

# Game Theory & Decision Theory

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# Savage as the foundation

Beliefs justify strategies

Is equilibrium relevant?

Prescriptive-descriptive approaches, but no signaling

Are randomizations subjective?

Purification, etc. [private randomization, as in poker]

Private information

Game tree  $\Rightarrow$  decision trees [perfect recall]

Or Markovian [stochastic game]

# Kohlberg-Mertens' program

Characterize "rational" behavior in a multi-person setting

Common knowledge of rationality [Aumann: correlated eq.?)

Strong belief in rationality [Battigalli-Siniscalchi]

Critiques:

They reject sufficiency of Nash eq., and require

Backward induction, forward induction

Invariance: Seq. eq. not invariant [but Nash is]

Axioms  $\Leftrightarrow$  Solution concept: Backward & Forward Induction and:

(a) ordinality, player-splitting, admissibility, (& continuity)

(b) no presentation effects: invariance, small worlds,  
decomposition

Mertens-stability, Metastability

Role of structure theorems. Refinements apply essentiality

# Backward induction

Is this a solved problem?

Reject implausible/non-credible threats:

expect future rationality

Subgame-perfect eq., sequential eq., (perfect-Bayes eq.)

Beliefs, sequential rationality (or weakly sequential)

Normal-form: proper  $\Rightarrow$  quasi-perfect  $\Rightarrow$  seq. eq. + admissible

# Forward induction

This has been an unsolved problem

Reject implausible beliefs

Examples: outside-option and signaling games

Ad hoc criteria: Intuitive criterion, divinity, etc.

Definition: beliefs confined to strategies optimal in equilibria with same outcome

Motivation stems from multiple eq. with same outcome

Existence  $\Leftarrow$  essentiality, or Mertens-stability

Invariance sufficient for 2-player games with generic payoffs/cheap talk

Entirely "decision theoretic"

Also true for a normal-form version

# Game theory in the social sciences

Must have complete theory of rational behavior to be useful

Even if considerable weight given to non-rational theories

Example: absence of presentation effects

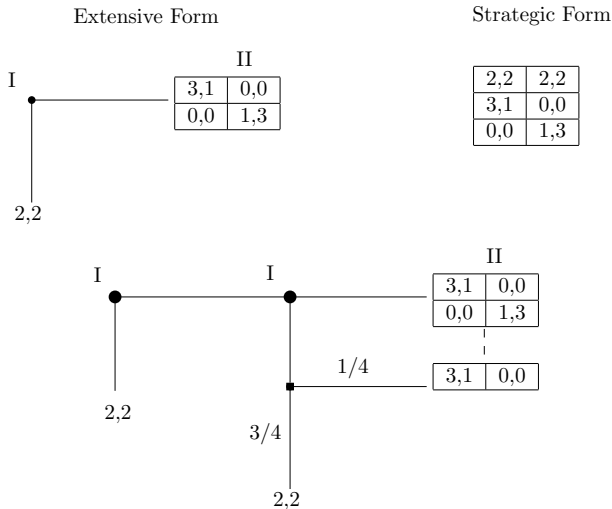
To get decision-theoretic properties, general versions of invariance axioms are required:

