β has autocorrelation only for Europe, oscillatory and not significant.

Figure 5 has shown that q and β vary somewhat independently, often correlated positively when $\sigma \ll 1$ ($\kappa \ll 1$, recalling that β is a negative slope and q varies inversely to that slope) but negatively when $\sigma>1$ ($\kappa>1$). But which affects which over time if the two are synchronously somewhat independent? In time-lagged correlation: Does the shape (q) of the body of the city affect the tail (β) in subsequent periods, or the reverse? β might shape q if long distance trade has an effect on the larger cities engaged in international trade, but q might shape β if it is the waxing and waning of industries in the smaller cities that feed into the export products for the larger cities, as we often see in China and Europe.

What lagged cross-correlations show for China and Europe – but not in Mid-Asia – is that, starting from the maximal correlation at lag 0, high q (e.g., over 1.5) predicts falls in Pareto β over time, reducing the slope of the power law tail below that of the Zipfian. This suggests that high q produces an urban system decline in β. This would contradict an hypothesis of long-distance trade as a driver of rise and fall *in the larger cities*. It would not contradict, however, the possibility that long-distance trade was directly beneficial to the smaller cities, with these effects feeding into the success of the larger cities with a time lag. For China and Europe, where successful long-distance trade was organized on the basis of the diffusion of effective credit mechanisms available to the smaller merchant cities, this seems a plausible explanation for the time-lag findings. These credit mechanisms were not so easily available in Mid-Asia where Islam operated to regulate interest rates to prevent excessive usury.

If there is a correlation between long-distance trade and the rise and fall of population pressure in the secular cycles of agrarian empires, our data might support Turchin's (2007) argument, formulated partly in response to our own studies of the role of trade networks in civilizational dynamics, that it is during the high-pressure (stagflation) period that long-distance trade flourishes. If so, the impact of trade should be reflected first in variations in q, which vary more slowly than β.

The overall pattern in the cross-correlations for the three regions together shows strong correlation synchronically between q and β at lag 0 ($p<.000001$) where high values of q predict falling values of β over time over three 50-year lags. Variables q and β have the least cross-correlate for Mid-Asia, but detailed examination of the Mid-Asia time-lags shows a weak cyclical dynamic of Hi-q→Lo-β →Lo-q→Hi-β that holds to 1950.

PART V: HISTORICAL NETWORK AND INTERACTION PROCESSES

H5 posited that intraregional and interregional trade is crucial for city system rise, and economic collapse may be involved in city system collapse. The data presented here is supportive of hypothesis, but we approach the question first as to whether, at the interregional level, there is synchrony between regions, of what sort, and whether trade is involved in this synchrony.

Turchin (2007) shows evidence for "a great degree of synchrony between the secular cycles in Europe and China during two periods: (1) around the beginning of the Common Era and (2) during the second millennium." We also find evidence for synchrony in city system rise and fall in common temporal variations in q for the second millennium. The correlations in q by time period follow a *s*ingle factor model, as shown in Table 5, with China contributing the most to the 47% common variance in *q* between the three regions.

Table 5. Communalities

	Initial	Extraction
MLEChina	1.000	.660
MLEEurope	1.000	.444
MLEMidAsIndia	1.000	.318

Extraction Method: Principal Component Analysis.

