Patterns of Matching and Systems with Internality

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ABSTRACT

In this work, we describe a theoretical model that predicts the existence of three patterns of subjects' behavior in an experimental chamber with concurrent schedules of reinforcements. Two of these patterns were found earlier by Baum (2002). The model is based on the assumption that an organism is a system with internality and that the alternatives are polarized for the subject: one of them plays the role of the positive pole and the other that of the negative pole. Polarization may be independent of the alternatives' richness with food. We demonstrate that the generalized matching law can be regarded as an empirical approximation of the theoretical equation, which connects the system's internal variable with the influence of the environment and probabilities of alternatives choosing.

The Generalized Matching Law (Baum, 1974) shows the relation between responses and reinforcements in the experiments with concurrent schedules of reinforcements:

$$\log\frac{N_2}{N_1} = c\log\frac{n_2}{n_1} + \log b\,, \qquad (1)$$

where N_1 and N_2 are numbers of responses and n_1 and n_2 are numbers of corresponding reinforcements in one experimental session. Parameters c and b are constant during the entire set of sessions in which schedule intervals vary from session to session for each key. It was found later (Baum et al., 1999) that when pigeons were exposed to a pair of concurrent VI schedules long enough, the behavior ratio deviated systematically from equation (1).

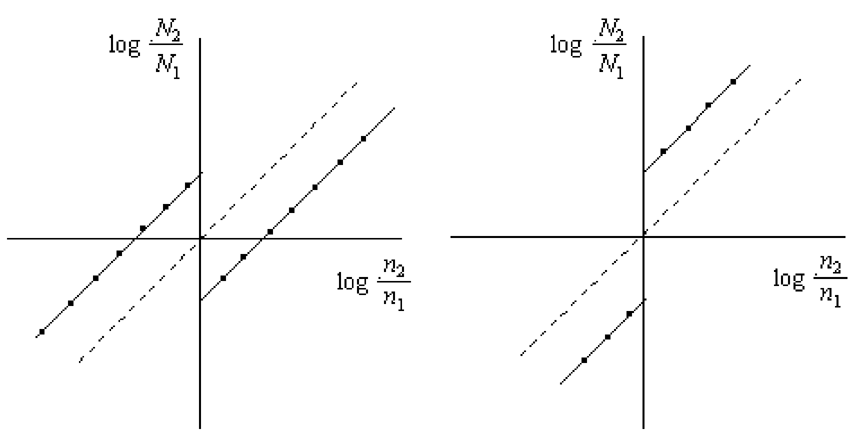

Fig.1. Baum's patterns.

An analysis of these deviations resulted in two-line behavior patterns (Baum, 2002, Fig.1). Baum noted that the left pattern corresponds to $c<1$, and the right one to $c>1$, if we approximate data with equation (1).

In this work, we describe a theoretical model, which predicts the existence of three patterns of behavior, two of which coincide with those in Fig.1. The following assumptions underlie the model: (a) an animal organism is a system with internality and (b) the alternatives are polarized: one plays the role of the positive pole and the other that of the negative pole.

The concept of bipolar choice was introduced by Osgood et al. (1957) and Kelly (1955) in the context of categorization. The most vivid example of bipolar choice is a human moral choice, in which one alternative plays the role of 'good' and the other that of 'bad', and their moral orientation may not coincide with their practical attractiveness for the subject. For example, in the choice between the truth and a lie, the alternative connected with a lie may be much more profitable for the subject than the one connected with the truth. Thus, the moral human choice has two aspects: deontological, in which the alternatives have bipolar orientation, and utilitarian where the alternatives have practical attractiveness. The idea of two aspects of choice was employed in the model of the subject with reflexion, which has explained several phenomena accompanying human choice (Lefebvre, 1980; 2001). In our previous work (Lefebvre, 2003), we demonstrated that the simplest version of the model of the subject with reflexion - a Reflexive Intentional Model of the Subject (RIMS) - allows us to predict the existence of three patterns of animal behavior in the experimental chamber. This finding underlies a hypothesis that choices made by mammals and birds also have two aspects: on the one hand, the alternatives have "material" attractiveness, on the other - one of them plays the role of the positive pole, and the other that of the negative. The alternatives' bipolar orientation represents their significance for the animal in a broader context than just richness in food within a short period of time. But from the point of view of the science of behavior, RIMS has an essential deficiency: it includes analogues of some introspective concepts - model of the self and intention.

