“Mathematical Economics: What’s Next”

Institute for Mathematical Behavioral Sciences

University of California, Irvine

How can mathematical economics help us understand the dynamic behavior of industries?

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May 12, 2006

Economics consists of models plural.

- Solow argues that economics is model building.*
  - "Ask a mainstream economist a question . . ., the response will be: suppose we model that situation and see what happens."

- Models come in families, but are seldom unified in any formal way.

- Reason: The economy is almost infinitely complex. Therefore to make any progress at all one must abstract from the situation in order to isolate the "one or two causal or conditioning factors, exclude everything else, and hope to understand how just these aspects of reality work and interact."

- Economists who work on a particular question do not have the concrete reality of a particular cell to focus on as a group of biologists might. Instead each constructs his or her little version of reality with a model and, not surprisingly given the necessary abstraction, each one's model is different.

*Robert M. Solow, "How did economics get that way & what way did it get? *Daedalus*, Fall 2005, 87-100.

Solow’s Negative Conclusion regarding Mathematical Economics

“If ‘formalist economics’ means anything, it must mean economic theory constructed more or less after the model of Euclid’s geometry. . . . The past fifty years have indeed seen formalist economics grow and prosper. But it has not grown very much. . . . To tell the truth, not many . . . pay any attention at all to formalist theory. Generally speaking, formalists write for one another.”

Implication (in my mind)

- Mathematical theory in economics, as employed by almost all economists, by and large is a set of tools that help economists analyze reliably and quickly the model's that they construct.

- My plan:
  - Present a model that I and several colleagues have been trying to understand.
  - Identify where theory has helped us.
  - Identify where theory has not helped us.
  - Briefly suggest how the rise of behavioral economics and computational economics reinforce the idea that mathematical theory is a collection of toolboxes, each of which relates to particular family of models.

The Example

Learning-by-Doing, Organizational Forgetting, and Industry Dynamics

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Learning-by-Doing

- A firm, as it produces a product, if there is learning-by-doing (LBD), learns to make it more and more cheaply. The out-of-pocket cost of each unit is a decreasing function of its serial number, i.e., of it cumulative experience.

- Empirical studies have found evidence that this is common:
  - Alchian (1963) (airframes);
  - Zimmerman (1982) (nuclear power plants);
  - Lieberman (1984) (chemical processing);
  - Irwin & Klenow (1994) (semiconductors);
  - Thompson (2001), Thornton & Thompson (2001) (Liberty Ships).

- Is LBD a source of market dominance?

- Theoretical literature (e.g., Cabral & Riordan 1994, Athey & Schmutzler 2001) focuses on dominance properties of pricing behavior: Does the leader charge a lower price than the follower?

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Organizational Forgetting

- Empirical work also suggests that organizations can "forget" the know-how developed through LBD:
  - Argote, Beckman, and Epple (1990) and Thompson (2003) (shipbuilding).
  - Benkard (2000, 2003) (wide-body jets).
  - Darr, Argote, and Epple (1995) (operation of pizza franchises).

- Organizational forgetting (OF) can arise due to:
  - Labor turnover.
  - Failure to institutionalize tacit knowledge gained from LBD $\Rightarrow$ inability to replicate know-how in future production periods and/or loss of know-how during periods of inactivity.

- Does OF undermine the economic power of LBD as a source of competitive advantage?

- We incorporate organizational forgetting into the Cabral and Riordan (1994) model.

What Do We Learn?

- We directly examine the industry dynamics implied by pricing behavior in Markov perfect equilibria.

- Dynamic competition with LBD and OF is akin to racing down an upward-moving escalator. A firm can move down the curve, but can as well drift back up.

- Organizational forgetting makes bidirectional movements through the state space possible. As a result it is a source of
  - aggressive pricing behavior;
  - market dominance;
  - multiple equilibria.

- Dominance properties are neither very informative nor particularly robust to organizational forgetting.

- Learning-by-doing and organizational forgetting are distinct economic forces.

Motivation I

- I want to teach a sensible story of dynamic competition when learning-by-doing exists, but the literature provided no persuasive theory beyond two or three period examples upon which backward induction can be applied.. Benkard (2004) is the lone convincing exception, but his analysis is only an instance: it gives no sense of the varieties of behaviors that may be possible in equilibrium.

- Some complaints
  - The theories of repeated games and stochastic games (including Markov games) is not much help. These theories focus on existence and the range of outcomes possible (e.g., folk theorems). The arguments used often involve carefully crafted equilibria that could not conceivably arise in a market.
  - Dynamic games often have multiple equilibria. A concern about much of the applied theory on dynamic games is that it carefully handcrafts a single equilibrium, but is silent about other possibilities even as it argues that there is, in many cases, great multiplicity.

Motivation II

- A model that represents more than a single, simple driver of dynamic behavior is likely to be too complicated to derive equilibria analytically. This may force an impatient investigator either to use computational methods or to trivialize the model.

- We seek to automate the computation of dynamic equilibria in a manner that maximizes the possibilities of identifying multiple equilibria. Existence is not enough. We need tools for understanding the dynamic performance of mechanisms, whether they be market mechanisms with implicit rules or designed mechanisms with explicit rules.

Research Overview

- Develop dynamic model of price competition in a differentiated product duopoly using the Markov-perfect equilibrium (MPE) framework of Ericson & Pakes (1995).

- Systematically compute the MPE for a large set of parameter values, checking for possibility of multiple equilibria by using a homotopy technique.

- Characterize and classify MPE based on properties of equilibrium policy function.

- Compute transitory and limiting distributions to characterize short-run and long-run industry dynamics.

- Develop intuition as to why multiple equilibria exist with differing degrees of aggressiveness.

Model, I

- Dynamic, stochastic game, with discrete time and infinite horizon:
  - $\beta \in (0,1)$ = discount factor.

- Two firms, each with a potentially different level of know-how $e_n \in \{1, ..., M\}$:
  - $(e_1,e_2) \in \{1, ..., M\} \times \{1, ..., M\}$ is the state of the industry.

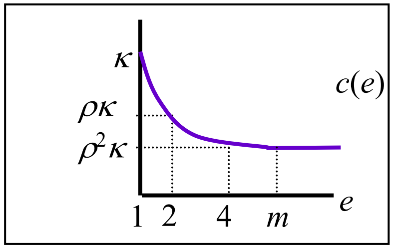
- Firms face a sequence of independent buyers, and in each period a buyer demands at most one unit of the good from one of the two firms.

- Timing:
  - State (e_1,e_2): firm's choose prices $\Rightarrow p_1(e_1,e_2), p_2(e_1,e_2)$.
  - One (or neither) firm makes a sale.
  - Learning/forgetting occurs.
  - Transition to new state (e'_1,e'_2).

- Restrict attention to Markov-perfect equilibria

Model, II

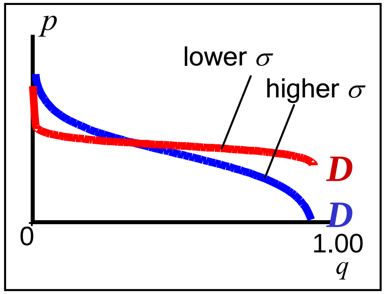
- Learning: marginal cost $c(e_n)$ decreases in e_n for $e_n \leq m$; constant for $e_n > m$.
  - m = "bottom" of learning curve.
  - κ = marginal cost at "top" of learning curve ($e_n = 1$).
  - ρ = slope of learning curve: for $e < m$, doubling e reduces c by 1 - ρ percent.



- Firms face a sequence of independent buyers, and in each period a buyer demands at most one unit of the good from one of the two firms. Demand is assumed to be logit. The probability that a given firm wins sale ($q_n = 1$) depends on current prices:

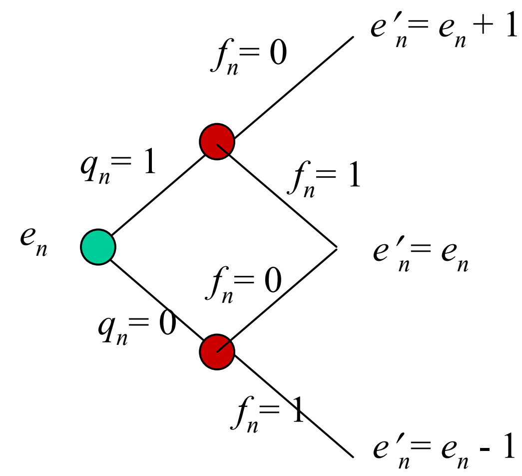
$$\Pr(q_1 = 1) = D_1(p_1, p_2) = \frac{\exp\left(\frac{v - p_1}{\sigma}\right)}{\exp\left(\frac{v_0 - p_0}{\sigma}\right) + \sum_{k=1}^{2} \exp\left(\frac{v - p_k}{\sigma}\right)}.$$

  - σ is product differentiation parameter: $\sigma \to 0 \Rightarrow$ no differentiation; $\sigma \to \infty \Rightarrow$ independent goods.
  - v = deterministic component of consumer utility for "inside goods."
  - v_0 = deterministic component of consumer utility for outside good, which is sold at its marginal cost c_0.



Model, III

- Law of motion: $e_n' = e_n + q_n - f_n$
  - e_n, e_n' = firm n's know-how in current and next period.
  - $q_n \in \{0,1\}$ = firm n's quantity sold in current period.
  - $f_n \in \{0,1\}$ = organizational forgetting.



- Organizational forgetting:
  - $\Pr(f_n = 1) = \delta(e_n)$, where $\delta(e_n) \in [0,1]$ and $\delta'(e_n) > 0$.
  - Probability of forgetting increasing in e_n is consistent with
    - experimental evidence on forgetting in industrial psychology literature.
    - decay function that is convex in time (also consistent with empirical evidence on forgetting from industrial psychology literature).
    - "capital stock" model of learning and forgetting used in empirical literature.
    - δ parameterizes rate of forgetting: higher δ higher rate of forgetting, and $\delta = 0 \Rightarrow$ no forgetting. $\delta(e) = 1 - (1-\delta)^e$.

- An interpretation of e and
  - Let e be the number of substantial, but tacit, techniques that are incorporated into the production process.
  - δ is the probability each period that a particular technique is forgotten. $(1 - \delta)^e$ is the probability that no techniques are forgotten.
  - $\delta(e)$ is the probability at least one technique is forgotten. It is increasing in concave in the firm's experience level.

Model, IV

- Key parameters in calculations:
  - State space: $M = 30$, with bottom of learning curve: $m = 15$.
  - Marginal cost at top of learning curve: $\kappa = 10$.
  - Product differentiation parameter in logit demand: $\sigma = 1$ (quite weak horizontal differentiation)
  - Attractiveness of outside alternative in logit demand (determines “size” of market for inside goods): $v_0 - c_0 = 0$ (outside alternative on par with inside goods at top of learning curve)
  - Discount factor: $\beta = \zeta/(1 + r) = 1/1.05 = 0.9524$ where ζ is the industry’s exogenous survival rate per period. This discount factor is compatible with one year period length, interest rate of 0.05 per year, and certain survival. It is also compatible with one month period length, interest rate of 0.01 per month, and monthly industry survival rate of 0.96.

- Computations are done for a wide range of LBD and OF rates
  - Slope of learning curve: $\rho \in \{0.95, 0.85, \ldots, 0.05\}$. Empirical estimates are in the range 0.70 to 0.95. We focus on 0.85.
  - Organizational forgetting parameter: $\delta \in \{0, 0.01, 0.02, \ldots, 0.10, 0.30, \ldots 0.90\}$. Empirically estimated rates range from 4% to 25% per month.

- Our central case is $\rho = 0.85$ and $\delta = 0.03$.

Model, V

- $V_1(e_1,e_2)$ is the expected NPV to Firm 1 of being in the industry in state (e_1,e_2). Bellman equation is

$$V_1(e_1,e_2) = \max_{p_1}\left\{(p_1 - c(e_1)D_1(p_1, p_2(e_1,e_2)) + \beta\sum_{k=0}^{2} D_k(p_1, p_2(e_1,e_2))\overline{V}_{1k}(e_1,e_2)\right\}$$

where

$$\overline{V}_{1k}(e_1,e_2) = E_{e_1',e_2'}[V_1(e_1',e_2') \mid e_1,e_2, \text{ buyer purchases good } k], \quad k \in \{0,1,2\}.$$

- The pricing strategy $p_1(e_1,e_2)$ is solution to this maximization. Doraszelski and Satterthwaite (2005) implies existence of symmetric MPE in pure strategies.

- We compute symmetric MPE:
  - Value function $V_1(e_1,e_2) = V(e_1,e_2)$ and $V_2(e_1,e_2) = V(e_2,e_1)$.
  - Pricing function $p_1(e_1,e_2) = p(e_1,e_2)$ and $p_2(e_1,e_2) = p(e_2,e_1)$.
  - Pricing function used to compute transient and ergodic distributions over states. These distributions used to characterize industry dynamics

- Computation:
  - Iterative algorithm (Pakes and McGuire 1994): map a given $V^0(e_1,e_2)$ and $p^0(e_1,e_2)$ into an updated $V(e_1,e_2)$ and $p(e_1,e_2)$ until convergence.
  - Homotopy methods: to trace out equilibrium manifold starting at the equilibria found using the iterative algorithm.

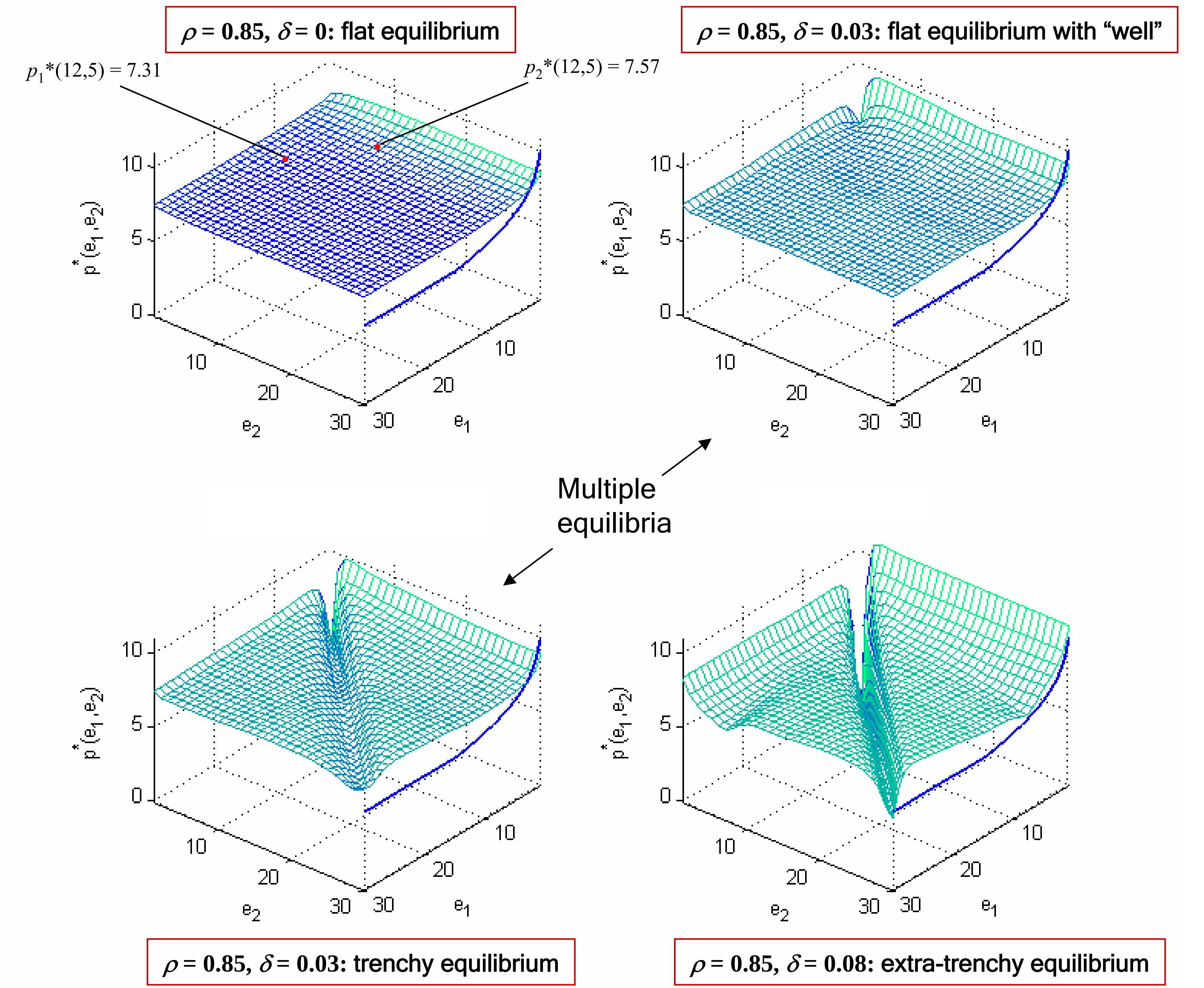
Key Constructs for Presentation of Results