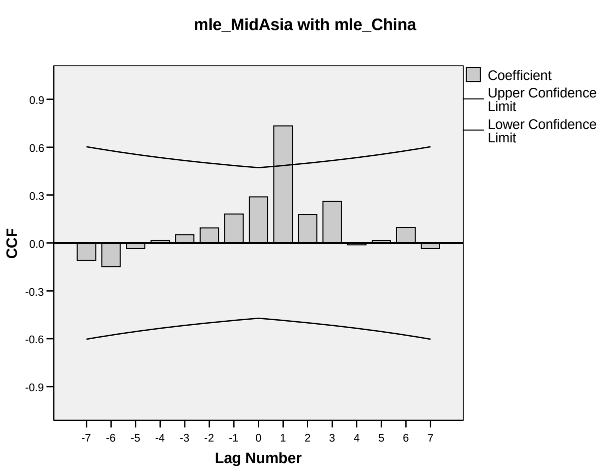
Total Variance Explained

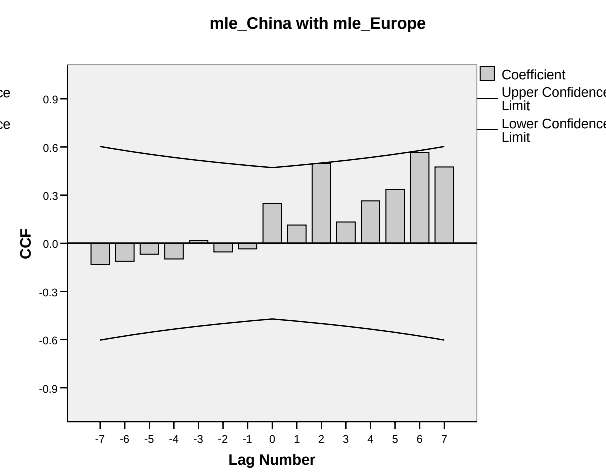
Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.422	47.403	47.403	1.422	47.403	47.403
2	.944	31.467	78.870			
3	.634	21.130	100.000			

Component Matrix(a)

	Component
	1
MLEChina	.812
MLEEurope	.667
MLEMidAsIndia	.564

The evidence from city sizes adds detail on dynamical interaction to that of interregional synchrony for the last millennium, supporting H5. Figure 7 shows that changes in *q* for Mid-Asia lead those of China by 50 years with a hugely significant correlation at 50-year lag 1; Mid-Asia leads Europe by 150 years (lag 3) but the cross-correlation is not quite significant; the cross-correlation for China's *q* leading Europe by about 100 years (lag 2) is also not quite significant although several earlier estimates of *q* did show significance (recall that the true values of *q* may vary somewhat even for MLE estimates).





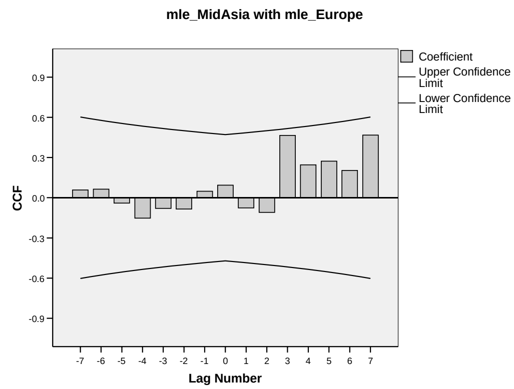


Figure 7: Cross-correlations for temporal effects of one region on another

H5 posits historical specificity that Eurasian synchrony has been partly due to trade, particularly that between China and Europe (noting as well that the practice of Islam in Mid-Asian region during this period tended to restrict the full employment of credit mechanisms). The cross-correlation in Figure 8, showing an effect on growth of β in Europe, sustained by the Silk Road trade, for example, suggests that trade is one of the factors causing the growth of power-law tails in urban size distributions, again supporting H5.

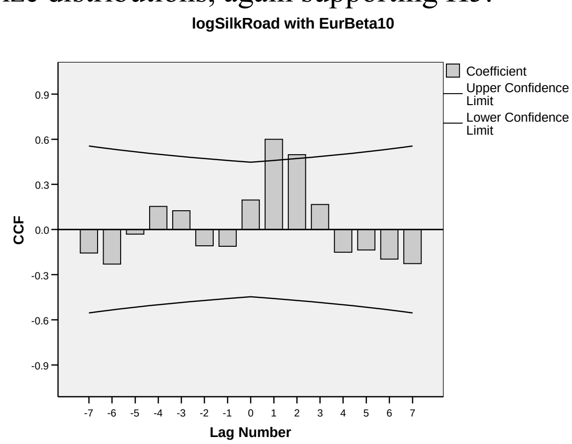


Figure 8: Time-lagged cross-correlation effects of the Silk Road trade on Europe

Figure 9 shows, from European data contributed by Turchin, synchrony between higher values of q (normal urban hierarchy) and the percent of French population attracted to Paris as a regional capital and economic center, with this percentage falling after the peak in q.