In this work, we offer a more general model based only on the fundamental property of a system with internality - the existence of a formal relation between the system's internal variable, the environment's influence, and the probability with which the system chooses each pole.

Here is the outline of the further consideration. First, we introduce a concept of a system with internality. Then we formulate an Axiom of Repeated Choice, whose essence is that with a small increase of the internal variable value, a small probability appears that the system would reconsider its decision and repeat its choice if during a preliminary stage the negative pole was chosen. This axiom is the key assumption underlying the entire theoretical construction. It allows us to write a differential equation whose solution is a Choice Function, which describes probabilistic behavior of a system with internality in a situation of a bipolar choice. Next, we obtain a general equation, which reflects relation between variables. Then by assuming that the system keeps the values of the variables within the limits set by the general equation (under conditions that the value of the internal variable is constant), we formally deduce three patterns of behavior. After that, we construct a hypothetical cognitive mechanism that connects the frequencies of reinforcements and responses with the state of the subject, in which it is 'ready' to choose alternatives with the probabilities given by the Choice Function.

A System with Internality

The central idea in the science of behavior, if we try to represent it with the help of mathematics, can be reduced to the description of an organism by a simple function:

$$B = \varphi(z), \tag{2}$$

where B corresponds to the organism's behavior, and z is the external influence. In the early 1930s, it has been already realized that this idea reflects only a very simplified view of an organism (Tolman, 1932). The behavior of well-developed organisms depends not only on the external world, but on the inner "environment" as well. The ways of processing information by an organism are very complex, and the inner environment can be considered as a special factor, to certain extent independent of the external world. Let S be a variable corresponding to this internal factor, then the organism's behavior is given by the following function of two variables:

$$B = \Psi(z,S).$$

Considering S to be the main variable and z a parameter we can represent this function as

$$B = \Psi(z,S) = \Phi_z(S). \tag{3}$$

Function (3) can be specified for the bipolar choice in which one alternative plays the role of the positive pole and the other that of the negative pole. Let us connect variable B with the probability of choosing the positive pole (X), and parameter z with the probability of the environmental 'push' toward the positive pole (x_1). We will write (3) as

$$X(S) = \Phi_{x_1}(S), \tag{4}$$

where $X(S)$ is a differentiable function, $0<X<1$, $0<x_1<1$, and $S\geq 0$. Let

$$X(0) = x_1.$$

This condition means that if S=0, the probability of choosing the positive alternative is equal to the probability of the environment's push toward it.

The Axiom of Repeated Choice

In order to specify function (4), we introduce an assumption about a procedure of choice when an internal variable increases for ΔS. The probability with which the system chooses the positive pole when the internal variable is equal to S will be designated as X_S. The following axiom makes the distinction between the positive and negative alternatives operational.

> When the internal variable grows from S to $S+\Delta S$ ($0<\Delta S<1$; ΔS is considered small) and x_1 does not change, the procedure of choice is as follows. First, the system makes a *preliminary* choice with the probability of choosing the positive pole equal to X_S. If the positive alternative is chosen, the system realizes its choice. If the negative alternative is chosen, then, with a small probability P equal to ΔS, the system *cancels* its choice and *repeats* the procedure of choice (with the probability of choosing the positive alternative equal to X_S). The result of the repeated choice is realized no matter which alternative is chosen. (Lefebvre, 2004).

It follows from the Axiom of Repeated Choice that for the values of the internal variable equal to S and $S+\Delta S$, the following trees correspond to the choices (Fig.2).

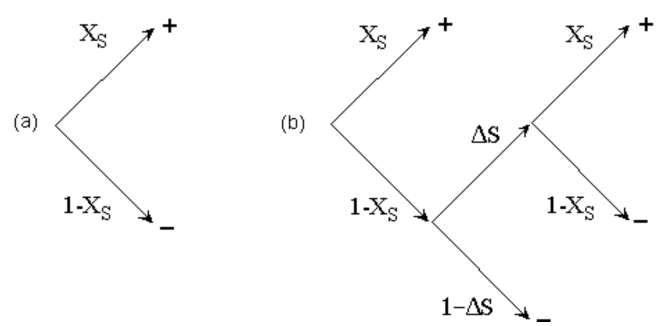

Fig.2. The trees of the choice of the alternative. (a) For the value of the internal variable equal to S. (b) For the value of the internal variable equal to $S+\Delta S$.

