

*MBS 99-06*

Semigroup and chromatic Number of a (-Closed Family

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We investigate the structure of a family  $F$  of finite subsets of a set  $Y = \cup F$ , closed under union, from the standpoint of a particular semigroup of transformations acting upon  $F$ . The effect of a transformation on a set in  $F$ , if any, consists in adding or removing some minimal set, which results in forming some other set in  $F$ . In this framework, we show that a sensible parameter of 'dimension' of  $F$  can be defined, whose value is equal to the chromatic number of a distinguished graph associated with  $F$ . If the value of that chromatic number is  $n$ , then the set of all transformations can be partitioned into  $n$  classes, such that within each class, the transformations are partially ordered in a consistent manner. This construction leads to an appealing coordinate representation of the family  $F$ .