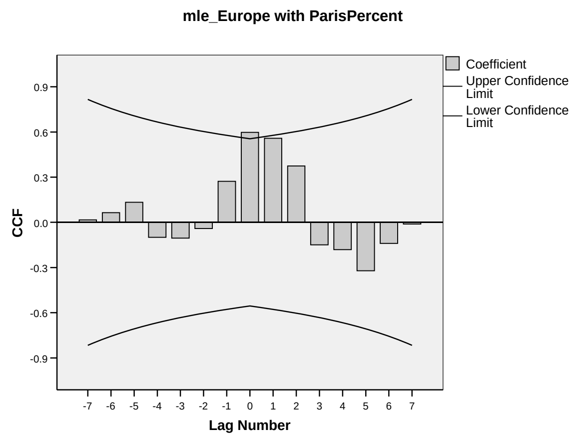


Figure 9: One time-lagged effect of regional *q* on primate city population

Our choice of the last millennium to test the interaction of the city size fluctuations with historical dynamics was motivated by the evolution of globalization in Eurasia in this millennium. Key elements in the transition to market-driven globalization occurred in China starting in the period of 10^{th} century invention of national markets, with currencies, banks and market pricing, a historical sequence that leads, through diffusion and competition, to the global system of today (Modelski and Thompson 1996).

The data on credit and liquidity in the Chinese economy also follows closely the rise and fall of *q*, as shown crudely in Figure 10, supporting H5. Rise of monetization, growth of credit, and development of banking accompany the early Zipfian *q*~1.5 of Song China, and these mechanisms of liquidity plummet with the Jin conquest of Kaifeng. Circa 7-800 years from 1100 BCE, with long periods of inflation, are required to regain liquidity and banking favoring international trade. During the Qing dynasty the Chinese money was silver coin. The first modern bank, the Rishengchang (Ri Sheng Chang) was established in 1824. It broadened to include banks in every major city, folding in bankruptcy in 1932. (the right end of the liquidity graph, estimated qualitatively in the lower figure, should be higher).

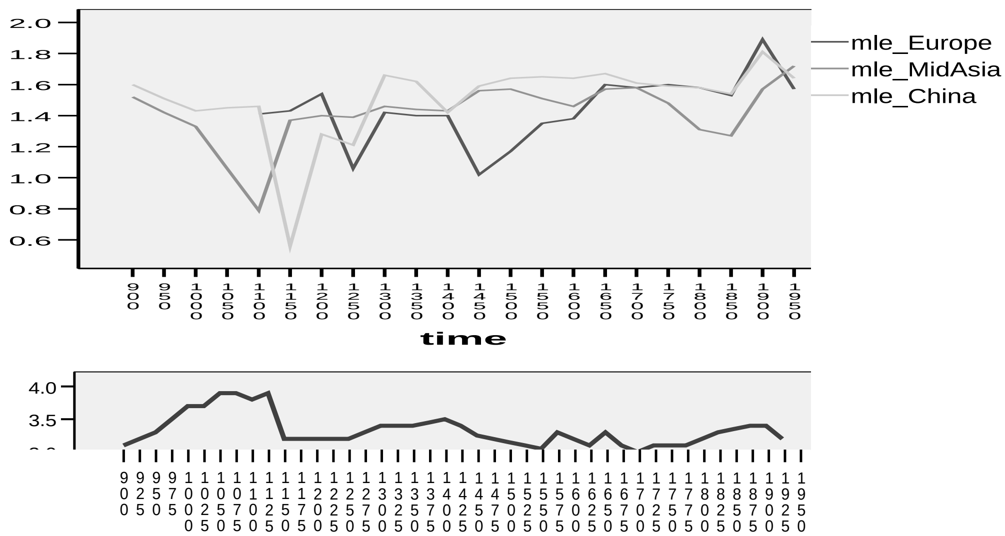


Figure 10: A crude long-term correlation between Chinese credit and liquidity (lower graph), estimated from Temple (1986), and Chinese (European, Mid-Asian) *q* (upper graph)

Figure 11 shows support for hypothesis H6, the coupling between urban system dynamics and structural demographic historical cycles (Turchin 2005a). The three insets show Lee's (1931) data on sociopolitical instability (SPI: specifically, internecine wars) for the periods of the Han, Tang, and Song dynasties (broken lines) against data on detrended population (solid lines).[13] J. S. Lee interpreted his data on Chinese internecine wars from 200 BCE (Han period) to 1930 as showing 800-year cycles of internecine conflict, weakly separated into two 400-year periods. Figure 11 breaks up his data into agrarian empire historical periods in which there are no major disruptive external wars. The relation of population pressure phasing to that of SPI is similar in each case, with SPI lagging population pressure at a roughly generational interval. This produces four phases: population growth (and certain types of innovation) in phase 1; rising resource scarcity and SPI, rising to a population peak in the stagnation phase, 2; a phase 3 of falling SPI and falling population pressure; and a 4th phase with minimum population pressure and SPI. Each period repeats a similar endogamous dynamics defined by the negative time-lagged feedback between *P*, population density per resource, and *SPI*, fitting a 2-equation model (with appropriate constants):

$SPI_{t} <\approx\approx + P_{t-1}$ (Population change drives change in SPI) (1)

[13] Han and Tang regional populations are dividing by estimation agricultural food supply (Turchin 2005a), with the Ming regional population (Zhao and Xie 1988) divided by growth trend.