- Pricing function describes equilibrium pricing behavior
  - Plot $p^*(e_1,e_2)$ as 3D surface plot and characterize special features

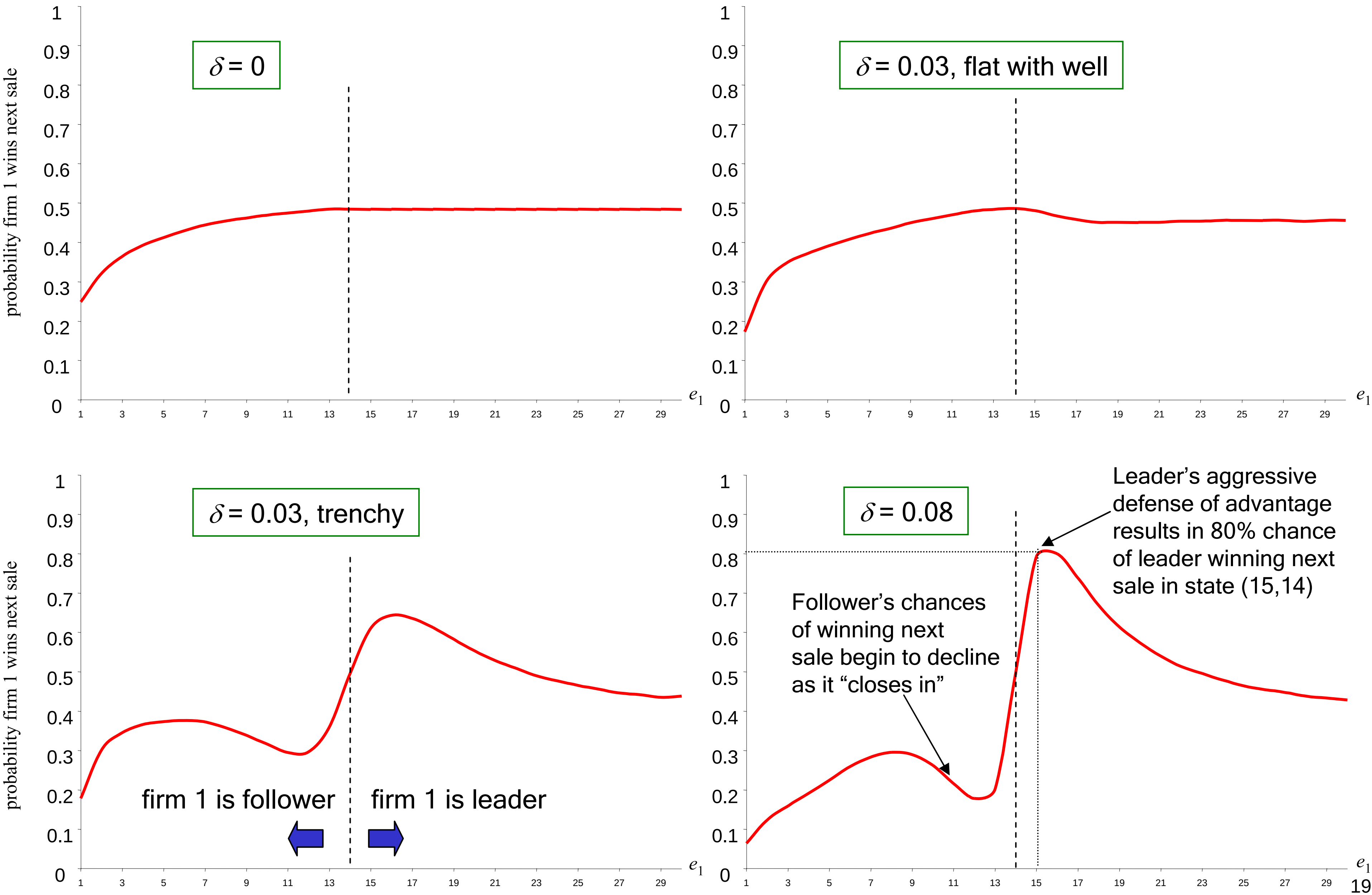
- Transient and limiting distributions characterizes industry dynamics
  - Using $p^*(e_1,e_2)$, compute transition kernel (an $M^2 \times M^2$ matrix):
    $$\Pr\{(e_1',e_2')|(e_1,e_2)\}$$
  - Compute transient distributions of industry states for any period T, assuming industry starts at state (1,1) at $T = 0$
  - Compute limiting (ergodic) distribution.

- Herfindahl index summarizes equilibrium market structure
  - At any state, Herfindahl index is given by a function of market shares:
    $$H(e_1,e_2) = \frac{D_1^2 + D_2^2}{\left(D_1 + D_2\right)^2}, D_n = D_n(p^*(e_1,e_2), p^*(e_2,e_1))$$
  - Given a distribution over the state space, we can compute expected Herfindahl for a period T or limiting case.
  - Provides a single-number descriptor of the equilibrium.
    ($H = 0.5 \Rightarrow$ symmetry, $H = 1.0 \Rightarrow$ monopoly)

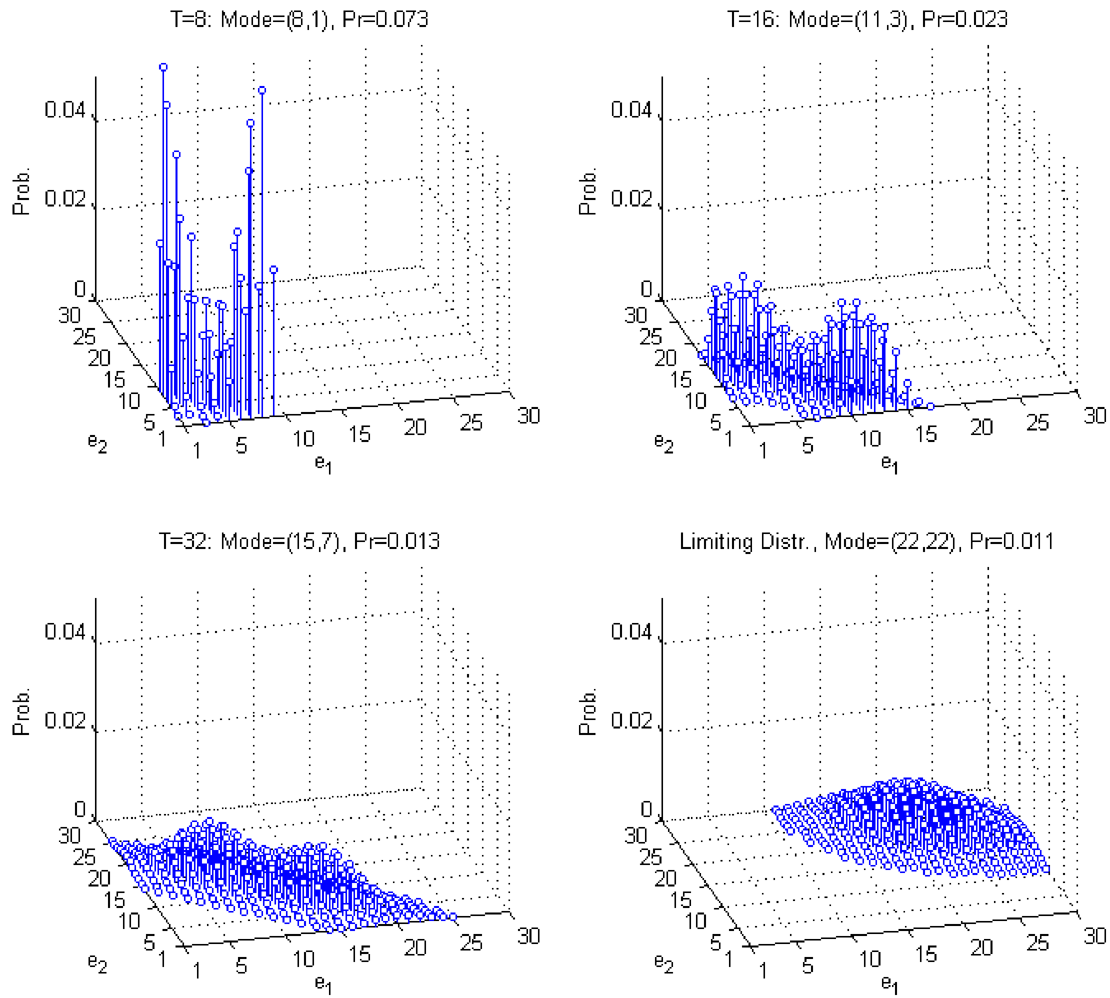
Firm 1's Equilibrium Pricing Function for $\rho = 0.85$ and $\delta \in \{0, 0.03, 0.08\}$



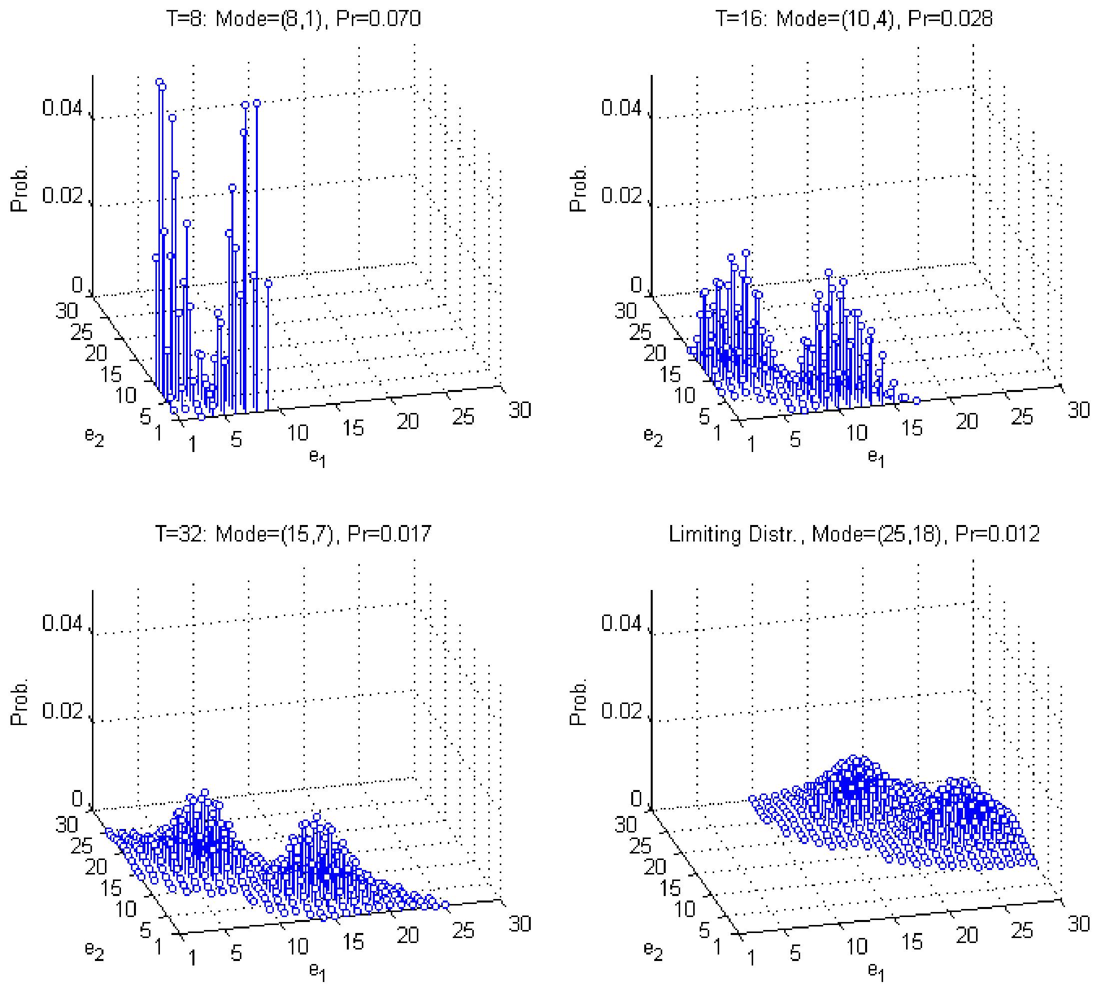
Equilibrium probability firm 1 wins next sale: firm 2 close to bottom of learning curve (e_2 = 14).



Transient and Limiting Distributions for $\rho = 0.85$ and $\delta = 0.03$, Flat Equilibrium with Well



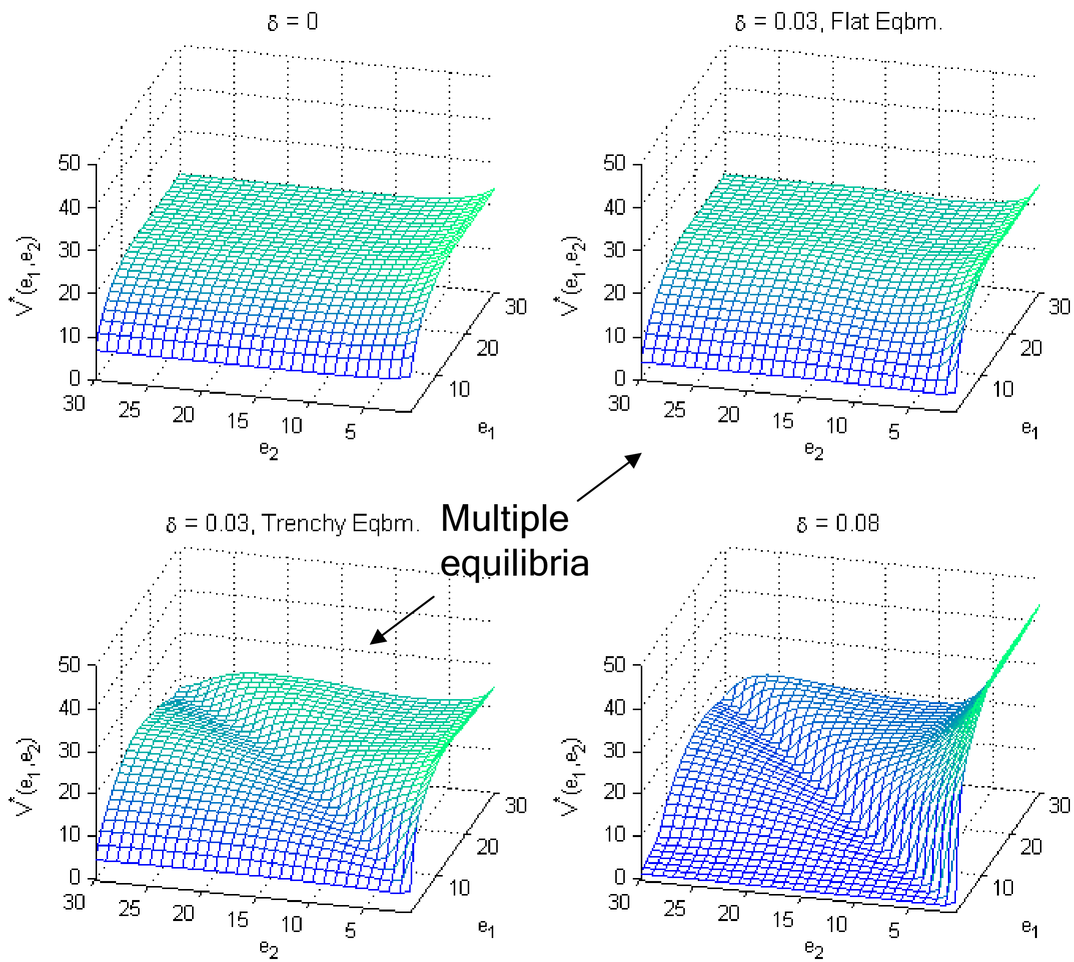
Transient and Limiting Distributions for $\rho = 0.85$ and $\delta = 0.03$, Trenchy Equilibrium



Overview of Equilibrium: Value Function for $\rho = 0.85$ and $\delta \in \{0, 0.03, 0.08\}$

