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Compatibility of gain-control and power law representations -- a János Aczél connection

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Detection probabilities in sensory psychology are sometimes represented by a gain control equation, such as P(x,y)=F[(u(x)-g(y))/h(x)]. Here, x and y are positive real numbers denoting ratio scale values of physical energies. P(x,y) is the probability of detecting stimulus x over background y, and u, g, h and F are real valued, continuous, strictly increasing functions. In some situations (e.g., in psychophysics), the researchers investigate empirically the detection phenomenon via the function μ satisfying A reasonable model to consider for the function μ is offered by the power law representation in which and K are non-constant functions of p. In this paper we study the consistency of these gain control and power law representations. The main result is that, under some background conditions, if the gain control and the power law representations jointly hold, then the detection probability P takes necessarily the form P(x,y)=F[(((((ln y(()/(ln x+())(]. The form of the function F is arbitrary. Our proof is based on the solutions of the functional equation

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