$P_t <\approx\approx - SPI_{t-1}$ (SPI has a revere effect on Population) (2)

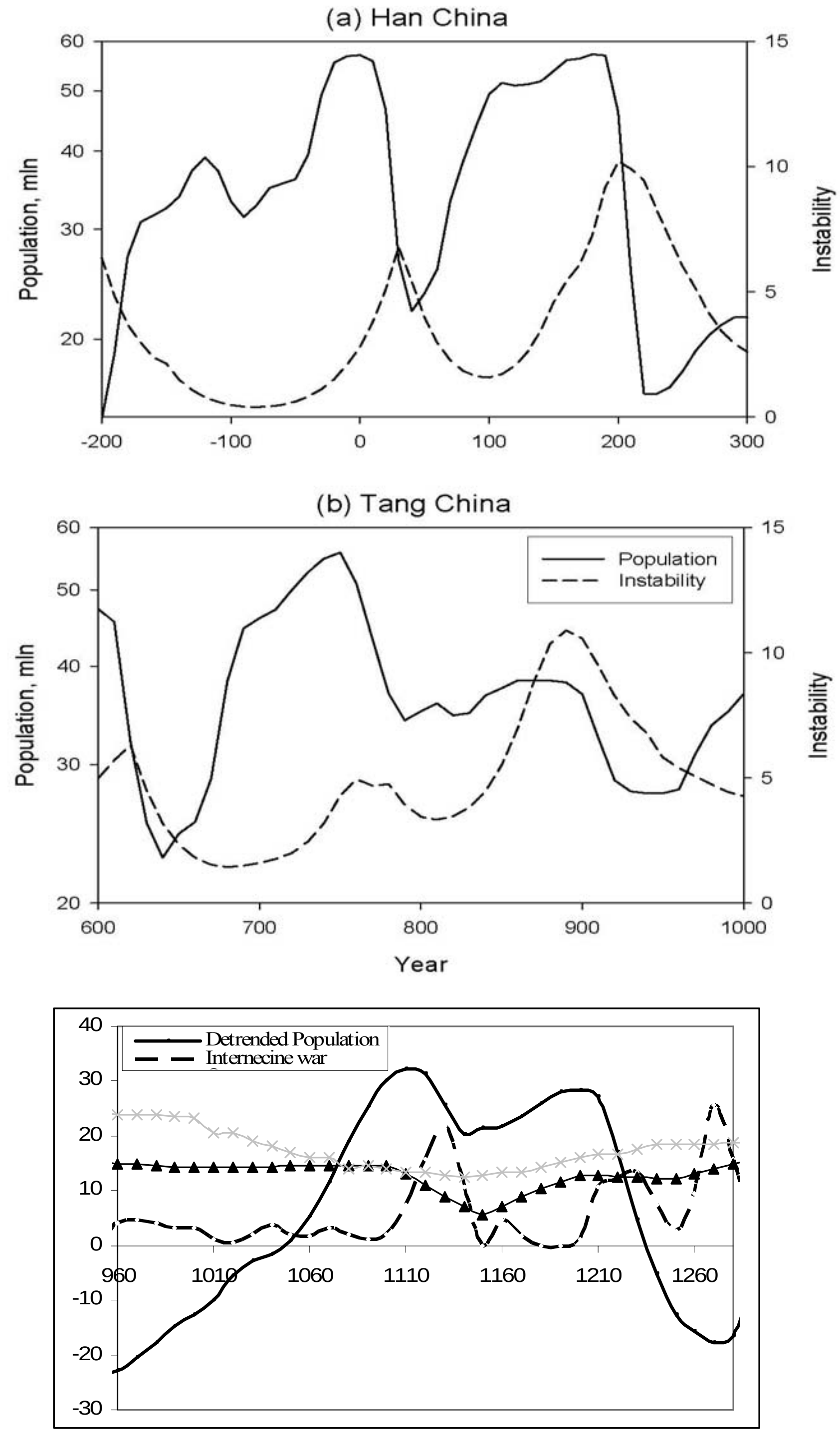


Figure 11: China's interactive dynamics of sociopolitical instability (broken curve for internecine wars, J.S. Lee 1931) and population (solid curve): figures (a) Han (200 BCE to 300 CE) and (b) Tang (600 to 1000 CE) from Turchin (2005a), with population detrended by bushels of grain; (c) Song Dynasty population (960 to 1279 CE) divided by successive trend values.