Flat equilibrium

Flat equilibrium with "well



Multiple equilibria

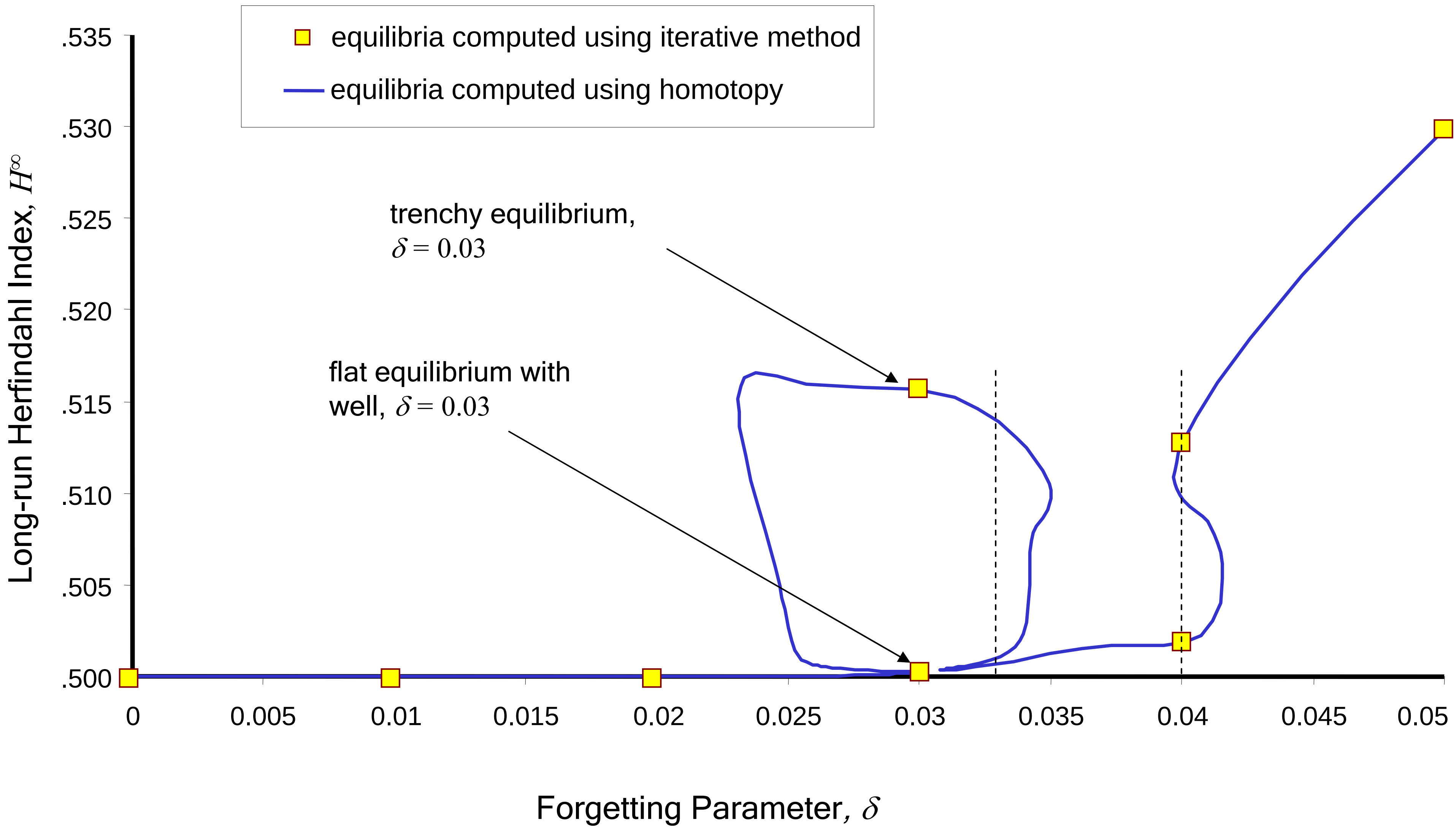
Trenchy equilibrium

Extra-trenchy equilibrium

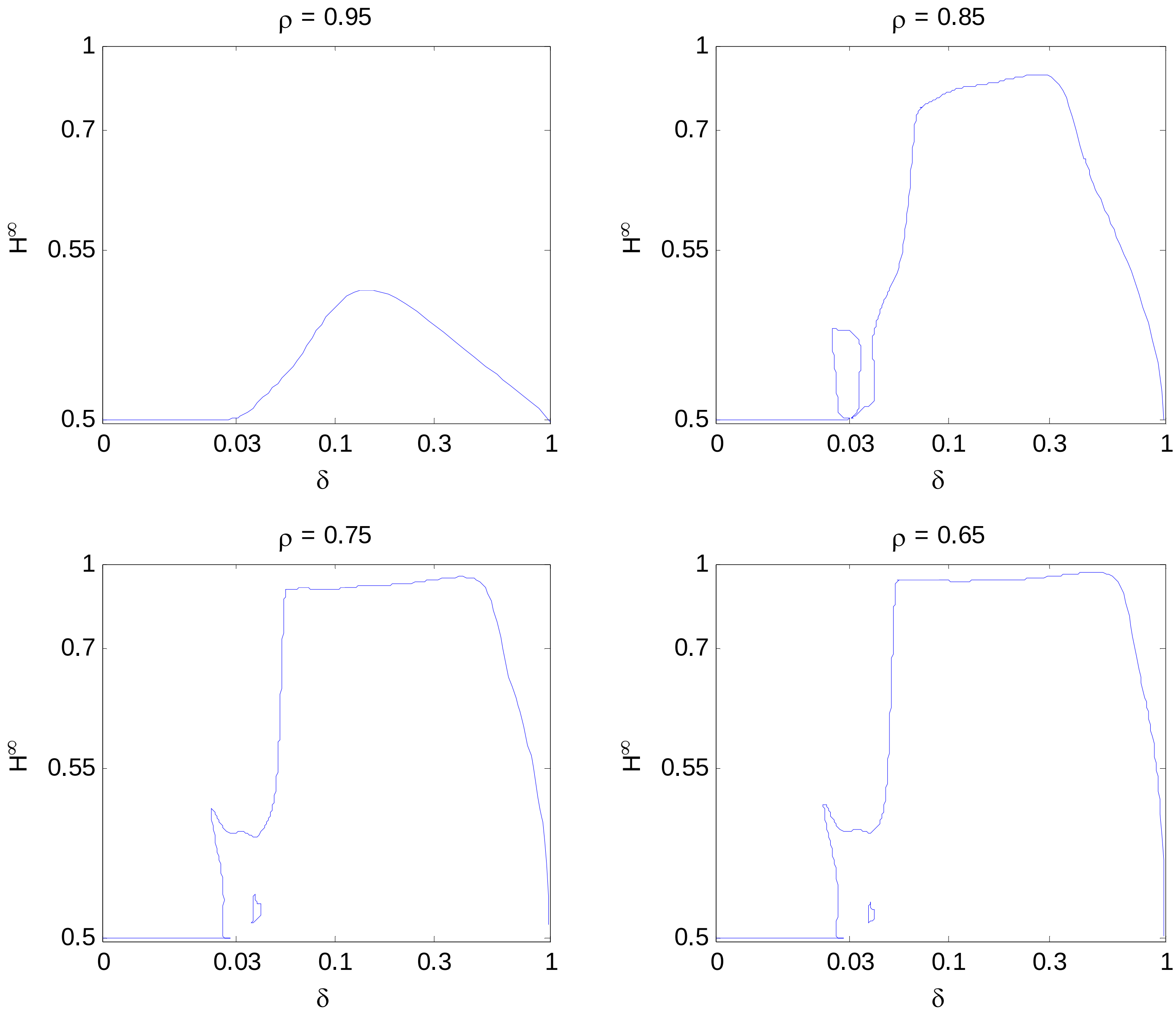
Summary: Equilibrium Classification

Equilibrium type	Price war for initial cost advantage?	Leader defends advantage if follower gets "close" to catching up?	Leader defends against "marginal" follower?	Market dominance in ... short-run?	Market dominance in ... long-run?
Flat	No	No	No	No	No
Flat with "well"	Yes	No	No	Yes	No
"Trenchy"	Yes	Yes	No	Yes	Yes, modest
"Extra-trenchy"	Yes	Yes	Yes	Yes	Yes, significant

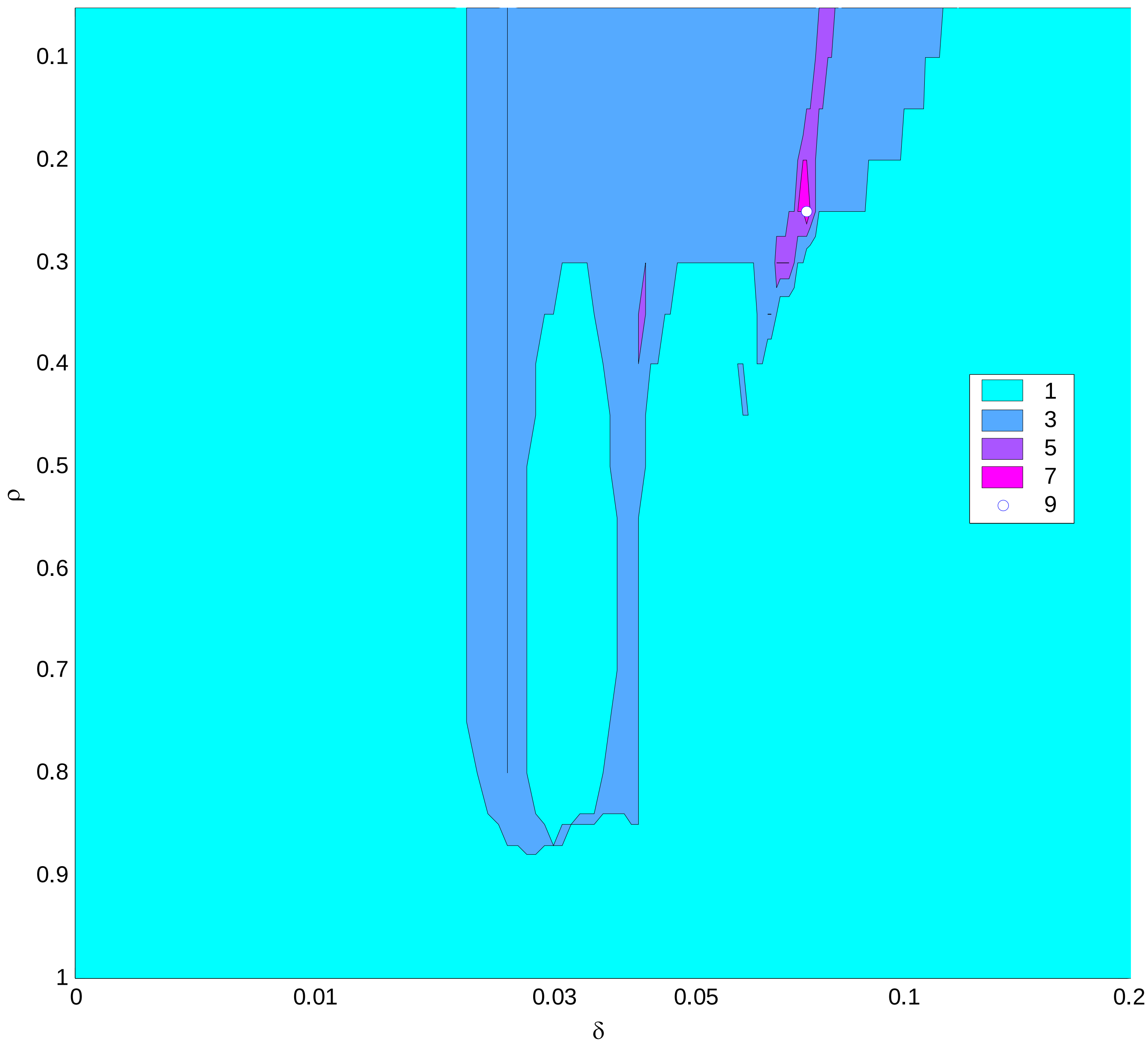
Equilibrium Correspondence (Expected Long-Run Herfindahl) for $\rho = 0.85$ and $\delta \in [0, 0.05]$



Equilibrium Correspondence (Expected Long-Run Herfindahl)



Number of equilibria



Multiple Equilibria: Role of Organizational Forgetting

- Multiple equilibria do not arise in absence of OF, but they can arise when there is OF. Why?

- OF makes it difficult for two firms to co-exist in long run in symmetric state (e,e) for e sufficiently large.

- OF can therefore lead to self-fulfilling hypothesis of market dominance:
  - Suppose firms believe that symmetric co-existence is impossible and that an industry leader will emerge
  - Therefore, in symmetric states, each firm prices aggressively to become the leader.
  - The price wars in symmetric states ensure that no firm can earn much money in these states making symmetric co-existence is, in fact, impossible.

- OF does not compel aggressive behavior:
  - Suppose firms believe that symmetric co-existence is possible and that symmetry will prevail in the long run.
  - Therefore, in symmetric states, each firm prices softly since any lead secured through aggressive price will short lived.
  - No price wars occur and the two firms both proceed down the learning curve together.

- Note: The multiple equilibria in this model is a purely dynamic phenomenon. In all equilibria that we have computed, at any pair of experience levels (e_1, e_2), each firm's optimal choice of price is unique.

Multiple Equilibria, OF, and Market Size

- Question: Why are equilibria unique for all $\delta < 0.025$. Why are they not unique in the neighborhood of $\delta = 0.03$.
- Neglecting the outside good, each period one unit of experience is earned and, in expectation, the two firms forget

$$\delta(e_1) + \delta(e_2) = \left[1 - (1-\delta)^{e_1}\right] + \left[1 - (1-\delta)^{e_2}\right].$$

  units of experience.
- In the long run the rate of new experience accumulation must equal stochastically the rate at which experience is forgotten:

  (∗) $$\delta(e_1) + \delta(e_2) = 1.$$
- Consider the flat-with-well and trenchy equilibria for the $\rho = 0.85$ and $\delta = 0.03$ parameter pair. At the mode of the industry's long run ergodic state distribution (∗) roughly holds.
  - For the flat equilibrium the mode of the limiting distribution is on the diagonal at $(e_1, e_2) = (22, 22)$. Solving (∗) at $\delta = 0.03$ for (e_1, e_2) symmetric gives $e_1 = e_2 = 22.76$.
  - For the trenchy equilibrium the mode of the limiting distribution is asymmetric at $(e_1, e_2) = (25, 18)$. Solving (∗) at $\delta = 0.03$ and $e_1 = 27$ gives $e_2 = 19.0$
- As δ increases each firm's scale in the sense of expected rate of production must increase if it is to be viable from a cost viewpoint.
  - Solving (∗) at $\delta = 0.025$ for (e_1, e_2) symmetric gives $e_1 = e_2 = 27.4$, i.e., both firms can easily coexist.
  - Solving (∗) at $\delta = 0.080$ and $e_1 = 26$ gives $e_2 = 1.46$, i.e., the follower is not viable in the long run.

Summary: Substantive Results

- The objective of the paper is to explore how LBD and OF interact to affect pricing behavior and the evolution of industry structure.

- Some specific insights:
  - OF breaks the uni-directionality of the traditional learning curve models. Because firms can forget, prize from winning a sale is “turbo-charged” and so symmetric firms are more predisposed to compete hard on price in states where, without OF, price competition would have been softer.
  - As a result:
    - leader has incentive to aggressively defend its advantage in states in which, without OF, leader would not be aggressive.
    - asymmetric market structures and even dominance by one firm in the long run.
    - multiple equilibria.
    - predatory pricing.
  - Multiple equilibria, some of which may involve aggressive behavior, occurs when two firms of minimum economic scale can not comfortably coexist in the market.

Summary: Techniques for Computing MPE

- The homotopy technique we have developed allows tracing out of connected parts of the equilibrium set and the systematic discovery of at least some of the instances of multiple equilibria.

- Our techniques for computation of MPE can accommodate the non-convexities that entry/exit and other investment behaviors naturally induce.

- Limitations: solving for MPEs with moderate numbers of firms is subject to the curse of dimensionality and no theories exist as to how firms select among equilibria and how they learn MPE behavior.

- Unsolved problem: we do not know how to relax the assumption of complete information. In particular, we have no idea how to solve for equilibria with incomplete information concerning, for example, each firm's marginal cost of production.

What theory helped us?

- Dynamic programming

- Caplin and Nalebuff's theorem that a unique price equilibrium exists when demand is logit.

- The theory of stochastic games did not help beyond existence.

- Standard existence theorems for stochastic games with discounting do not apply.
  - But the strategy of proof used in standard existence theorems, if it is combined with Harsanyi's technique for purifying mixed strategies, can be adapted to prove that symmetric equilibria exist for this class of games. See Doraszelski and Satterthwaite (2003).
  - Observe that Pakes and Ericson published the computational MPE model of industry dynamics in 1994 with a flawed proof of existence. Theorists specializing in stochastic games apparently knew it was incorrct, but did not provide a resolution of the problem.