It follows from tree (b) that

$$X_{S+\Delta S} = X_S + (1 - X_S)\Delta S X_S \,. \tag{5}$$

Now we will search for such a differentiable function $X(S)$ (where $X(0)=x_1$) that can be represented as

$$X(S + \Delta S) = X(S) + (1 - X(S))X(S)\Delta S + o(\Delta S), \tag{6}$$

where $\frac{o(\Delta S)}{\Delta S} \to 0$, if $\Delta S \to 0$. Expression (6) can be rewritten as

$$\frac{\Delta X(S)}{\Delta S} = (1 - X(S))X(S) + \frac{o(\Delta S)}{\Delta S} \,.$$

For the limit at $\Delta S \rightarrow 0$, we obtain the following differential equation:

$$\frac{dX(S)}{dS} = (1 - X(S))X(S). \qquad (7)$$

By solving (7) under condition $X(0) = x_1$, we find that

$$X = \Phi_{x_1}(S) = \frac{x_1}{x_1 + (1 - x_1)e^{-S}}. \qquad (8)$$

Function (8) corresponds to (3) for bipolar choice. This function is the Choice Function.

There is an inverse function for (8):

$$S = \Phi_{x_1}^{-1}(X),$$

which can be found by transforming (8):

$$S = \Phi_{x_1}^{-1}(X) = \ln\frac{1 - x_1}{x_1} - \ln\frac{1 - X}{X}, \qquad (9)$$

where $\ln a = \log_e a$.

Equation (9) can be rewritten as

$$S - \ln\frac{1 - x_1}{x_1} + \ln\frac{1 - X}{X} = 0 \qquad (10)$$

or

$$W(x_1, X, S) = 0. \qquad (11)$$

Function (11) describes the connection between variables x_1, X, and S. If a system maintains relation (11), its choice is described by function (8).

Relations between frequencies and probabilities

For the sake of convenience of the consequent analysis we rewrite (9) as follows:

$$\ln\frac{1 - X}{X} = \ln\frac{1 - x_1}{x_1} - S. \qquad (12)$$

We introduce now variables p and q:

$$p = \frac{N_1}{N_1 + N_2} \quad \text{and} \quad q = \frac{n_1}{n_1 + n_2}, \qquad (13)$$

where N_1 and n_1 correspond to the positive pole, and N_2, and n_2 to the negative pole. Let $X = p$ and $x_1 = q$, then we can rewrite (12) as

$$\ln\frac{N_2}{N_1} = \ln\frac{n_2}{n_1} - S\,. \tag{14}$$

There is a formal similarity between (14) and (1), if we assume that $S = -\ln b$, $c = 1$, and the logarithm base in (1) is e.

It is important to keep in mind that the values of X and x_1 were interpreted as *probabilities*, and that variables p and q have the meaning of *frequencies*. We will return later to this difference, and for the time being we will assume that frequencies are equal to probabilities.

Prediction of behavioral patterns

Suppose that the value of S is fixed for each subject on the entire set of sessions. We will call an alternative *richer with food*, if a subject chooses the line of behavior such that this alternative is reinforced more often. Consider now three possible relations between the alternative's 'richness' and its polarity.

A. One alternative plays the role of the positive pole independently from being more or less rich in a given session.

B. In each session, the alternative that is less rich, plays the role of the positive pole.

C. In each session, the alternative that is *richer* plays the role of the positive pole.

Independently from polarization, one alternative will be called 'right', and the other 'left'. Let K_1 and K_2 be the numbers of "pecks" to the right and left keys, and k_1 and k_2 be the numbers of reinforcements from the right and left alternatives. With the help of (14) we construct graphs which connect $\ln\frac{K_2}{K_1}$ and $\ln\frac{k_2}{k_1}$ for each case.

(A) Let the right alternative play the role of the positive pole in all sessions. Then $K_1=N_1$, $K_2=N_2$, $k_1=n_1$, $k_2=n_2$, and equation (14) can be represented as

$$\ln\frac{K_2}{K_1} = \ln\frac{k_2}{k_1} - S\,. \tag{15}$$

If the left alternative plays the role of the positive pole in all sessions, $K_1=N_2$, $K_2=N_1$, $k_1=n_2$, $k_2=n_1$, and equation (14) is:

$$\ln\frac{K_1}{K_2} = \ln\frac{k_1}{k_2} - S$$

or

$$\ln\frac{K_2}{K_1} = \ln\frac{k_2}{k_1} + S\,. \tag{16}$$

The graphs in Fig.3 correspond to equations (15) and (16):