The third inset in Figure 11 falls within our period of study for China and adds our measures of city system parameters q (diamonds) and β (light line) each multiplied by 10 for easy visibility.

Fall in β (light line) in Figure 11(c), which indexes a fatter tail for large cities, caused for example by people leaving the largest cities to go to smaller ones, occurs with rise of population pressure. Decline in q (triangles) signals a change in city size curve shape that affects the smaller cities and works to their advantage in indexing disproportional change in size of smaller cities "up" into the thicker tail of the city size distribution: this occurs in 11(c) just before the population peak. The trough of β co-occurs with the peak of the SPI crisis of sociopolitical violence. The trough of q occurs once the SPI crisis has ended, followed by rising q with renewed growth of population pressure, p, then leveling of q before the next population peak. As in the endogenous dynamic of sustained fluctuation modeled by Turchin's equations (1) and (2), upward p leads upward SPI by a generation and downward SPI leads upward p by a generation.

A 4-variable dynamic including q and β, however, is not that simple. Two-equation time-lagged regressions with constants fitted to each term behave similarly for China and Europe, 925-1970, as shown in equations (3) and (4), except that the SPI index affects q without a time lag. Sociopolitical instability tends to have an immediate effect on the relation between smaller and larger cities that affects q but not β:

$$\beta_t <\approx\approx -q_{t-1} + q_{t-1}\beta_{t-1} \quad \text{(overall } R^2 \sim .79\text{, China} \sim 0.75\text{, Europe} \sim 0.69) \qquad (3)$$

$$q_t <\approx\approx -\beta_{t-1} + q_{t-1}\beta_{t-1} - SPI_t \quad \text{(overall } R^2 \sim .57\text{, China} \sim 0.54\text{, Europe} \sim 0.66) \qquad (4)$$

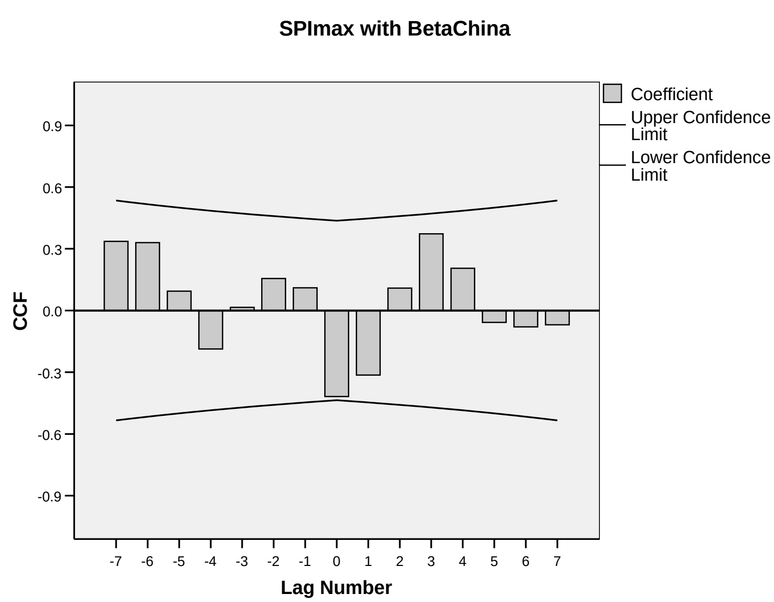
Further, without the effect of SPI, these two equations, unlike (1) and (2), would predict positive feedback between β and q that would result in either a convergent or a divergent time series.[14] It is only the SPI index, given the locations of SPI events (which we also estimated for Europe from historical data) with the Turchin dynamics that acts as an external shock and that makes the predicted time series oscillatory, and often clocked with Turchin's endogenous dynamic (modeled by equations 1 and 2). Significantly, then, there is support for the idea that it is the conflict events within Turchin's endogenous dynamic that drive q in the city dynamic, which in turn drives β (equation 3) in that dynamic. Figure 11 the case for Song China, where q and β are more stable over time than population pressure and SPI, but their periods of collapse are affected

[14] A two-equation reciprocal time-lag model such as equation s(1) and (2) produces fluctuations if the signs of the right hand elements are opposite, but convergence or divergence if they are the same. This can be verified in difference equations using initial values that generate a full time series.

synchronically with generational time lag according to the dynamics of Turchin's model of agrarian empires.