Where to go?

- Conjecture. The insights that we have obtained from our computational exploration of LBD and OF could not have been obtained analytically (at least by us) over any reasonable time horizon.

- Question. Under what circumstances is theoretical knowledge gained through numerical experiments substitutable for knowledge gained through mathematical proof? This question can only grow more acute over time as the cost of conducting numerical experiments drops due to improved computers, better algorithms, and higher level programming languages.

- Observation. Behavioral economics is identifying a host of situation contingent biases in the way individuals make economic decisions. Understanding how these biases affect organizational and market performance requires that these biases be inserted into models and the resulting equilibria be analyzed. Theoretical knowledge in microeconomic knowledge may then become even more inductive than it is currently.

Conclusion

- Mathematical economics, if it is to be relevant to mainstream economics, should focus on establishing theorems and developing techniques that are applicable to classes of models that naturally arise in economics per se.

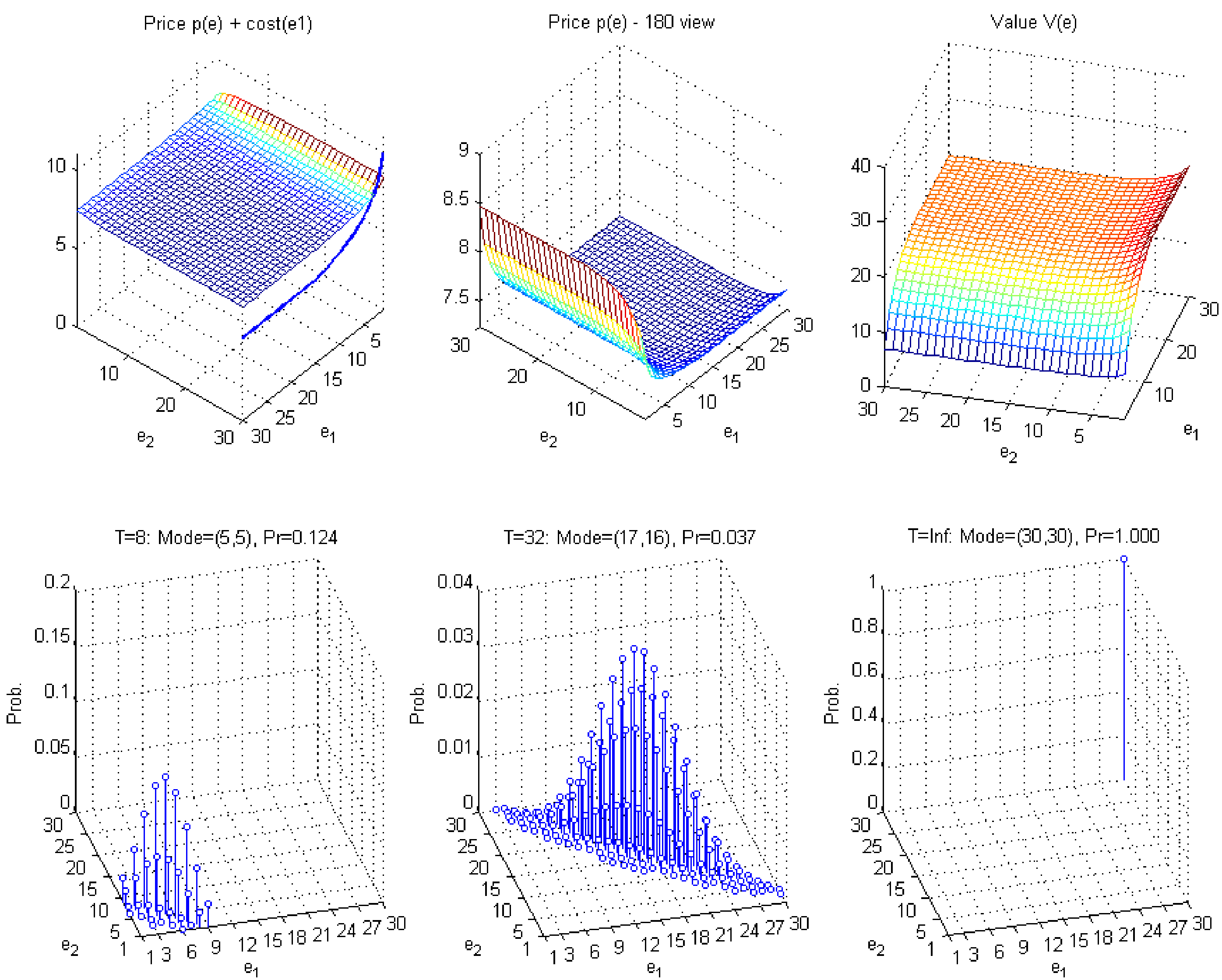
- Two examples that naturally arise in the LBD and OF example:

  - How can firms possibly learn to play dynamic MPE equilibria? Does this learning select among equilibria and therefore function as a refinement. Any credible answer to this, I would guess, will have to be informed by work in behavioral economics.
  - What are sufficient conditions and what are necessary conditions in a full information Markov game for equilibria to arise in which the long run outcome is asymmetric and is supported by aggressive behavior (e.g., trenches)?

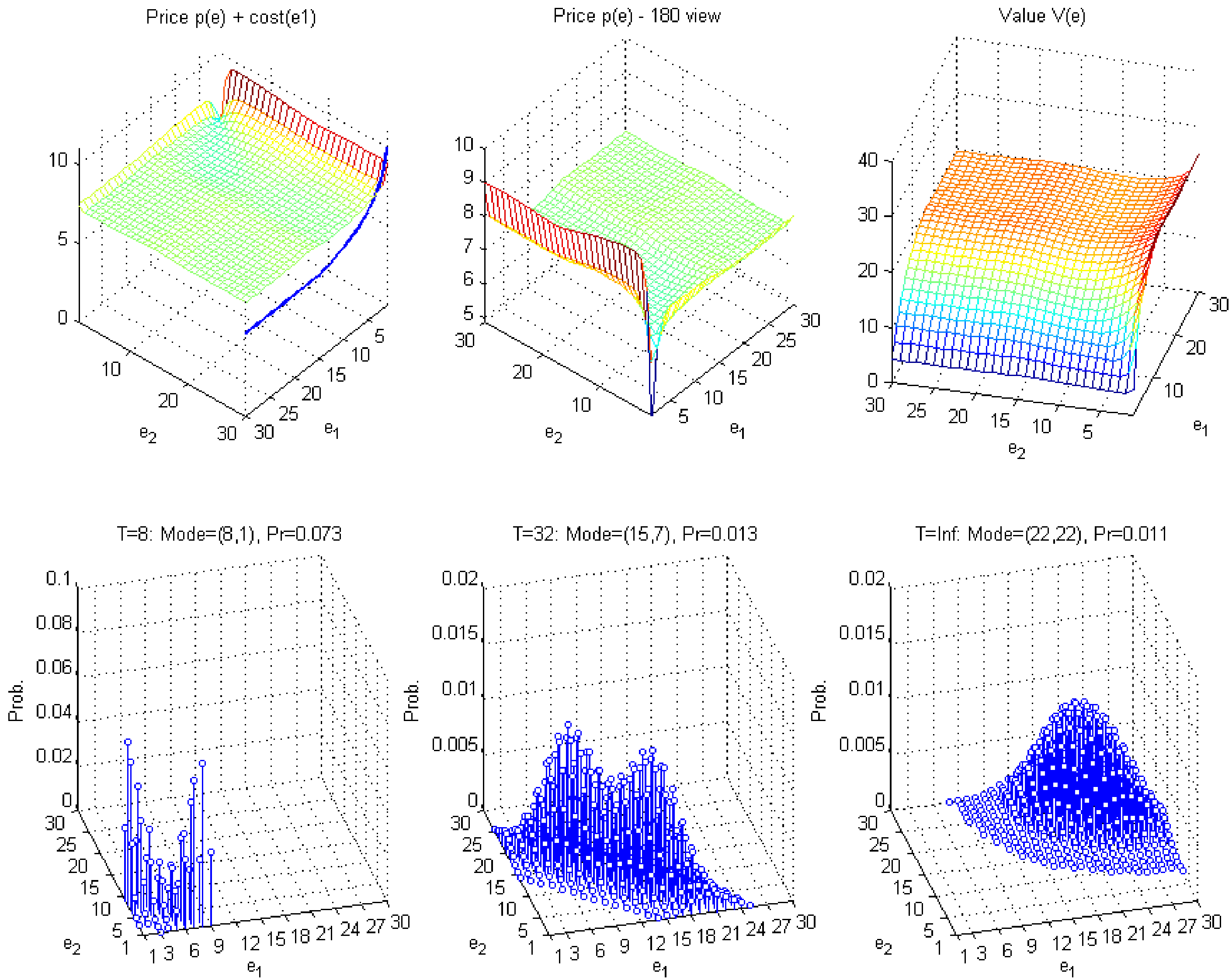
Thanks for listening.

Back-Up Slides

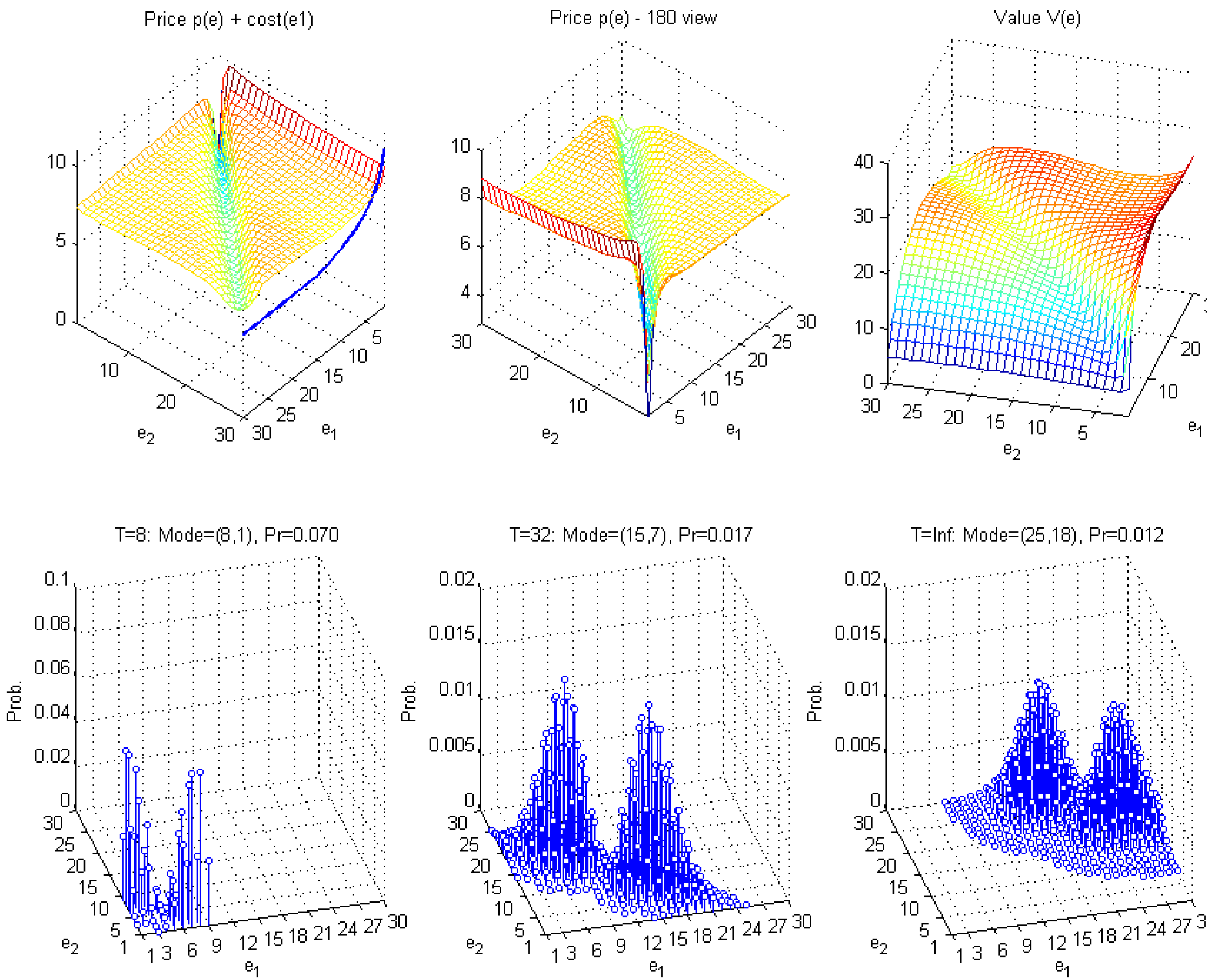
Summary Plot: $\rho = 0.85$, $\delta = 0$



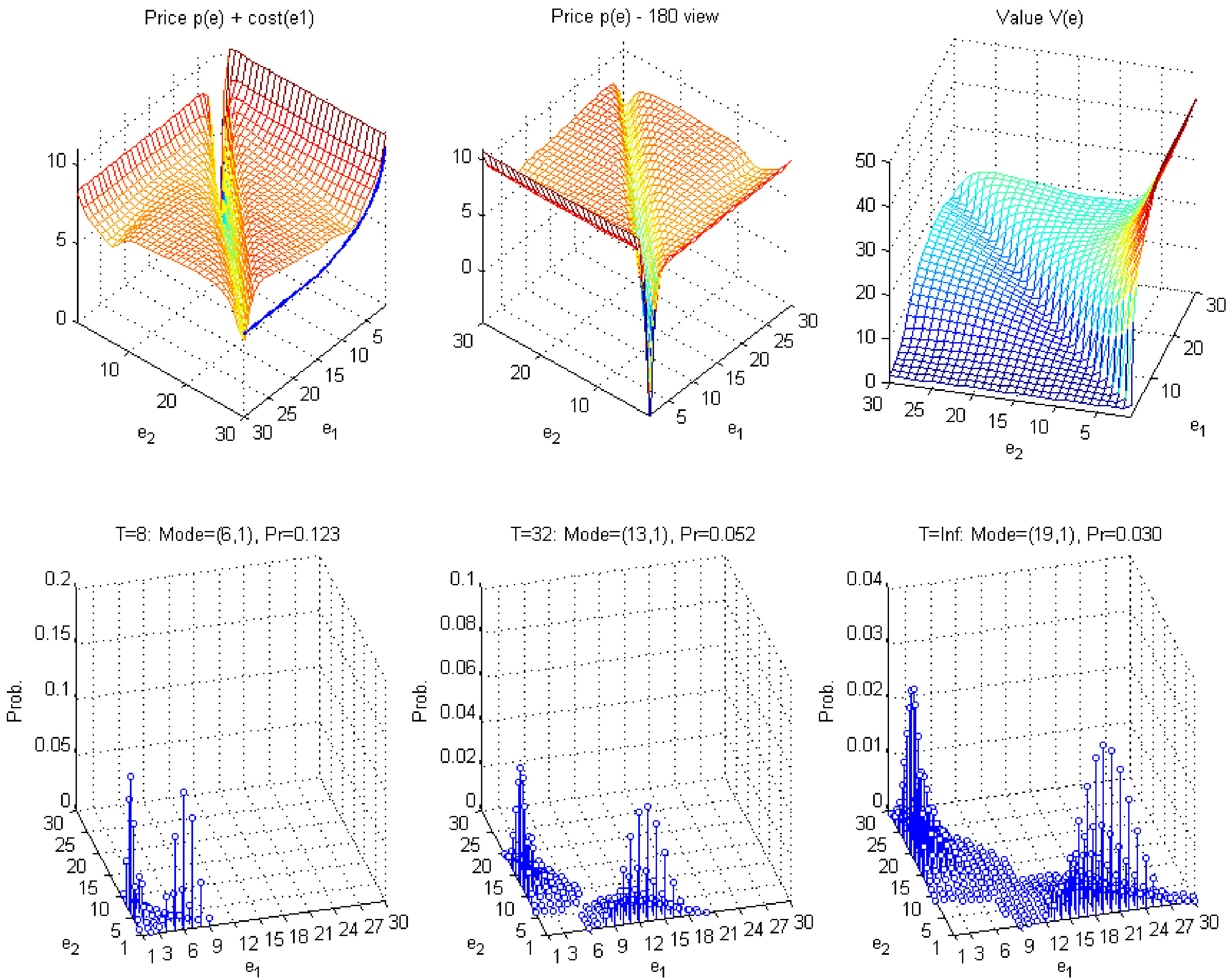
Summary Plot: $\rho = 0.85$, $\delta = 0.03$ (Flat Equilibrium)



Summary Plot: $\rho = 0.85$, $\delta = 0.03$ (Trenchy Equilibrium)