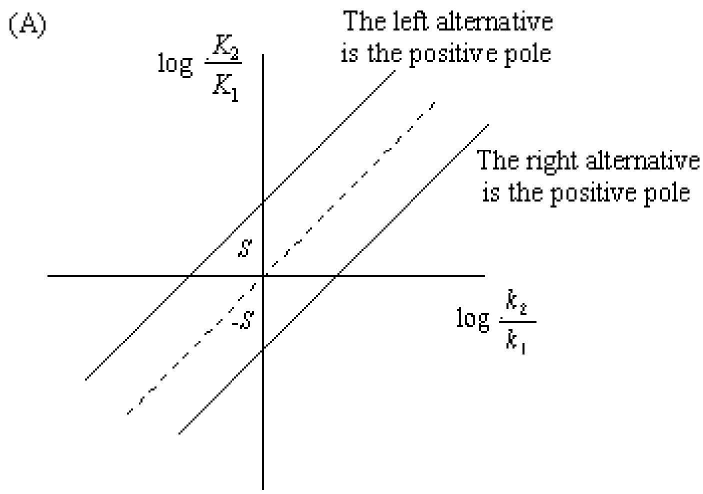

Fig.3. An alternative plays the role of the positive pole independently from being more or less rich in a given section.

(B) The right-hand side of the horizontal axis corresponds to $k_2>k_1$, and the left-hand side to $k_2<k_1$. When $k_2>k_1$, the right alternative plays the role of the positive pole, and when $k_2<k_1$, the left one does. Therefore, case B corresponds to the following graph (Fig.4):

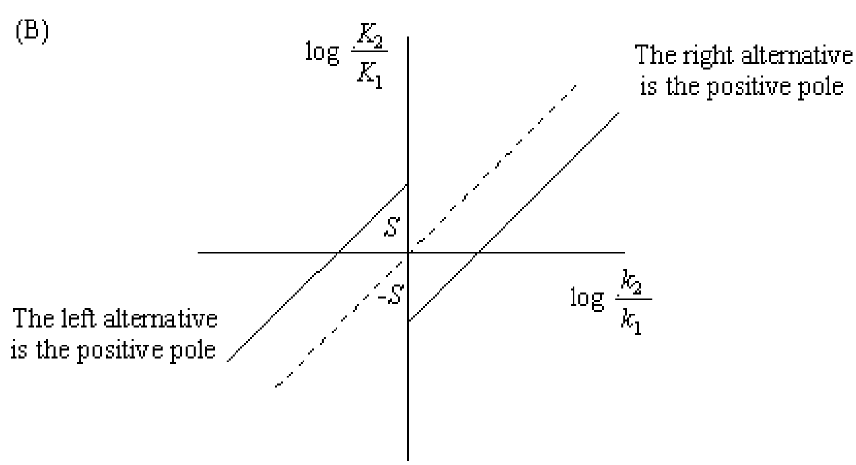

Fig. 4. In each session, the alternative that is less rich plays the role of the positive pole.

(C) When $k_2>k_1$, the left alternative plays the role of the positive pole, and when $k_2<k_1$, the right one does it (Fig.5):

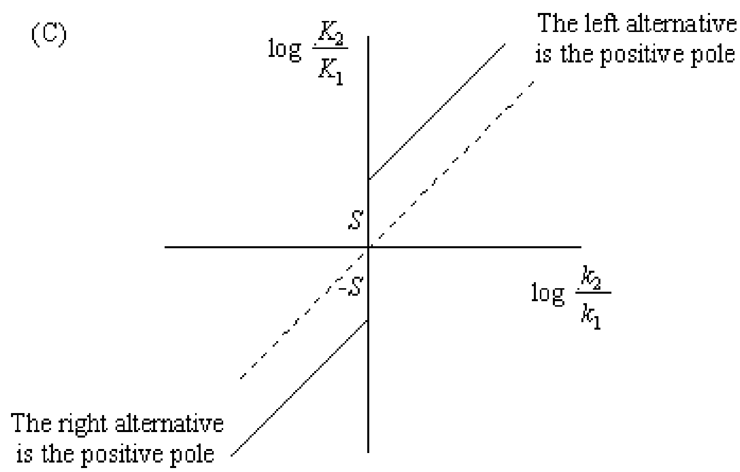

Fig. 5. In each session, the alternative that is richer plays the role of the positive pole.

