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The Sum of the Parts Can Violate the Whole

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It is not unusual for concerns from political science to be decentralized into disconnected parts; e.g., with different initiatives on a ballot. But the shifting majority phenomenon, where support all passed outcomes. At the other extreme, a number of issues may be bundled into a single bill; this may occur to avoid "shifting majority" difficulties or even to pass "pork" by combining it with desired alternatives. But, any analysis requires understanding what fraction of the voters support what portions of the bill. We develop a geometric approach to identify all possible profiles which support specified votes for separate initiatives or for a bundled bill. This disaggregation allows us to compute the likelihood of different scenarios describing how voters split over the alternatives, and to offer new interpretations for pairwise voting. The source of the problems, which is an unanticipated loss of available information, also explains a variety of other phenomenon such as Simpson's Paradox (a statistical paradox where the behavior of the "parts" disagrees with that of the "whole") and Arrow's theorem from social choice.