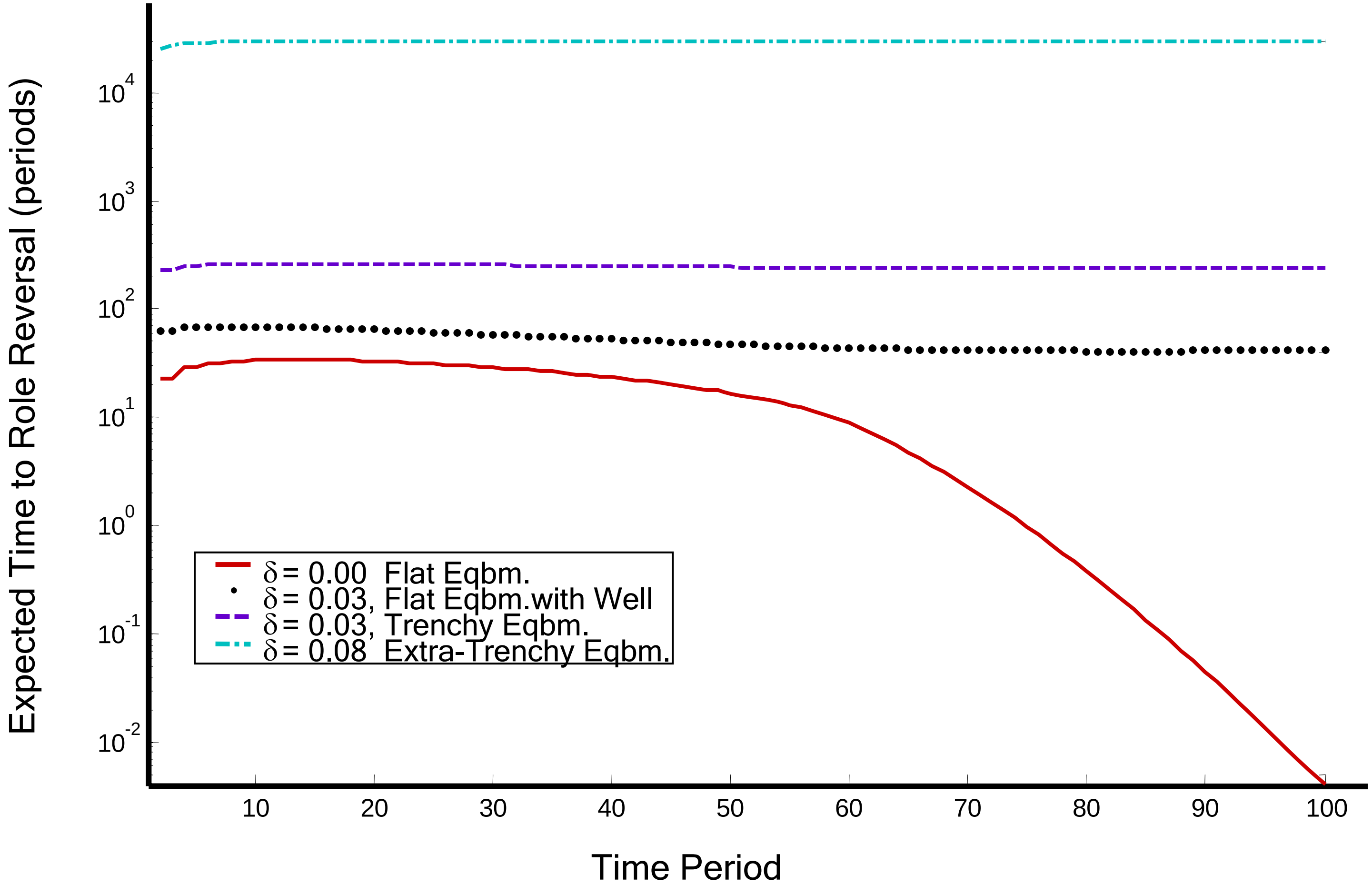


Summary Plot: $\rho = 0.85$, $\delta = 0.08$

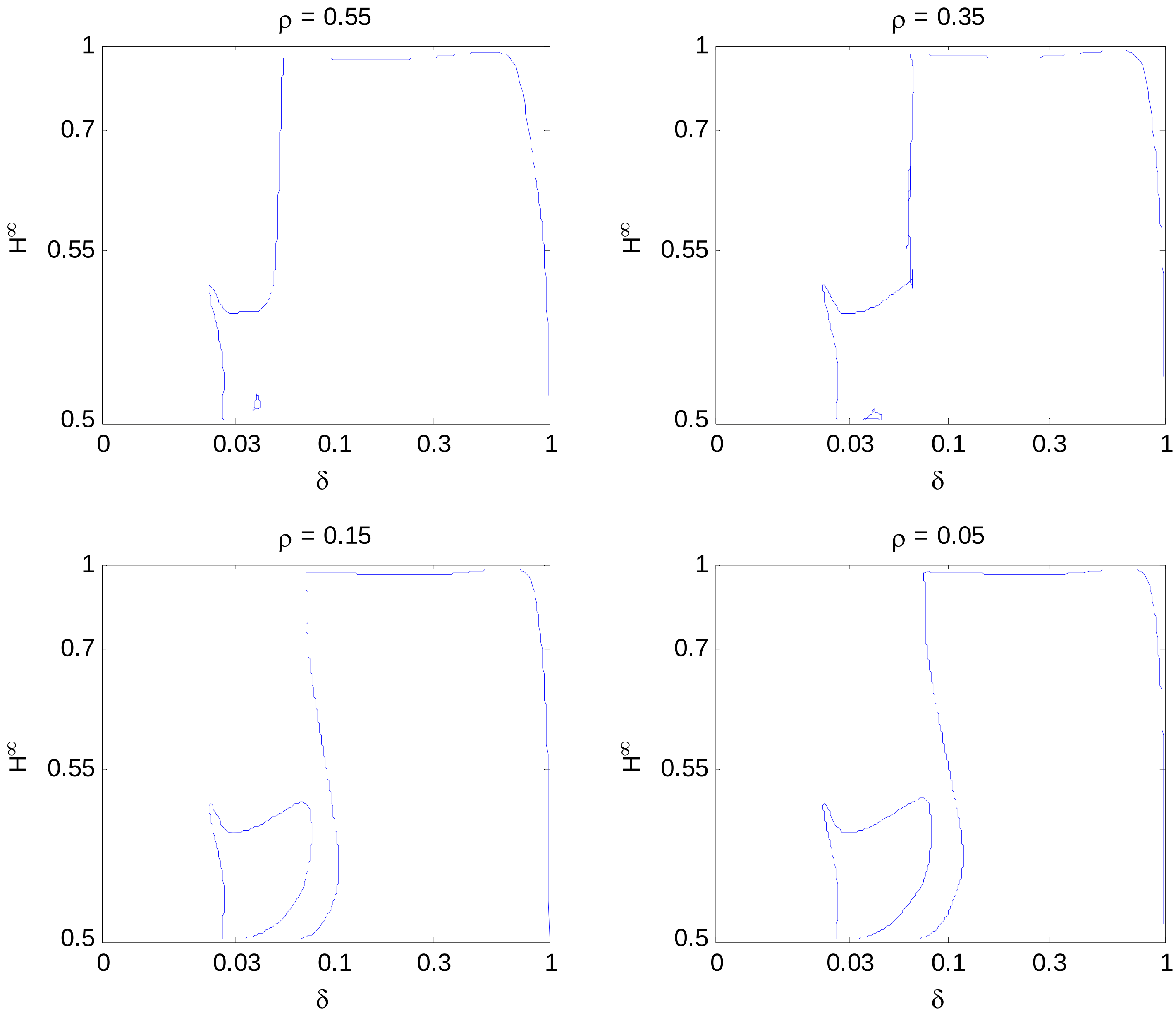


Stability of Leadership: Expected Role Reversal Time for $\rho = 0.85$ and $\delta \in \{0, 0.03, 0.08\}$

This plot illustrates the sustainability of the leader's cost advantage .



Lim-Herf correspondence, $\rho = \{0.55, 0.35, 0.15, 0.05\}$



A Selection Story
Trenchy dominates flat-with-well.

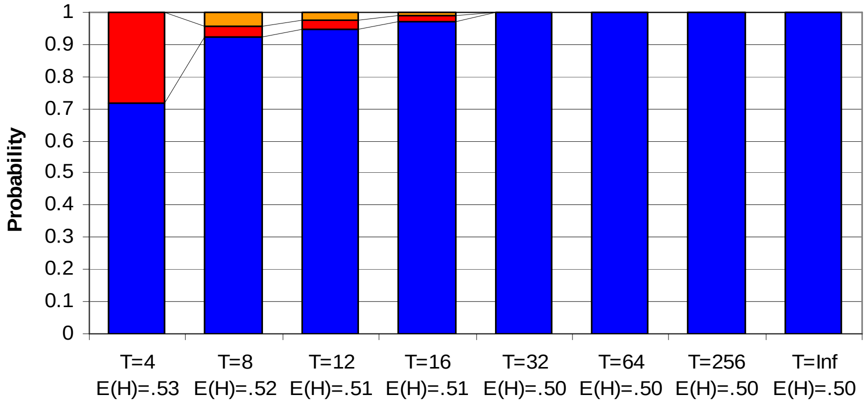
- Consider again the $\rho = 0.85$ and $\delta = 0.03$ case and its two equilibria.

Flat-with-well				Trenchy		
(e_1, e_2)	(p_1, p_2)	(V_1, V_2)		(e_1, e_2)	(p_1, p_2)	(V_1, V_2)
(1,1)	(4.9, 4.9)	(4.8, 4.8)		(1,1)	(2.9, 2.9)	(5.1, 5.1)
(2,1)	(6.8, 8.5)	(11.6, 3.9)		(2,1)	(6.6, 8.7)	(14.1, 4.2)
(3,1)	(7.5, 9.3)	(15.3, 3.9)		(3, 1)	(7.8, 9.8)	(18.1, 4.3)

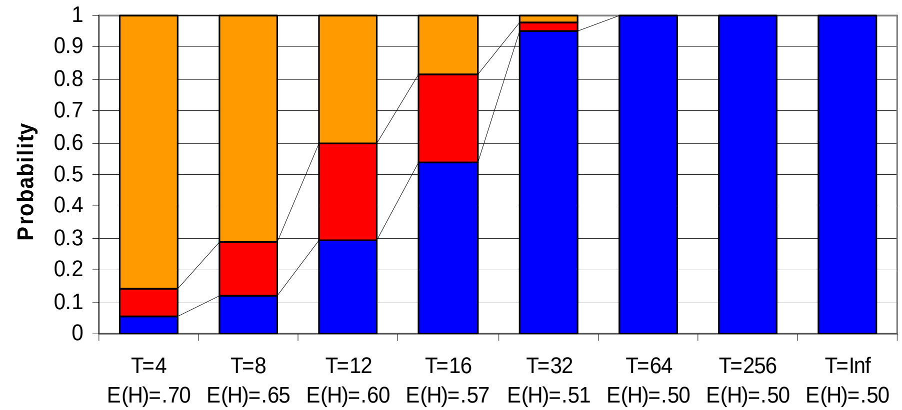
- Costs are 10.0 at $e = 1$, 8.5 at $e = 2$, and 7.7 at $e = 3$.
- In this model a firm enters at state 1. If entry is simultaneous, entry occurs at state (1,1), values are symmetric, and necessarily a dominant equilibrium exists. If entry is not simultaneous, then the incumbent firm can be at experience level 1, 2, 3, . . ., 30 and the firms may disagree as to what equilibrium is preferable.
- At each of these 3 critical points both firms' values are greater in the trenchy equilibrium than in the flat-with-well equilibrium. This is particularly pronounced at state (2, 1), which may be a more relevant starting point for dynamic competition than the symmetric state (1,1)

Overview of the Equilibrium: Distribution and Expected Value of Herfindahl Index for $\rho = 0.85$ and $\delta = 0, 0.03, 0.08$

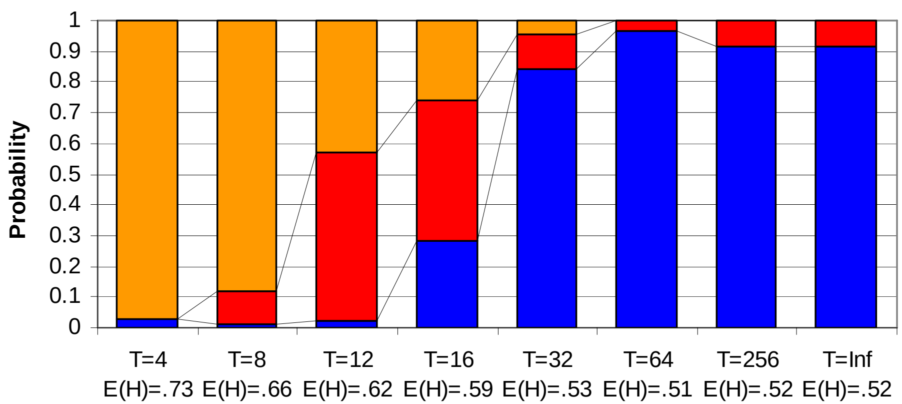
Flat equilibrium, $\delta = 0$



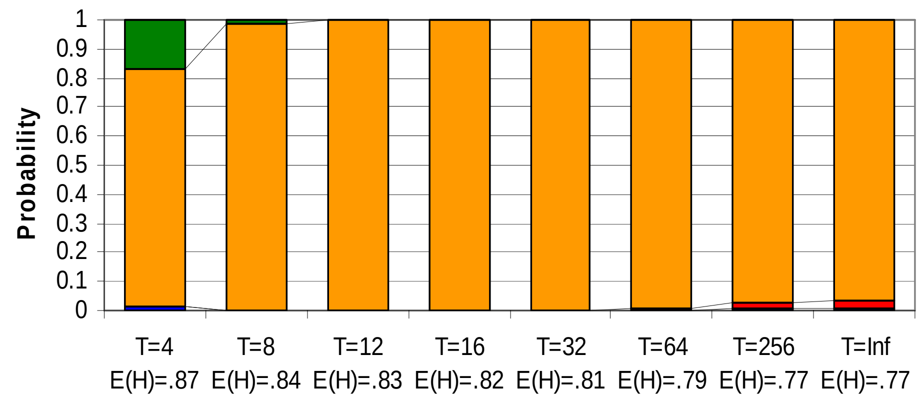
Flat equilibrium with "well," $\delta = 0.03$