Independence of redundant strategies

Small Worlds

But the behavioral evidence is that most people violate invariance

# Basic Forward Induction



# Signaling Game

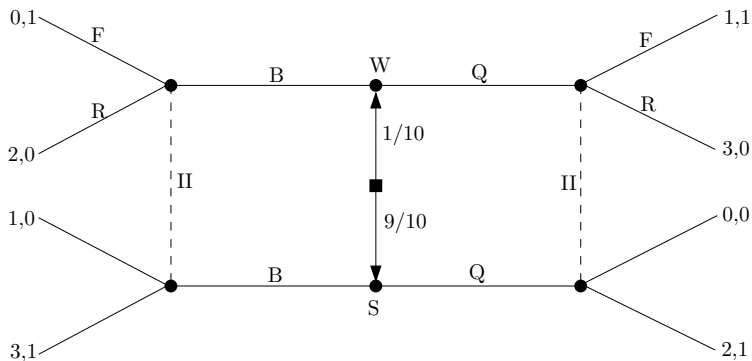


Figure: The Beer-Quiche game



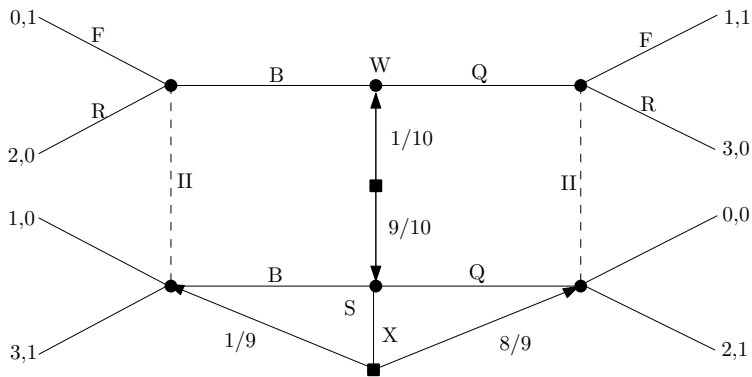
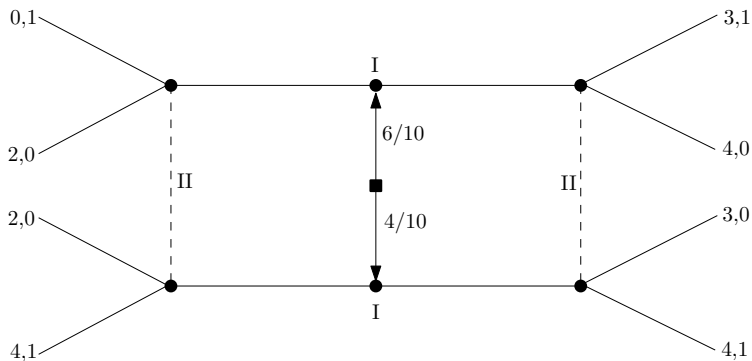


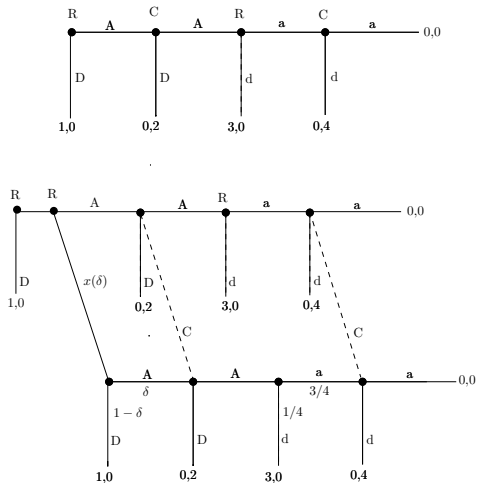
Figure: Invariance Applied to The Beer-Quiche game

# Spencian Signaling



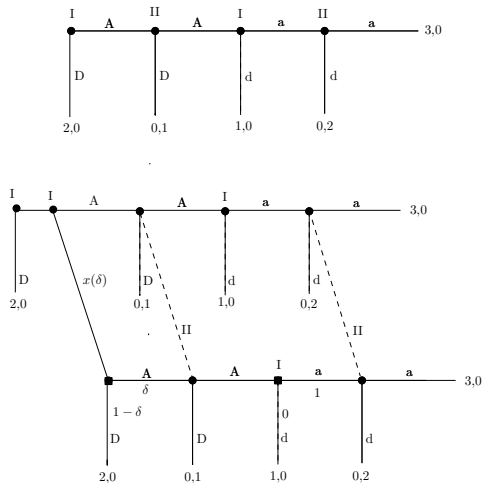
**Figure:** A signaling game with pooling and separating equilibria

# Centipede Game



**Figure:** Top: A game between players R and C. Bottom: The game modified so that player R can commit to the mixed strategy  $x(\delta)$  after rejecting  $D$ .

# Reny's Example



**Figure:** Top panel: Reny's example of a game between players I and II. Bottom panel: The game modified so that player I can choose the redundant strategy  $x(\delta)$  after rejecting  $D$ .