MS 98-14

Are social equivalences ever regular? Permutation and exact tests

John P. Boyd, Kai J. Jonas

A regular equivalence on a relation induces matrix blocks that are either 0-blocks or regular-blocks, where a regular-block contains at least one positive entry in each row and column. The authors devise both a permutation test and an exact statistical test that separates these two aspects of regular equivalence for square matrices. To test for the regular-block property, the natural test statistic is the number of rows and columns within each purported regular block that fail to meet the criteria of having at least one positive entry, while the permutation group is the group of all permutations that fix each regular-block as a whole (alternatively, within each sub-row), except for diagonal blocks, for which the diagonal entries are individually fixed. The exact test is derived by assuming that the number of zeros in each block is fixed and that each permutation of zeros is uniformly distributed. This implies that the probability of finding, say, k zeros in a given set of rows and columns follows the hypergeometric distribution. These results from the separate blocks are combined by convolution to give the distribution of k zero vectors in the matrix as a whole. These tests were applied to data sets from Sampson's Monastery, Wasserman and Faust's Countries Trade Networks, Krackhardt's High-tech Managers, and B.J. Cole's Dominance Hierarchies in Leptothorax Ants. In all four cases, the 0-blocks were very significant, having only a tiny fraction of permutations with fewer errors than was found in the data. With the regular blocks, however, there was no significant relation in the Countries data and a significant overall tendency in the other three data sets toward having more departures from regular 1-blocks in the data than in the permuted matrices!