Also significant, the three agrarian empire periods (Han, Tang and Song) in Figure 11 are separated by periods of external wars and instabilities that interrupt the endogenous dynamics between population pressure and SPI fluctuations. Each period resumes fluctuation modeled by the two-equation time-lagged dynamics of *P* and *SPI*, which tends to obtain only when major external disturbances are absent. The external wars in Lee's (1931) analysis show up as large SPI fluctuations between dynasties (usually leading to their termination and a later successor empire), while internal SPI fluctuations occur within the periods of relatively more endogeneity. These various cycles couple to form the larger 800-year cycles of multiple successive empires, observed by Lee, marked by the most violent transitions. Overall, there is support in the Chinese and European examples for the aspects of hypothesis H6 initially designated 6a, 6b, and 6c.

Further evidence for the coupling of urban system dynamics with population/instability (structural demography) historical dynamics is presented in Figure 12. Much as SPI leads population declines in historical dynamics, the top graph in the figure shows that SPI is synchronously correlated with low β (reduced urban hierarchy) in city distributions for China. β recovers following peaks in SPI, just as population does. The bottom graph in Figure 12 shows a 50 year lag between a high q/β ratio and rise in SPI, much as population growth relative to resources predicts rise in SPI.



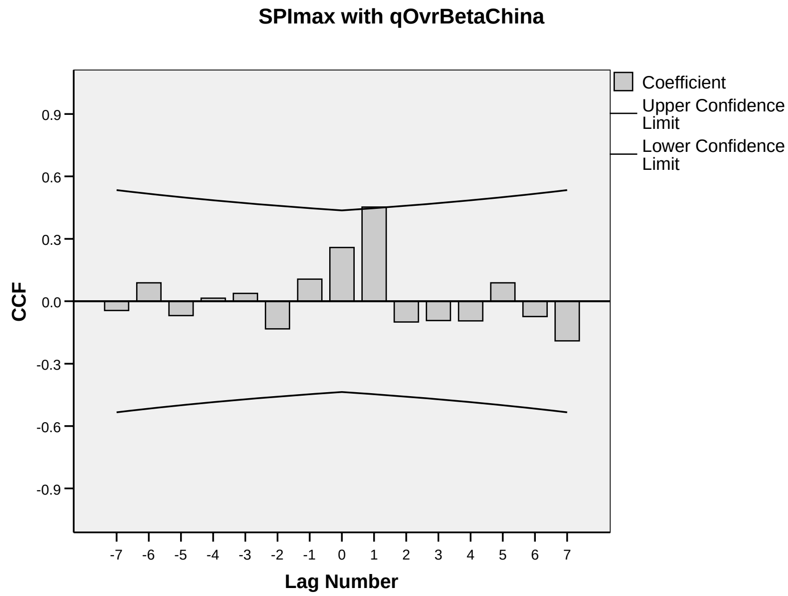


Figure 12: Cross-correlations of q and β and sociopolitical instability (SPI)

PART VI: CONCLUSIONS

This study and those that preceded it began as experiments in building from two sets of sources, one in quantitative history and the work of Goldstone (1991), Nefedov (1999), Turchin (2003), and Spufford (2002) and the other in more meaningful mathematical and measurement concepts that incorporate city size distributions as an object of study. The development of unbiased estimates of variations in city size distributions using maximal likelihood (MLE) allowed a level of precision and accuracy even with small samples that led to useful findings in this study that with MLE are likely to be reliable.

By focusing on the 75 largest cities over a series of time periods that go back to antiquity – closely enough spaced to get the quantitative variations of the full cycles of city system oscillations – Chandler (1987) made available a full run of data for studying how city system evolutions couple with agrarian sociopolitical dynamics. Initially, this will not be equally possible in every region at every time period, but only where the density of cities is sufficient for quantitative study. By focusing on Eurasia and its major regions, including China, we availed ourselves of some of the richest pockets of Chandler's comparative data on cities, especially for the early period of globalization, where China had the largest number of large cities. In refining the present model for comparison with other regions we will consider whether to adjust for

Chandler's possible underestimation of walled city populations for China, but the biasing assumption he made for China's walled cities (Appendix A) does not carry over to other regions.