Trenchy equilibrium, $\delta = 0.03$

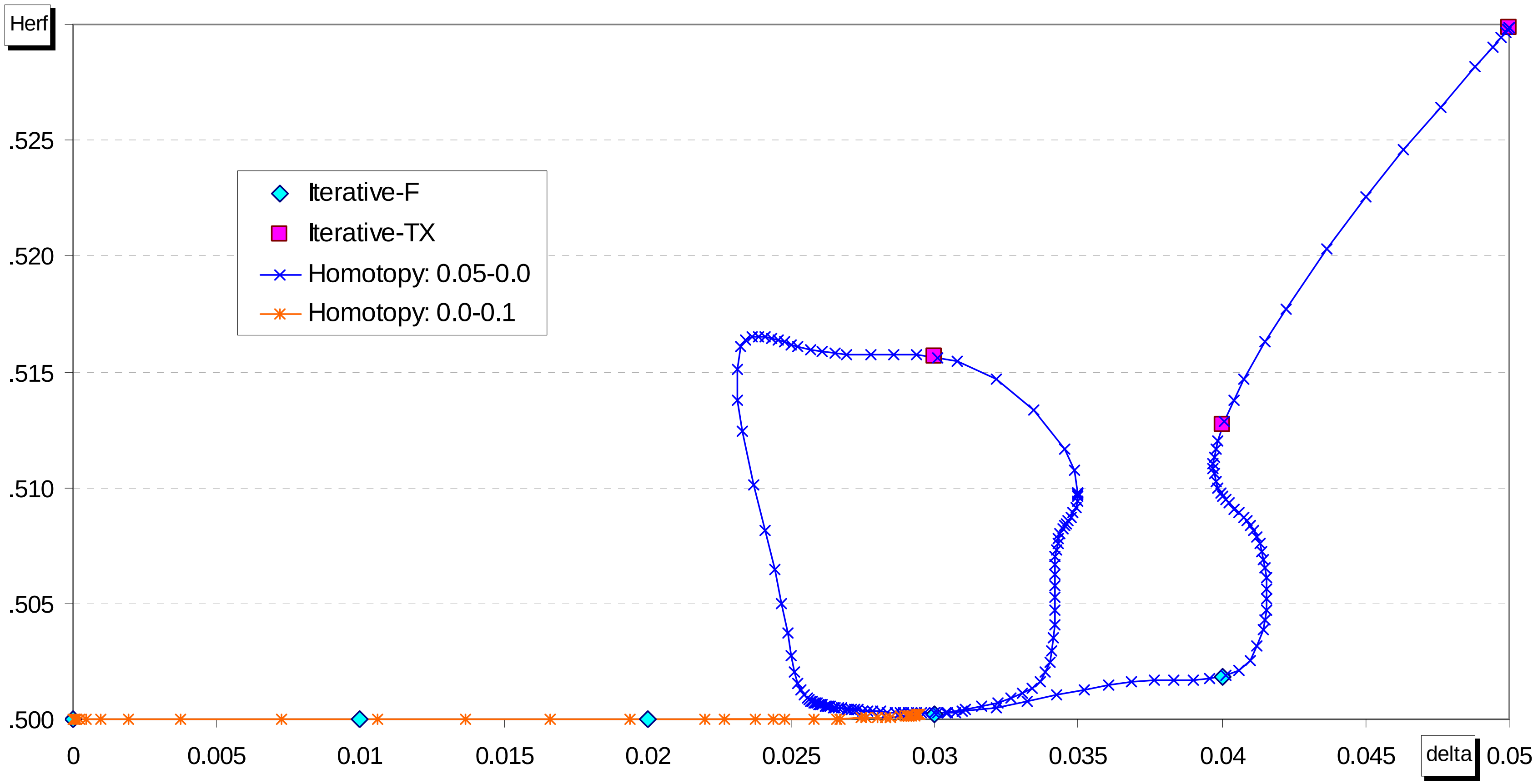


Extra-trenchy equilibrium, $\delta = 0.08$



$H = 0.50 - 0.55$ $H = 0.55 - 0.60$ $H = 0.60 - 0.90$ $H = 0.90 - 1.00$

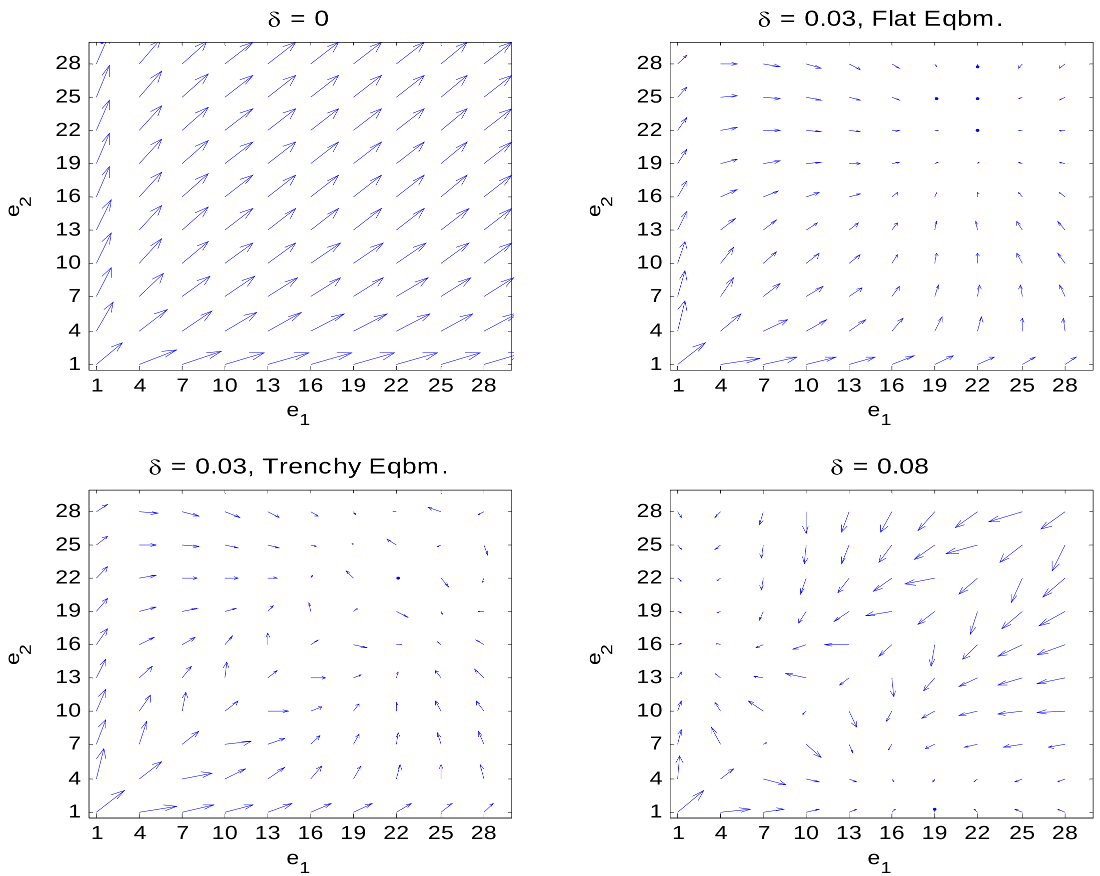
Equilibrium Correspondence (Expected Herfindahl) According to Homotopy Path for $\rho = 0.85$ and δ □[0, 0.05]



Industry Dynamics: Resultant Forces for $\rho = 0.85$ and $\delta \in \{0, 0.03, 0.08\}$

Flat equilibrium

Flat equilibrium with “well”



Trenchy equilibrium

Extra-trenchy equilibrium

Intuition: LBD Without OF

- In absence of OF, firm 1's marginal cost of selling an additional unit today is:

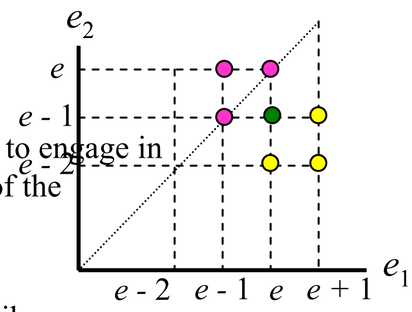
$$\overbrace{c(e_1)}^{\textit{current MC}} - \beta\overbrace{\{[V_1(e_1+1,e_2) - V_1(e_1,e_2)] + [V_1(e_1,e_2) - V_1(e_1,e_2+1)]\}}^{\textit{“prize” from improving future competitive position}}$$

$\Rightarrow$ differences in pricing incentives of two firms shaped by (1) differences in current MC; (2) differences in prizes.

- In absence of OF, prize has two components:
  - $V_1(e_1+1,e_2) - V_1(e_1,e_2)$ (“Advantage-building” motive)
  - $V_1(e_1,e_2) - V_1(e_1,e_2+1)$ (“Advantage-defending” motive).

- Key points:
  - With no OF, once firm reaches bottom of learning curve, advantage-building motive goes away ...
  - ... but advantage-defending motive creates potential incentive for firm to price aggressively, even though firm is at bottom of its own learning curve; i.e., $V_1(m,e_2) - V_1(m,e_2+1) > 0$.
  - Still, there are limits to how aggressive leader will be: leader at bottom of learning curve never prices below current MC (Proposition 3)
  - And for parameter value we study, leader's pricing at bottom of learning curve is typically quite soft

Intuition: Impact of OF on Price Competition



1. Desire to (i) prevent own forgetting (ii) increase odds that rival forgets predisposes firms to engage in aggressive price competition in symmetric states (e,e), even at the bottom of the learning curve.

2. Leader's "prize" in state (e,e–1) is bigger than follower's, reinforcing leader's MC advantage (kind of like "efficiency effect" in R&D literature)
   - ⇒ leader significantly under-prices follower, making its advantage more sustainable and valuable than in the absence of OF.

3. (1) + (2) ⇒
   - Firms are willing to fight harder on price to acquire advantage in (e–1,e–1) than would have been the case without OF.
   - Follower less likely to price aggressively to catch up in states like (e,e–2), (e,e–3), and so on, than without OF, because leader's advantage is so secure in state (e,e–1).

4. This logic folds back up state space to intensify price competition all along the diagonal ($e_1 = e_2$) ...
   - ... except when δ is *so large* that it is extremely doubtful that firms can move even one or two steps down the learning curve, in which case prices at top of learning curve are close to static Nash prices.

Related Literature

- Cabral and Riordan (C-R)(1994, *Econometrica*)
  - Fully analytical analysis of MPE with LBD.
  - Focus entirely on properties of pricing function. No explicit analysis of industry dynamics.
  - No OF.

- Lewis and Yildirim (2002, *AER*)
  - Focus on how single strategic buyer can optimal design a procurement auction when sellers face learning curve.
  - No OF.

- Benkard (2003, *Review of Economic Studies*)
  - Computational analysis of MPE in model with LBD and OF.
  - Focus on calibrating model to wide-body airframe market of 1970s/80s to show that predictions of MPE analysis fit the actual data.
  - Does not focus on specific role played by OF.

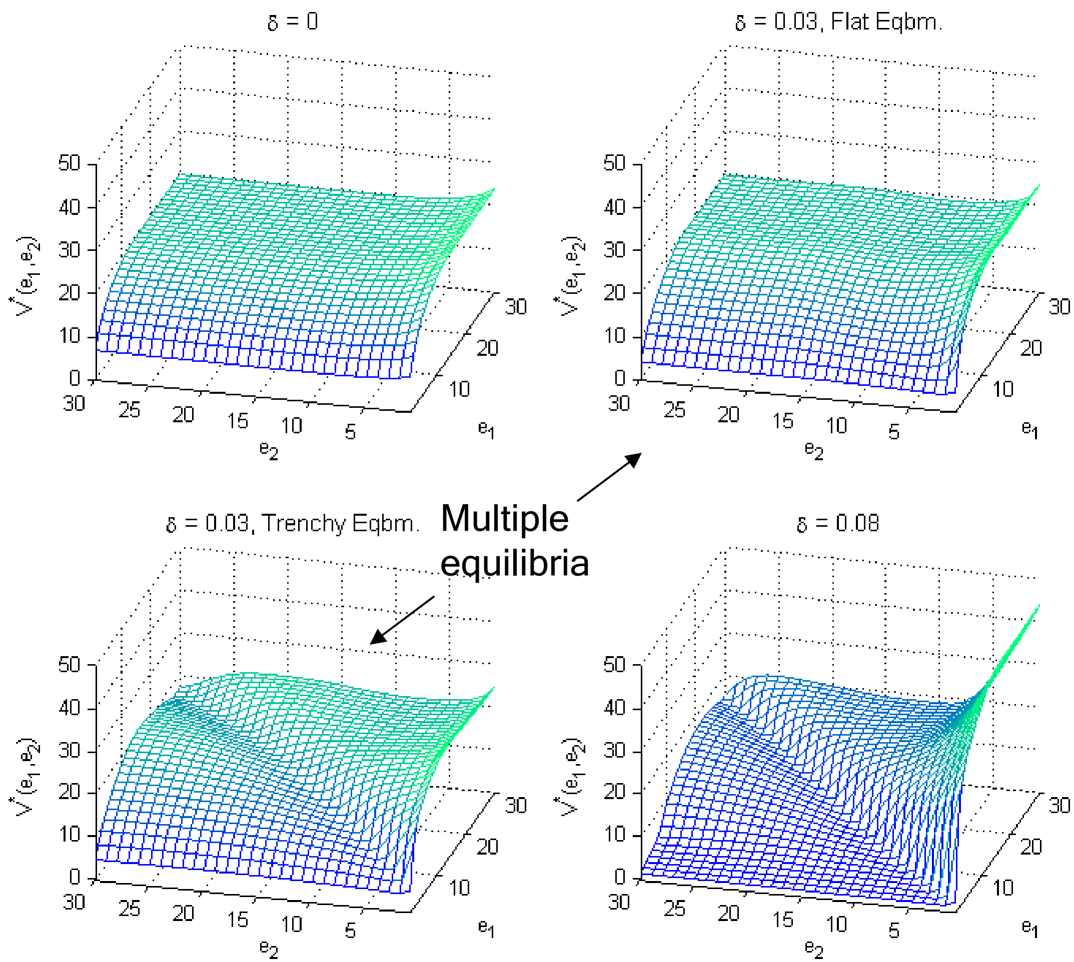
Overview of Equilibrium:
Value Function for $\rho = 0.85$ and $\delta \in \{0, 0.03, 0.08\}$

Flat equilibrium

Flat equilibrium with “well

Trenchy equilibrium

Extra-trenchy equilibrium



Overview of Equilibrium: Expected Herfindahl Index Under Limiting Distribution for (r, δ)□$R \times D$

Slope of learning curve

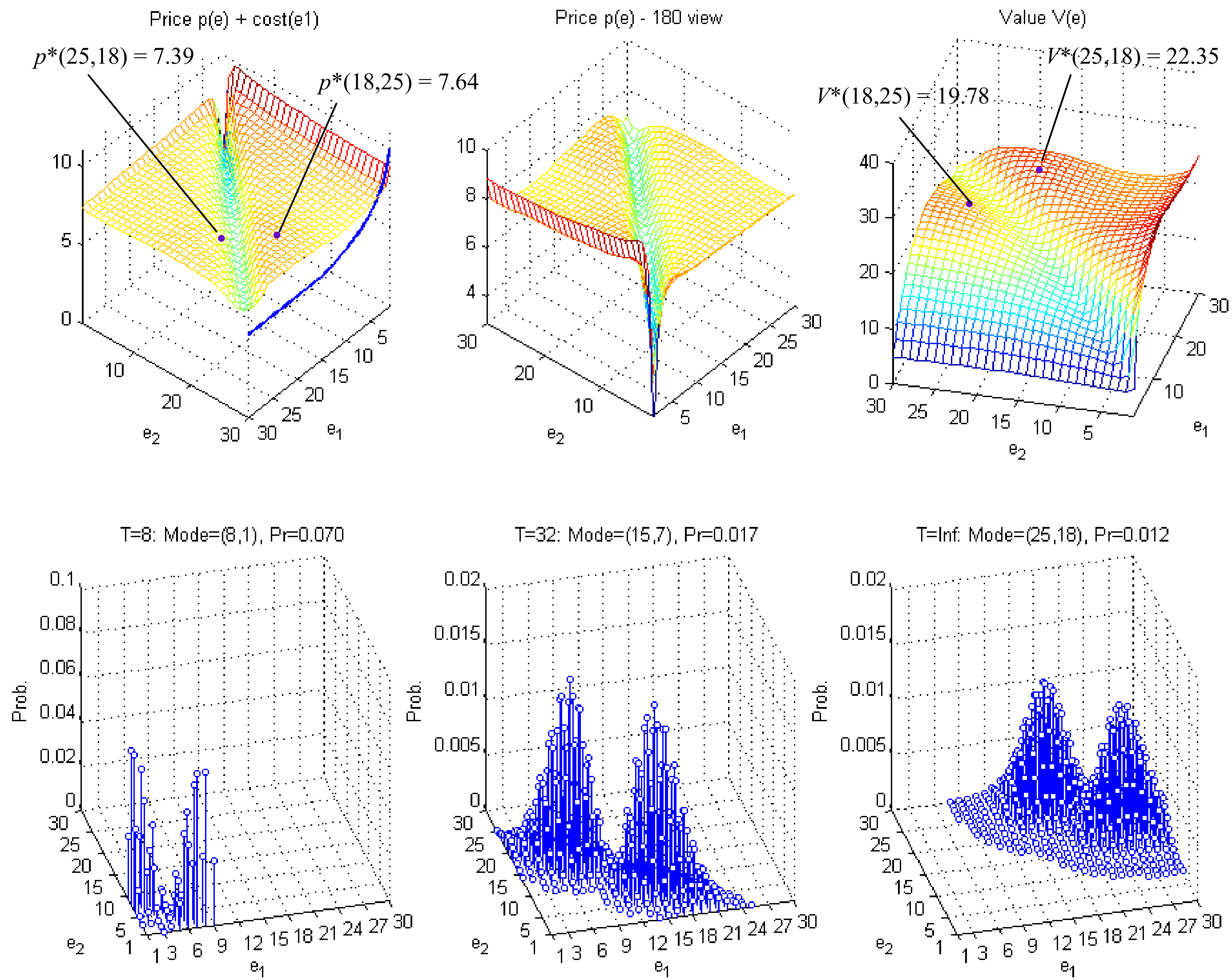
Organizational forgetting

ρ	0.95	0.85			0.75			0.65			0.55			0.35			0.15			0.05		
δ		*F*	*T*	*X*	*F*	*T*	*X*	*F*	*T*	*X*	*F*	*T*	*X*	*F*	*T*	*X*	*F*	*T*	*X*	*F*	*T*	*X*
0	.500	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-
0.01	.500	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-
0.02	.500	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-	.500	-	-
0.03	.500	**.500**	**.516**	-	-	.520	-	-	.521	-	-	.521	-	**.500**	**.521**	-	**.500**	**.521**	**-**	**.500**	**.521**	**-**
0.04	.502	**.502**	**.513**	-	**.502**	**.519**	-	**.502**	**.521**	-	**.501**	**.521**	-	**.500**	**.523**	-	**.500**	**.523**	**-**	**.500**	**.523**	**-**
0.05	.504	-	.530	-	-	.538	-	-	.580	-	-	.542	-	-	.527	-	**.500**	**.527**	**-**	**.500**	**.527**	**-**
0.06	.507	-	.559	-	-	-	.882	-	-	.923	-	-	.940	**.500**	**.531**	-	**.500**	**.530**	**-**	**.500**	**.530**	**-**
0.07	.511	-	-	.744	-	-	.884	-	-	.921	-	-	.938	-	-	.954	**.500**	**.532**	**-**	**.500**	**.534**	**-**
0.08	.515	-	-	.770	-	-	.883	-	-	.920	-	-	.937	-	-	.952	**.501**	**-**	**.959**	**.501**	**.530**	**.961**
0.09	.519	-	-	.787	-	-	.882	-	-	.918	-	-	.935	-	-	.950	**.502**	**-**	**.957**	**.502**	**-**	**.960**
0.10	.523	-	-	.803	-	-	.882	-	-	.916	-	-	.933	-	-	.949	**.505**	**-**	**.956**	**.504**	**-**	**.958**
0.30	.517	-	-	.860	-	-	.922	-	-	.936	-	-	.942	-	-	.945	-	-	.945	-	-	.945
0.50	.508	-	-	.597	-	-	.897	-	-	.956	-	-	.965	-	-	.969	-	-	.969	-	-	.968
0.70	.504	-	-	.539	-	-	.615	-	-	.762	-	-	.904	-	-	.965	-	-	.975	-	-	.977
0.90	.501	-	-	.511	-	-	.532	-	-	.561	-	-	.591	-	-	.672	-	-	.766	-	-	.803