In cases B and C, the graph discontinues at $k_2=k_1$. We can see that two Baum's patterns (see Fig.1) correspond to the graphs in Figures 4 and 5. The graph in Fig.3 represents an independent pattern that cannot be reduced to the two others.

On what does the alternatives' polarization depend?

Which factors determine positive-negative orientation of the alternatives? An analysis of the available information allows us to suggest some hypotheses.

Pattern A reveals itself in those experiments in which the alternatives have significant differences not related to the rate of reinforcements. For example, one alternative is connected with VI, and the second one with VR. In this case, the VR alternative plays the role of the positive pole, and the VI alternative that of the negative pole (see experimental data in Baum, 1974, and Baum & Aparicio, 1999).

Pattern B is realized under conditions that the alternatives differ only by the rate of reinforcements, and the alternative which is less reinforced plays the role of the positive pole, and the one which is more reinforced plays the role of the negative pole.

Pattern C is observed in the experiments in which there is 'punishment' or a factor making it difficult to reach the alternatives. Under this condition, the alternative which is more reinforced plays the role of the positive pole, and the one which is less reinforced that of the negative pole; for example, the experiment by Aparicio (2001), in which the alternatives were separated by a barrier.

Pattern B poses a question: why does the alternative that is less useful for the subject play the role of the positive rather than the negative pole? Before answering this question, let us note that an assumption about mammals and birds ability to polarize alternatives suggests a hypothesis that human ability to code one alternative as 'good' and the other as 'bad' is not a purely social phenomenon but appears as a result of long biological evolution. In a context of opposition between 'good' and 'bad', the alternative which is more beneficial from the practical point of view, may be related to the negative token, for example, 'dirty money'. We may speculate that a similar mechanism of denigration works in animals as well, and as a result the alternative that is more useful at this moment in some cases acquires the role of the negative pole, and the less useful alternative becomes positive. Pattern B may reflect such cases.

Frequencies as traces of probabilities regulation

Variable X and x_1, which characterize system with internality have the meaning of probabilities and can refer only to an *instant* state of the system. Using the quantum-mechanical metaphor, we can say that they correspond to a mixed state of a system in a given moment. On the other hand, variables $p = N_1/(N_1 + N_2)$ and $q = n_1/(n_1 + n_2)$ describe not an instant state but the entire history of the subject's behavior in the course of one session. In order to explain the relation between frequencies and probabilities, which set with the equations $X=p$ and $x_1=q$, we will consider a hypothetical mechanism of information processing by the subject's cognitive system.

Consider one session. A sequence of white and black circles corresponds to the subject's responses to the positive and negative alternatives. A sequence of white and black triangles corresponds to reinforcements, The triangle color depends on the color of the alternative from which the reinforcement is received: white designates the positive alternative, and black designates the negative alternative. The circles without triangles above are the responses without reinforcement. (In the real experiments, reinforcements appear more rare than in Fig.6).

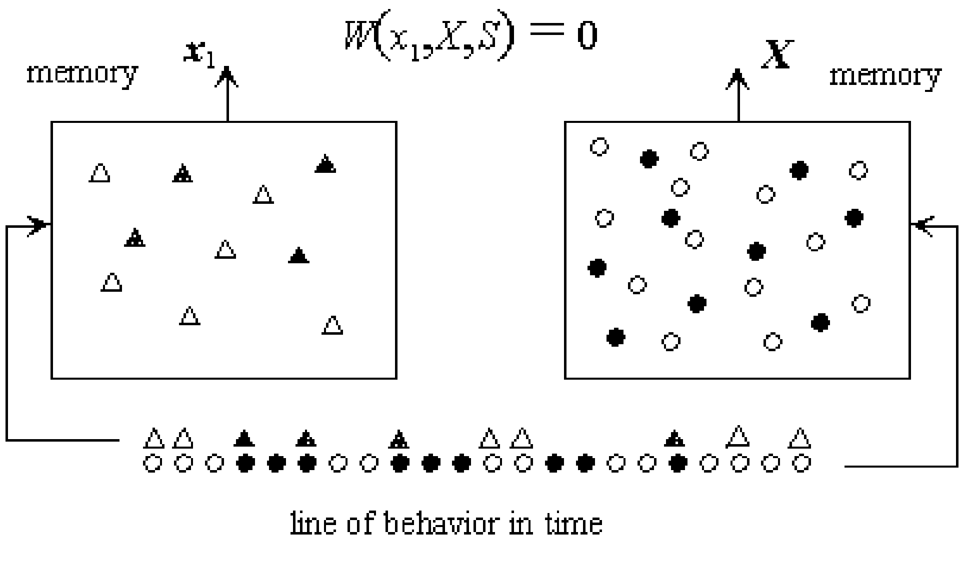

Fig. 6. A hypothetical scheme of information processing. The triangles correspond to the reinforcements: the whites from the positive alternative, the blacks from the negative one. The circles correspond to the responses: white to the positive and black to the negative alternatives. For fixed S, the system maintains relation $W(x_1, X, S) = 0$ in each session, and as a result the value of function (8) is being computed.

