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Almost Connected Orders
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One of the standard axioms for semiorders states that no three-point chain is incomparable to a fourth point. We refer to asymmetric relations satisfying this axiom as 'almost connected orders' or 'ac-orders.' it turns out that any relation lying between two weak orders, one of which covers the other for inclusion, is an ac-order (albeit of a special kind). Every ac-order is bracketed in a natural way by two weak orders, one the maximum in the set of weak orders included in the ac-order, and the other minimal, but not necessarily the minimum, in the set of weak orders that include the ac-order. The family of ac-orders on a finite set with at least five elements is not well graded (in the sense of Doignon and Falmagne, 1997). However, such a family is both 'upgradable' and 'downgradable,' as every nonempty ac-order contains a pair whose deletion defines an ac-order on the same set, and for every ac-order which is not a chain, there is a pair whose addition gives an ac-order.