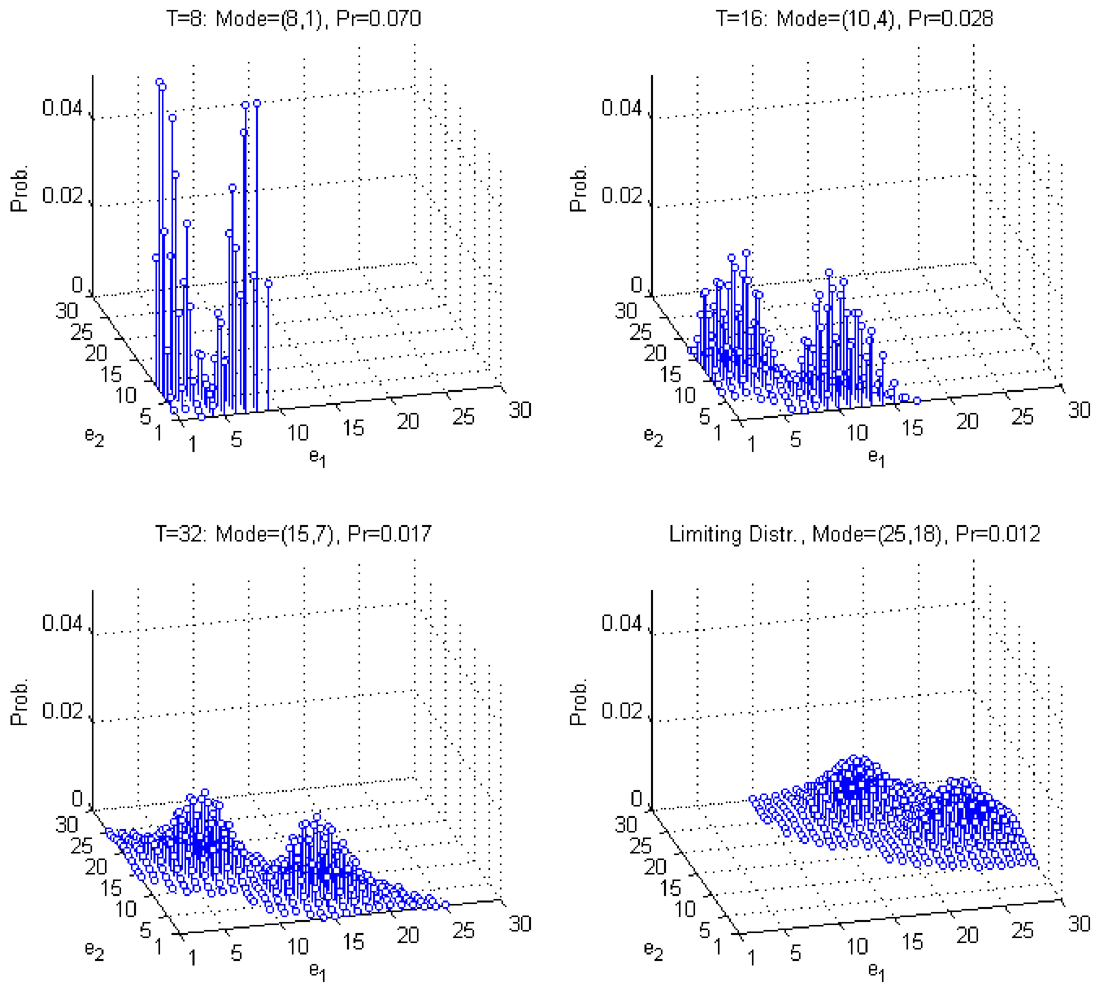
Indicates nature of equilibrium:
F = flat equilibrium and flat equilibrium with well.
T = trenchy equilibrium.
X = extra-trenchy equilibrium.

Closer Look at Trenchy Equilibrium for $\rho = 0.85$, $\delta = 0.03$

- Firms are likely to end up in different know-how positions (though modal position in long run implies both firms achieve marginal cost $c(15)$).
- Trench extends all along the diagonal: price war will occur if follower moves into a position to challenge the leader.
- Threat of price war supports asymmetric structure



Transient and Limiting Distributions for $\rho = 0.85$ and $\delta = 0.03$, Trenchy Equilibrium



Entry and Exit

- Maximum of two firms in industry. Duopoly states (e_1,e_2), $e_n \geq 1$; monopoly states $(e_1,0)$, $e_1 \geq 1$ and $(0,e_2)$, $e_2 \geq 1$.

- If firm exits, it leaves for good; its "slot" is taken by another potential entrant.

- Timing
  - Each incumbent draws a random scrap value X from a distribution $F(X)$ and potential entrant (if any) draws a set-up cost S from a distribution $G(S)$.
  - Incumbents and potential entrants make entry and exit decisions based on privately observed scrap values and set-up costs.
  - Price competition takes place between active firms.
  - Entry and exit decisions generate equilibrium operating probability $\lambda^*(e_1,e_2)$, e.g.,
    - $\lambda^*(0,5)$ = entrant's probability of entry when incumbent rival is in state 5.
    - $\lambda^*(3,5)$ = incumbent's probability of not exiting when incumbent rival is in state 3 and rival is in state 5.

Predatory Pricing

- Equilibrium with entry and exit yields behavior that “feels” like predatory pricing
  - An industry leader’s equilibrium price is below its marginal cost
  - If follower subsequently exits, leader raises price substantially
- Does equilibrium give rise to predatory pricing?

Economic Definitions of Predatory Pricing

- Ordover-Willig (1981): "A response to a rival that sacrifices part of the profit that could have been earned under competitive circumstances, **were the rival to remain viable**, in order to induce exit and gain consequent monopoly profit" (emphasis added).

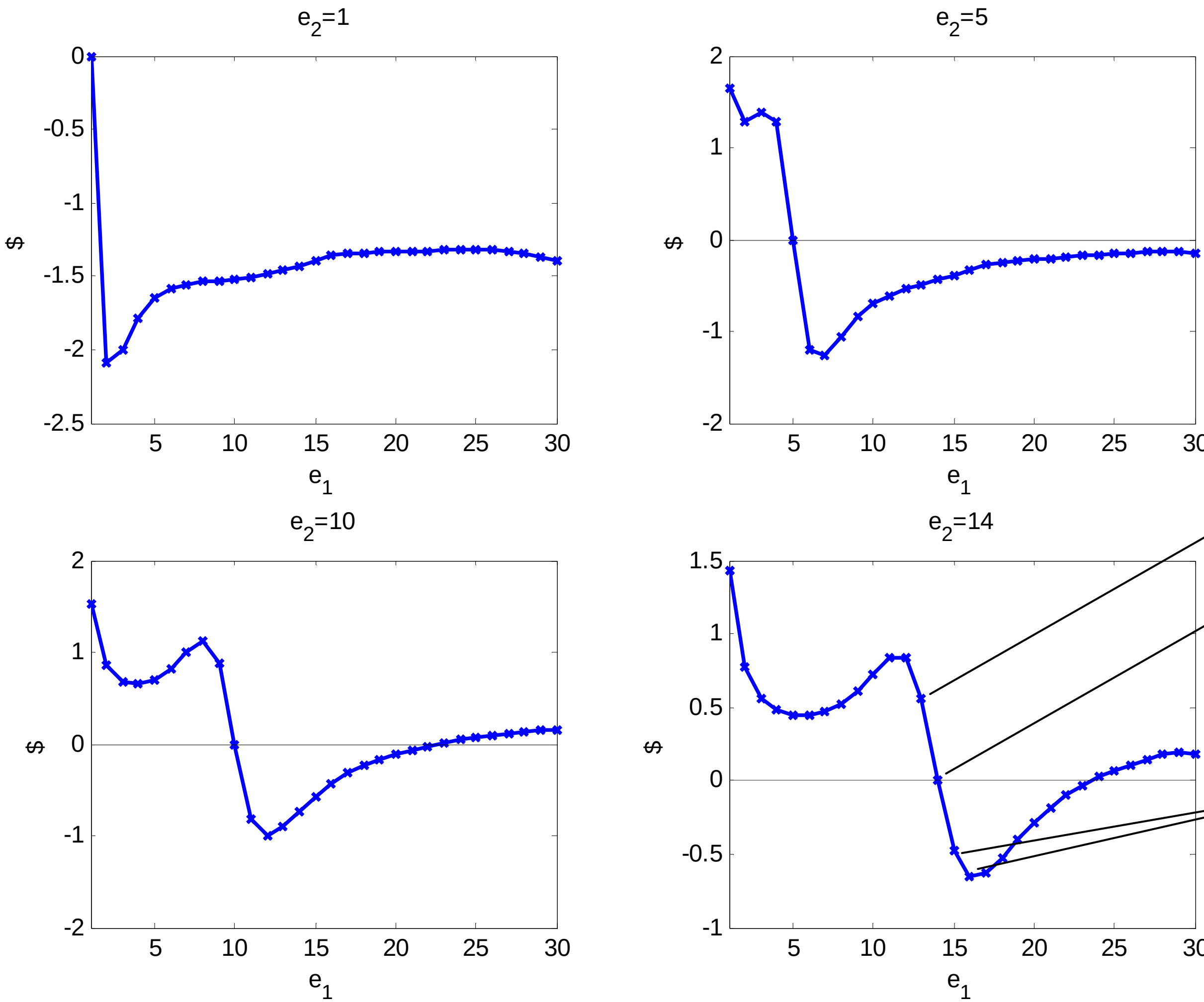
- Cabral-Riordan (1997): "An action is predatory if (1) a different action would increase the likelihood would remain viable and (2) a different action would be more profitable under the counterfactual hypothesis **that the rival's viability were unaffected**" (emphasis added).

- Imply different counterfactuals:
  - Ordover-Willig: compare actual price to price that would prevail in a world in which firms are assumed not to exit.
  - Cabral-Riordan: compare actual price to price that would prevail in a world in which the industry leader acted as if its price had no effect on the likelihood of its rival exiting.

Testing for Predatory Pricing in Our Model

- Compute counterfactual equilibrium:
  - No exit (Ordover-Willig).
  - Firms act "as if" industry cannot transition to a state in which rival might exit next period $\Rightarrow$ firms "ignore" impact of current pricing decision on next-period probability of exit (Cabral-Riordan).
- Consider asymmetric states $e_1 > e_2$ and compare equilibrium price of the low-price firm in model with entry and exit to the price the firm would have charged in the counterfactual equilibrium:
  - If actual equilibrium price of low-price firm in state (e_1,e_2) < counterfactual price of that firm in state (e_1,e_2), we have predatory pricing.

Another Way to Look at Trenches: Price Differences for Selected States for $\rho = 0.85$, $\delta = 0.03$ (Trenchy Equilibrium)



$p^*(e_1,14) - p^*(14,e_1)$

Firms charge same price in state (14,14)

Leader aggressively defends its advantage by under-pricing follower in states (15,14) and (16,14).

Homotopy path following: overview

- Have system of n equations with n unknowns, parameterized by δ:

  $$F(x;\delta)=0 \quad ,F:\Re^n\times[0,1]\to\Re$$

  and a solution for (wlog) $\delta = 0$, denoted x_0
- Homotopy traces out the level set of F in (x,δ) space:

  $$F^{-1}(0)=\{(x,\delta):F(x,\delta)=0\}$$

  by using the Jacobian of F w.r.t. both x and δ.
  The path starts at $(x_0, 0)$ and continues until $\delta = 1$.
- If the path intersects itself, Jacobian becomes singular, and homotopy cannot continue.
- In our case:
  - x = price and value for each state
  - F = residuals of FOC and Bellman equations for each state
  - n = 1800 = 30*30*2
- We used a version of HOMPACK package to perform path-following

Homotopy Methods

- Describe equilibrium by a system of $2M^2$ equations with $2M^2 + 1$ unknowns:

$$H : \Re^{2M^2+1} \to \Re^{2M^2}$$

$$H(p,V,\delta) = \begin{bmatrix} FOC_{e_1,e_2}(p,V,\delta) \\ Bellman_{e_1,e_2}(p,V,\delta) \end{bmatrix} = \begin{bmatrix} 0 \\ V \end{bmatrix}.$$

  Note that the system is indeterminate because of the inclusion of the "extra variable" δ.

- Parameterize the $2M^2 + 1$ unknowns with a new scalar s: $Y(s) = [p(s), V(s), \delta(s)]$. Trace out the level set of equilibrium values as a function of s by implicitly differentiating the system with respect to s and solving the resulting set of differential equations using a known equilibrium as a starting point. Since the system is indeterminate and only one solution is needed, this amounts to solving

$$\frac{\mathrm{d}y_i}{\mathrm{d}s} = (-1)^i \det\left(\mathrm{d}H_{-i}\right), i = 1 : M^2 + 1$$

  where $\mathrm{d}H_{-i}$ is the Jacobian of the system with the i-th column removed

- We use L.T. Watson's public domain package Easy_HOMPACK.

Why Do We Get a Flat Equilibrium with no OF?

- Consider subgame beginning in state $(m,m-1)$: leader at bottom of learning curve with cost $c(m)$; follower one state back with cost $c(m-1)$:

Leader wins wins sale

$(m,m-1)$

$(m,m-1) \Rightarrow V(m,m-1)$

$(m,m) \Rightarrow p(m,m)$ = static Nash price for $c(m)$

$V(m,m) = \sigma/(1 - \beta)$

Follower wins sale

- For parameters in our calculations, in state $(m,m-1)$ leader's prize is modest:
  - Key point: with some degree of product differentiation, follower eventually *will* catch up ...
  - ... and if β is close enough to 1, the value of "delaying the inevitable" is modest.
  - Implication: leader under-prices follower, but not by very much in state $(m, m\text{-}1)$.

- Folding back: winning the next sale in state $(m-1,m-1)$ is not that valuable $\Rightarrow$ prices close to static Nash prices.
  - $\Rightarrow$ Leader not too aggressive in state $(m-2,m-1)$ ... and so on.
  - Leader's non-aggressiveness at bottom of learning curve keeps follower "in the game," i.e., willing to keep price close to leader's so that it eventually catches up.

OF Predisposes Firms to Fight Hard on Price in Symmetric States ...

- Why? LBD with OF is like racing down an up escalator.

- In state $(e,e) \geq (m,m)$, firms set static Nash price without OF ...

- ... but with OF, they set prices below the static Nash prices. With OF, winning next sale is valuable because if you don't win, you might forget (worsening your competitive position), and if you do win, your rival might forget (improving your competitive position).

- In symmetric states near bottom of the learning curve, OF increases the prize (relative to no OF), which intensifies price competition to gain an advantage. This effect becomes stronger as δ increases.

