Solutions to Three Functional Equations Arising from Different Ways of Measuring Utility János Aczél, R. Duncan Luce, Gy. Maksa

Utility of gains (losses) can be measured in four distinct ways: riskiness vs. risky choices and gains (losses) alone vs. the gain-loss trade-off. Conditions forcing these measures all to be the same lead to functional equations, three of which are: $\mathrm{F}-1[\mathrm{~F}(\mathrm{X})+\mathrm{F}(-\mathrm{Y}) \mathrm{Z}=\mathrm{F}-1[\mathrm{~F}(\mathrm{XZ})=\mathrm{F}(-\mathrm{YZ})] \mathrm{F}:]-\mathrm{k}, \mathrm{k}^{\prime}\left[(]-\mathrm{K}, \mathrm{K}^{\prime}\left[; \mathrm{k}, \mathrm{k}^{\prime}, \mathrm{K}, \mathrm{K}^{\prime}>0\right)\right.$ (i) $\mathrm{F}(\mathrm{X}-\mathrm{R})[1-\mathrm{F}(\mathrm{Y})]+\mathrm{F}(\mathrm{Y})=\mathrm{F}[\mathrm{F}-1(\mathrm{~F}(\mathrm{X})[1-\mathrm{F}(\mathrm{Y})]=\mathrm{F}(\mathrm{Y})-\mathrm{S}](\mathrm{F}:[0,1[([0,1[)$ (ii) F-1[F(X) + F(Y) $\mathrm{F}(\mathrm{X}) \mathrm{F}(\mathrm{Y})] \mathrm{Z}=\mathrm{F}-1[\mathrm{~F}(\mathrm{XZ})=\mathrm{F}[\mathrm{YP}(\mathrm{X}, \mathrm{Z})]-\mathrm{F}(\mathrm{XZ}) \mathrm{F}[\mathrm{YP}(\mathrm{X}, \mathrm{Z})]] \mathrm{F}:[0,1[([0,1[, \mathrm{P}:[0,1[\mathrm{x}[0,1]([0,1])$ (iii) We determine all strictly increasing, subjective al(and thus continuous) solutions of (i) and (ii) and all strictly increasing, subjective solutions of (iii) that are differentiable on ( 0,1 [ as are their inverses (thus, $\mathrm{f}^{\prime}(0$ on ]0,1[).

