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A Schur-Concave Characterization of Risk Aversion for Non-Expected Utility Preferences

S. H. Chew, M. H. Mao

For expected utility preferences, Rothschild and Stiglitz (Journal of Economic Theory, 1970) showed that aversion to mean-preserving increases in risk is equivalent to two additional definitions of risk aversion based respectively on the behavior of the decision maker's risk premium and on his demand for risky asset. This result has been extended in Machina (Econometrica, 1982a) for non-expected utility preferences which are Frechet differentiable with respect to the L-metric. He showed that aversion to mean-preserving increases in risk is equivalent to two strengthened definitions of risk aversion based on the decision maker's conditional risk premium as well as his conditional demand for risky asset. This paper further extends Machina's characterization of risk aversion to continuous non-expected utility preferences without imposing any differentiability requirement. The necessary and sufficient condition for risk aversion is derived in terms of the Schur-concavity of the preference functional when evaluated on finite lotteries with equal probabilities. The latter is characterized by its marginal-rate-of substitution between a high income state and a low income state being not less than unity. Correspondingly, the more risk averse the preference ordering, the greater is its marginal-rate-of-substitution between a high income state and a low income state.