We did find strong evidence of historical periods of rise and fall in the city systems of different regions, and time lagged effects of changes in city size distributions in one region on other regions. These are weak and slow from Mid-Asia to China, and strong and fast from China to Europe, which makes sense in terms of the Silk Roads trade. This provides additional evidence of synchronies missing from Chase-Dunn, Niemeyer, Alvarez, Inoue, Lawrence and Carlson (2006) and the studies of Eurasian synchrony. Many of the correlations, however, are time-lagged rather than temporally synchronous. The effects run in the directions suggested by Modelski and Thompson (1996).

We are reasonably confident in concluding that the Pareto I and II (*q*-exponential) measures of city hierarchies through time, especially when used in combination, can provide a measurement paradigm of standardized methods and tests of replication in historical comparisons. The attractive features of the *q*-measure gained added benefit from the precision of our measures with the use of MLE method.

Richness of supporting data would logically take us next to Middle Asia, its subregions, and the larger world from 700 CE that embraced the rise of Islam, the Mongol use of the Silk Roads and development of new towns and cities on those routes to link China and the rest of Middle Asia into a global system. Such a study, modeled on this one, would include the role of the Indic subcontinent, and that of the Mongols (Barfield 1989, Boyle 1977) in trade and conquest, the Arab colonization of North Africa and Spain, and the feeding of urban developments in the Mediterranean, Russia, and Europe.

When we compare our results, measurements, and mathematical models to those of the structural demography or secular cycles studies of Goldstone (1991), Nefedov (1999), and Turchin (2003, 2005), we find several novelties that separate our findings from that of the standard Lotka-Volterra oscillation model for historical fluctuations. Turchin (2003, 2004), for example, argues the Lotka-Voltera dynamic works optimally when one of the interactive variables (say, population/resource ratio measure of scarcity and sociopolitical violence) is offset by ¼ cycle. Our cycle of city-size oscillations might be two to four times as long as Turchin's secular cycles (J.S. Lee divides his 800 year periods into two periods of 400, seeming to do with an early growth of early forms of "empire" in a region, then a time of turbulence in the second period; then a new cycle of empire). It is possible that the city cycle operates at one or both th4se time-scales, and at the spatial scales of larger alternating civilizational networks of states and forms of empire. Long city-size system oscillations of ca. 800 years would not be offset by a ¼ cycle but by $\frac{1}{8}$th of a cycle, which is a long period of instability (vulnerable to conquest from the outside following internal instabilities). From our perspective, however, sociopolitical instability is not smoothly cyclical but episodic. Rebellions, insurrections, and all sorts of protest are events that mobilize people in a given time and generation, and that impacts that, when repeated frequently, have massive effects. We see this in long-term correlations with SPI, such as internecine wars in China.

We have been able to discern some of the effects of trade fluctuations (if not trade network structure) in these models. The monetary liquidity variable for China, in one of our tests, showed the effect of a trade-related Silk Roads variable on q. We think it possible to reconstruct trade routes as historical time series, and to do ordinal ranking of trade volumes on these routes. We think that these have strong effects, along with disruptive conflicts and political or empire boundaries, on the economies of individual cities and regions, and that these variables could be shown to have dynamical interactions within the context of secular cycle and urban systems rise and fall.

We have not done forward-looking prediction, but with MLE expect to be able to do better estimation, and possibly to correct biases in a reconstruction of Chandler's China dataset. Our hypotheses must remain so, as well supported and not yet contradicted. Some of patterns we to see in our data as concerns globalizing modernization are consistent with prior knowledge and others are startling. To be expected are the developmental trends of scale – larger global cities, larger total urban population, and larger total population. We can also now investigate whether, with time, the crossover to power-law parameter (Pareto II "scale" or σ) moves further down the tail, so that more and more of the city distribution becomes power law, and more consistent with much of the previous work on power-law scaling.

What is startling is that there are some long-wave oscillations in q that are very long. Hopefully, a long-term trend and contemporary structure of Zipfian city distributions is an indicator of stability, but even the 20th century data indicate that instabilities are still very much present and thus likely to rest on historical contingencies (somewhat like the occurrence of a next earthquake larger than any seen in *x* years prior), and very much open to the effects of warfare and internal conflicts that are likely to be affected by population growth, and as opposed to stabilization of trade benefiting per-capita-resources ratios.