The two “boxes” correspond to two sections of the memory. The traces of reinforcements are saved in the left section, and the traces of responses are saved in the right one. We assume that system *remember neither* the *order* of responses and reinforcements *nor* the *correspondence* between them. For example, the system cannot determine which white circle corresponds to a particular white triangle. Therefore, a ‘lump’ of white and black triangles is located in the left section, and a ‘lump’ of white and black circles is in the right one (see Fig.6).

Let randomized flashes of “recollection” appear in a cognitive system. Each reminiscence corresponds either to a flash of one circle or of one triangle. Flashes of triangles and circles do not depend on each other. White triangles flash with the probability x_1, white circles flash with the probability X; probability x_1 is a characteristic of an input, and probability X is that of an output of the system with internality.

The value of S is constant on the set of all sessions. In each session, by controlling the value of X, the system maintains relation $W(x_1, X, S) = 0$ and, in this way, computes the value of function (8), with fixed S and changing x_1.

The subject in the experimental chamber receives very scarce reinforcements, that is, x_1 changes rarely. Thus, when x_1 changes, the organism can add to the right box either white circles or black ones until equation (11) is restored. We see that regulation in this system is such that the *frequencies* of quasi-random sequences of reinforcements and responses automatically turn into *probabilities* of memory “flashes,” since the number of white circles in the memory box and in the behavior line equal to each other; the same is true for black circles and for white and black triangles (see Fig.6).

Function of the uncertain state

In accordance with our hypothesis, the relation between reinforcements and responses given by (14), is a consequence of the process of holding equation (11) by the system with internality. As a result the subject is in the uncertain state corresponding to the *probability X*. What is the functional meaning of the uncertain state? An answer can be as follows: “collapse” of this state with the probability of choosing the positive pole equal to X is an *instantaneous choice* of the alternative in an important moment of an animal’s life. For example, let the positive pole correspond to the better protected alternative that is poorer in food supply. In a moment of sudden danger, the animal would choose the positive alternative with the probability X. Thus, continuous run of the animal between the food hoppers is a cognitive computational process which involves the entire organism (Lefebvre, 2003). The goal of this process is not only a search for food, but also the formation and support of an uncertain state for a possible instantaneous choice in the future.

How is an uncertain state connected to the procedure of repeated choice? Suppose there are small fluctuations of the value of the internal variable, and it can take on any value from the small interval $[M_1,M_2]$, where $M_1<M_2$. In the scheme given in Fig.6, the value of the internal variable is equal to the minimal value, that is, $S=M_1$. If in the moment of the instantaneous choice, the value of the internal variable is equal to $M_1+\Delta S$, then the system makes its preliminary choice with the probability $X(M_1)$. If the positive

alternative is chosen, the system realizes its choice, if the negative alternative is chosen, the system repeats its choice with the small probability ΔS.

Expenses and revenue

In the process of forming and maintaining an uncertain state, the alternatives play the role of special agencies to which the subject appeals for food. Since each appeal requires spending energy, we consider numbers N_1 and N_2 as the subject's *expenses*, and numbers n_1 and n_2 as his *revenue*. Consider expression:

$$\frac{n_1}{N_1} = e^{-S}\frac{n_2}{N_2}, \tag{17}$$

which follows from (14); the fractions

$$\frac{n_1}{N_1} \text{ and } \frac{n_2}{N_2}$$

may be interpreted as mean *payments* required by the subject for one appeal to the positive and negative agencies. It follows from (17) that

$$\frac{n_1}{N_1} \le \frac{n_2}{N_2}. \tag{18}$$

Therefore, it turned out that on average, the subject never requires larger payment from the positive agency for a single appeal, than it requires for the single appeal from the negative agency. Something similar can be observed in human behavior (Lefebvre, 2003). Everyone can bring an example, in which people agree to perform some work related to high values (as building a cathedral) for free or for a smaller reward, than they would require for a comparable task not connected with such values.

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