Our results allow us to consider the edge-of-chaos metaphor of complexity with respect of q as a first but largely insufficient approximation to an explanation for what we see historically. It is a truism to say that complexity, life, history, and complex systems generally stand somewhere between rigidity at one pole, which might be exemplified by $q>1.7$, and on the other, an exponential random distribution ($q\approx1$) of city sizes, or the heightened unpredictability of chaos ($0<q\ll1$).[15] But we do not see support for *equilibrium* on the "edge of chaos" in these data. The historical q-periods of China and other regions tend to cluster, somewhat like "edging on chaos," near an average of $q\approx1.5$, but they do so in terms of oscillations, not far from equilibrium, but not a stable equilibrium. From that average they may fall into decentralized chaos, here in the metaphoric sense (but with exceptionally low q), in which the power-law tails are absent (with larger cities seemingly crushed in size by internecine wars) and smaller cities are frequent relative to hubs, or rise into regimes are affected by massive external drains on the economy or political policies that seem to put q into abnormally rigid states (exceptionally high q). The directions of change in q are largely predictable as a function of the current-state variables (such as population/resource ratios and sociopolitical violence) in the historical dynamics models up to, but not yet including, the contemporary period. How to derive predictions for the contemporary era is not yet evident given the new configurations of industrial societies, but it is very probable that such predictions as do emerge for the present will contain processes operative in the past.

On the issue of the coupling of cycles, Turchin cycles seem to embed two leading polity cycles (Modelski and Thompson 1996) that average about 110 years. These are averages, and actual timings vary, but we give explanations elsewhere for why such these average cycle-lengths might tend to diminish by half as each embedded process tends to operate at successively smaller spatial scales. We hypothesize an embedding of dynamical processes that runs from

[15] Technically, the mathematics assigned to chaos is a deterministic departure from randomness in which a dynamic trajectory never settles down into equilibrium, and small differences in initial conditions lead to divergent trajectories. The link between empirical history and "edge of chaos" is typically done by simulation.

trading zone network sizes and city-size distributions that cycle roughly 200, 400, or 800 years, partly dependent on the severity of the declines.

APPENDIX A: Chandler's Chinese City Data.

Many Chinese historians question the estimates of Chinese city population densities given by Chandler (1987:7): "Chinese cities tend to have an especially low density because of the Chinese refusal to sleep below anyone, so their houses are of nearly all of just 1 story. Hence, inland Chinese cities had a density of only about 75 per hectare, and even in seaports or the imperial capital the density hardly exceeded 100." One such anonymous reviewer considered this an underestimate by half based on current knowledge.

To test the effects of these likely underestimates population, we examined the data from the first of our periods, as shown in Table A.3, after checking Chandler's (1987:417-451) text to note the basis for his estimates for each city. We adjusted the five city estimates affected by raising each by 50% (half the attributed error), recomputed the number of cities for each size category, and then compared the power-law coefficients for the original and the revised data as well changes in estimates of *q*. Changes were evident and significant in each case. They did not change the r^2 for goodness of fit, but for *q*, for example, the value change from 1.9 to 0.7. Similar tests can be made for other periods, but we may need independent evidence as to the extent to which the critique merits new density computations.

Possible errors of this sort in Chandler's estimation assumptions, then, could change our scaling results. These biases could change the patterns of variation in *q* seen in our results. Because Chandler uses the common assumptions throughout these historical periods up to 1950, however, it is possible that they may not affect our historical comparisons about relative changes in *q*, at least up through 1950. For 1962, 1968, and 1970 he has taken the city population data of Richard Forstall of Rand McNally and Co. To test the robustness of our results, however, would entail a project that builds on the Excel spreadsheet for the Chandler data as exemplified above. By consulting his book to pick out each variable used to make multiplicative estimates – such as size of urban area, number soldiers (e.g., times 6), number of streets, etc. – a spreadsheet of calculations with the multiples and their base numbers might be useful to improve Chandler's estimates using a Bayesian weighting by consistency.

Chandler (1987:6) also notes that "the large growth of suburbs outside city walls had not begun before 1850 except in the newly rising industrial conurbations of Britain." This might cause problems "since the presence of suburbs has been well documented in history with new walls built to enclose a population that had spread beyond the original walls" (Pasciuti and Chase-Dunn 2002:1). Chandler does include suburbs in his criteria for city boundaries, however, and in his estimates throughout the book.

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