... Which Causes the Leader to Aggressively Defend its Advantage Close to the "Diagonal"

$(e,e-1)$	*Leader wins*	$(e,\ e-1)$	$(e+1,\ e-1)$
		$(e,\ e-2)$	$(e+1,\ e-2)$
	Follower wins	$(e,\ e-1)$	$(e-1,\ e-1)$
		$(e,\ e)$	$(e-1,\ e)$

- Consider sub-game beginning in state (e, e-1):

- OF works to increase leader's prize from winning the next sale ...
  - By winning ... leader *avoids* state (e,e) which is worse with OF than without OF.
  - By winning, follower may slip to state e-2 which cannot occur without OF.

- OF works to decrease follower's prize from winning next sale … and tends to make it smaller than leader's ...
  - By winning, follower may move to state (e,e) where price competition is so intense that its value might actually be lower than in state ($e,e-1$).

- Implication: leader will significantly under-price follower in state ($e,e-1$)
  - Higher prize reinforces lower current MC, and leader's advantage might be more secure in ($e,e-1$) than without OF.

- Folding back: winning the next sale in state ($e-1,e-1$) is more valuable than without OF, so firms more likely to fight hard to win the next sale in this state, reinforcing the predisposition toward aggressive pricing implied by OF.

Background and Motivation (continued)

- Recent empirical work suggests that LBD can be uncertain and transitory: organizations can "forget" the know-how developed through LBD:
  - Argote, Beckman, and Epple (1990) and Thompson (2003) (shipbuilding).
  - Benkard (2000, 2003) (wide-body jets).
  - Darr, Argote, and Epple (1995) (operation of pizza franchises).

- Organizational forgetting (OF) can arise due to:
  - Labor turnover.
  - Failure to institutionalize tacit knowledge gained from LBD, leading to inability to replicate know-how in future production periods and/or loss of know-how during periods of inactivity

- Does OF undermine the economic power of LBD as a source of competitive advantage?
  - If firms do not inexorably move down a learning curve, is it reasonable to expect that a firm with a learning-based cost advantage would be able to extend, or even preserve, its advantage over time?

- This paper: studies fully dynamic model of price competition in a differentiated product oligopoly with LBD and OF.

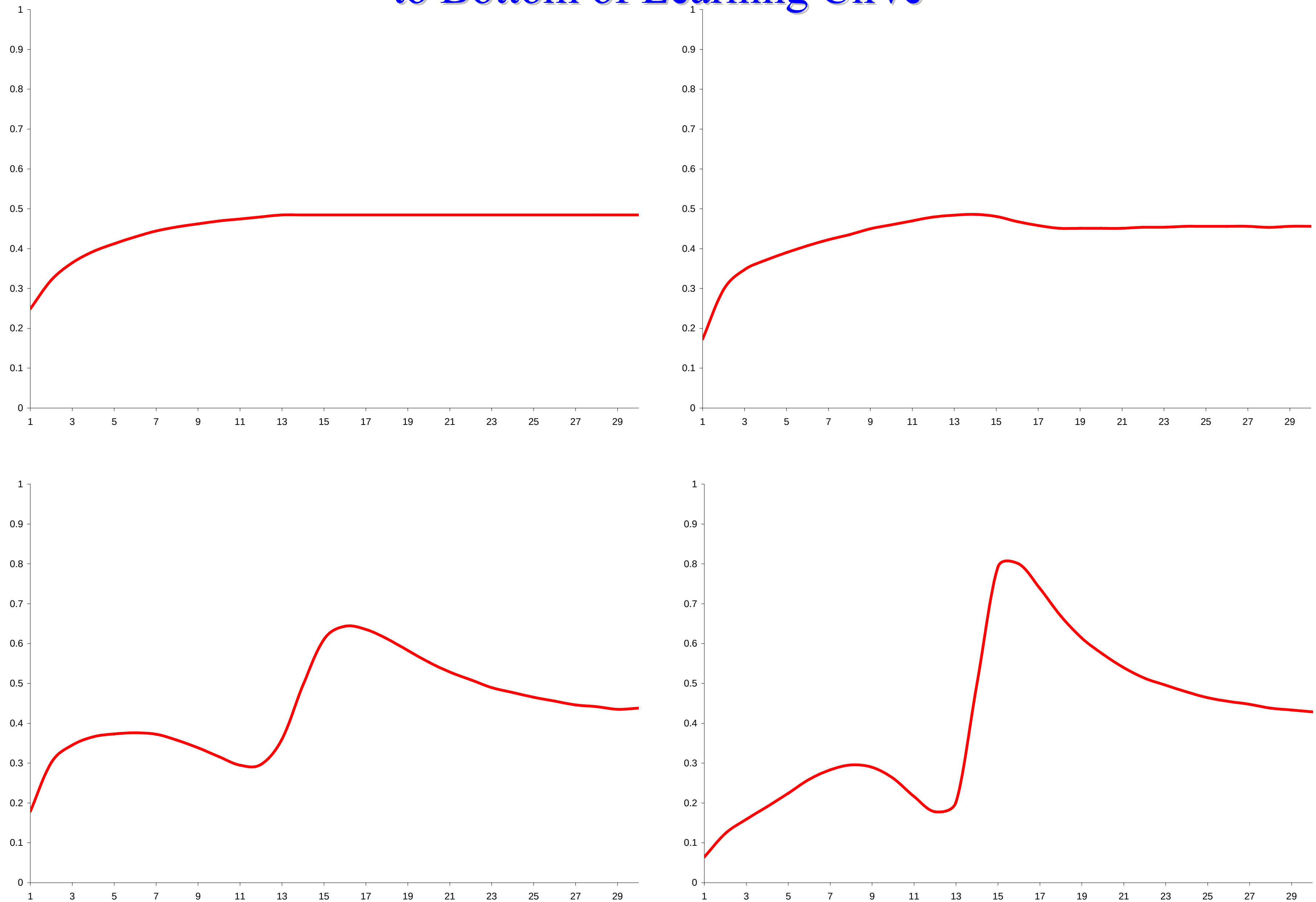
What We Also Did

- Exit and entry
  - When follower can exit & not re-enter, competition intensifies further
  - Get more multiple equilibria, including for no-forgetting case.

- Policy experiment: Ban below-cost pricing
  - For most parameterizations, cannot avert asymmetry or improve welfare.

- *N* firms
  - Hard to analyze (can’t plot, computing power limits *N* at 5)
  - Similar competition patterns

Equilibrium Probability Firm 1 Wins Next Sale, Firm 2 Close to Bottom of Learning Cirve



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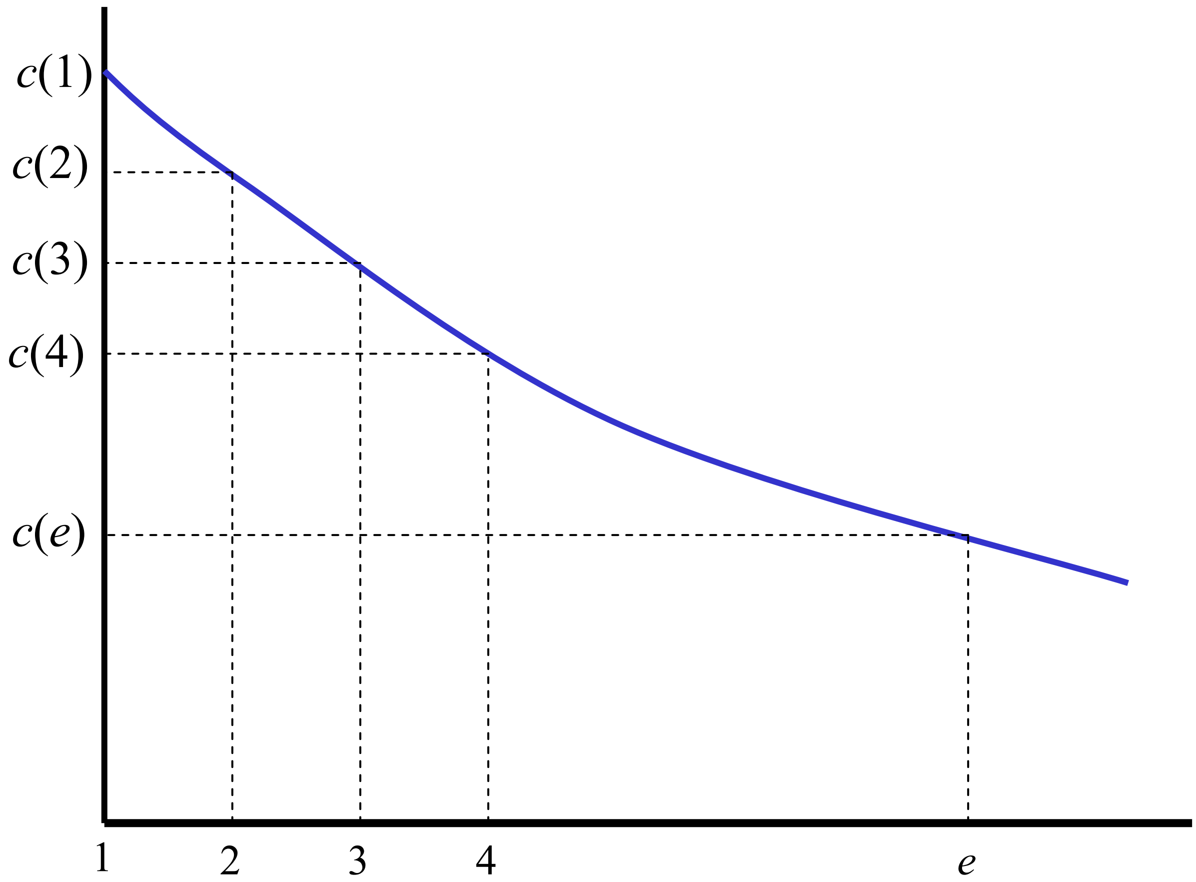

$c(1)$
$c(2)$
$c(3)$
$c(4)$
$c(e)$
1
2
3
4
e

1 unit of demand
per period (e.g., year)

$\beta = 1/(1+r)$

$c(e)$ (see below)

$\delta(e)$

learning curve
flattens at m

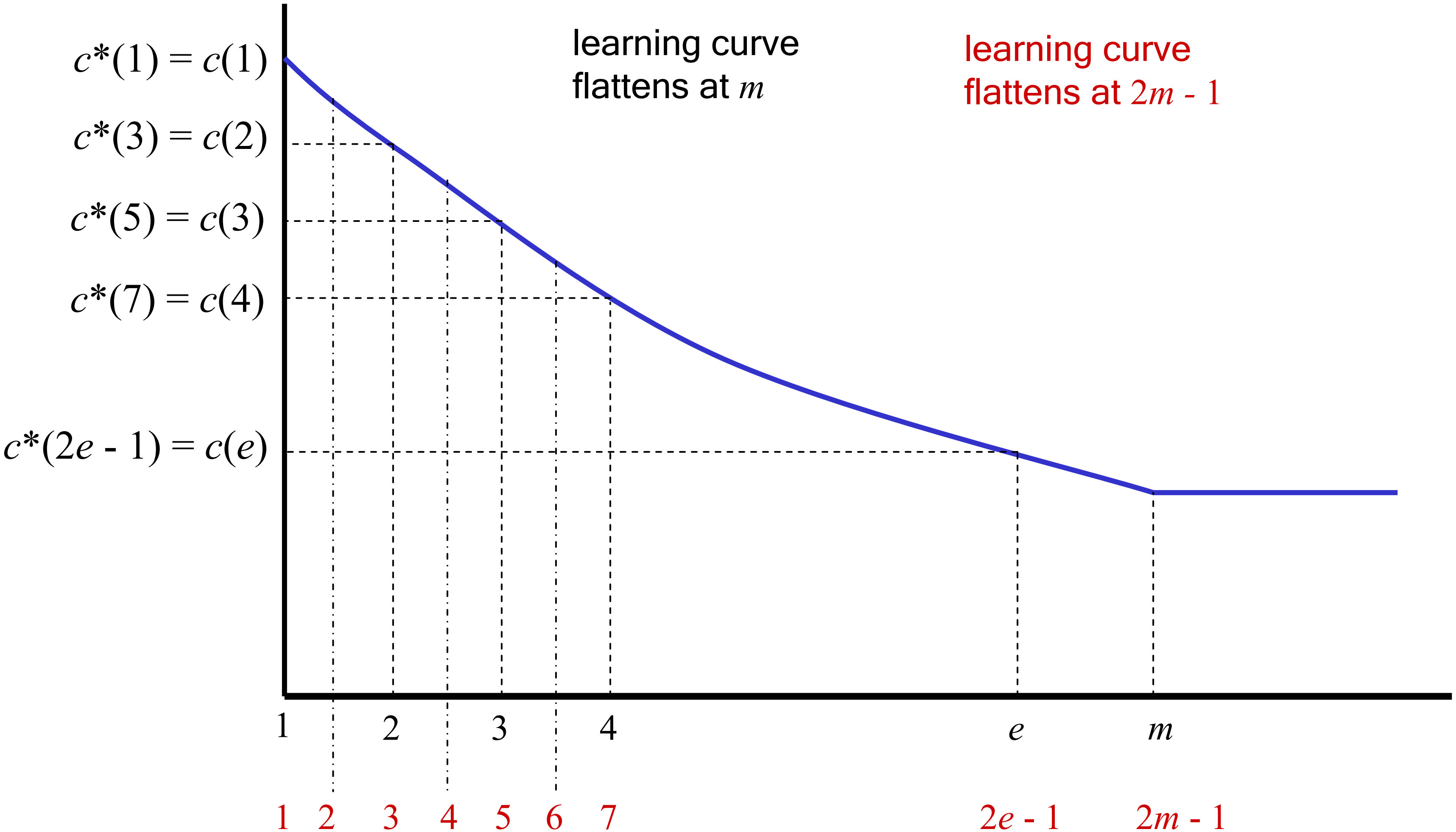
2 units of demand
per period (e.g., year)

$\beta^* = 1/(1+r/2)$

$c^*(e) = c(2e\text{-}1)$

$\delta^*(e) = \delta(2e\text{-}1)$

learning curve
flattens at $2m$ - 1



Entry, Exit, and Predatory Pricing

- Extend model to include entry and exit

- Predatory pricing: equilibrium price of low-price firm in state (e_1,e_2), $e_1 > e_2$, is less than the price that would prevail in a counterfactual model in which
  - firms never exit (Ordover and Willig, 1981)
  - firms act "as if" industry cannot transition to a state in which rival exits in the next period $\Rightarrow$ firms "ignore" impact of current pricing decision on next-period probability of exit (Cabral and Riordan, 1997)

Ordover-Willing counterfactual

$\rho = 0.85$

δ	Predatory pricing: above cost	Predatory pricing: below cost	Non-predatory: below cost
0	0%	0%	0.7%
0.01	0%	0%	0.7%
0.02	0%	0%	0.7%
0.03(F)	19.8%	0%	1.1%
0.03(T)	11.5%	7.8%	1.6%
0.04(F)	0%	0.2%	2.3%
0.04(T)	8.5%	14.3%	1.8%
0.05	23.7%	13.1%	1.4%
0.06	49.7%	8.5%	0.5%
0.07	50.6%	7.6%	0%
0.08	60.5%	9.2%	0%
0.09	63.7%	10.1%	0%
0.10	62.8%	10.6%	0.2%
0.30	38.9%	9.2%	0.7%
0.50	33.1%	7.1%	2.3%
0.70	31.5%	6.4%	3.0%
0.90	29.4%	6.4%	3.0%

Cabral-Riordan counterfactual

$\rho = 0.85$

δ	Predatory pricing: above cost	Predatory pricing: below cost	Non-predatory: below cost
0	0%	0%	0.7%
0.01	0%	0%	0.7%
0.02	0%	0%	0.7%
0.03(F)	15.6%	0%	1.1%
0.03(T)	11.5%	7.8%	1.6%
0.04(F)	0%	0.2%	2.3%
0.04(T)	3.4%	13.6%	1.8%
0.05	18.2%	10.6%	1.4%
0.06	40.5%	7.1%	0.5%
0.07	39.1%	7.6%	0%
0.08	48.5%	9.2%	0%
0.09	52.6%	10.1%	0%
0.10	53.8%	10.6%	0.2%
0.30	38.2%	9.2%	0.7%
0.50	32.9%	7.1%	2.3%
0.70	31.3%	6.4%	3.0%
0.90	29.2%	6.4%	3.0%

Economic Definitions of Predatory Pricing

- Ordover-Willig (1981): "A response to a rival that sacrifices part of the profit that could have been earned under competitive circumstances, **were the rival to remain viable**, in order to induce exit and gain consequent monopoly profit" (emphasis added).

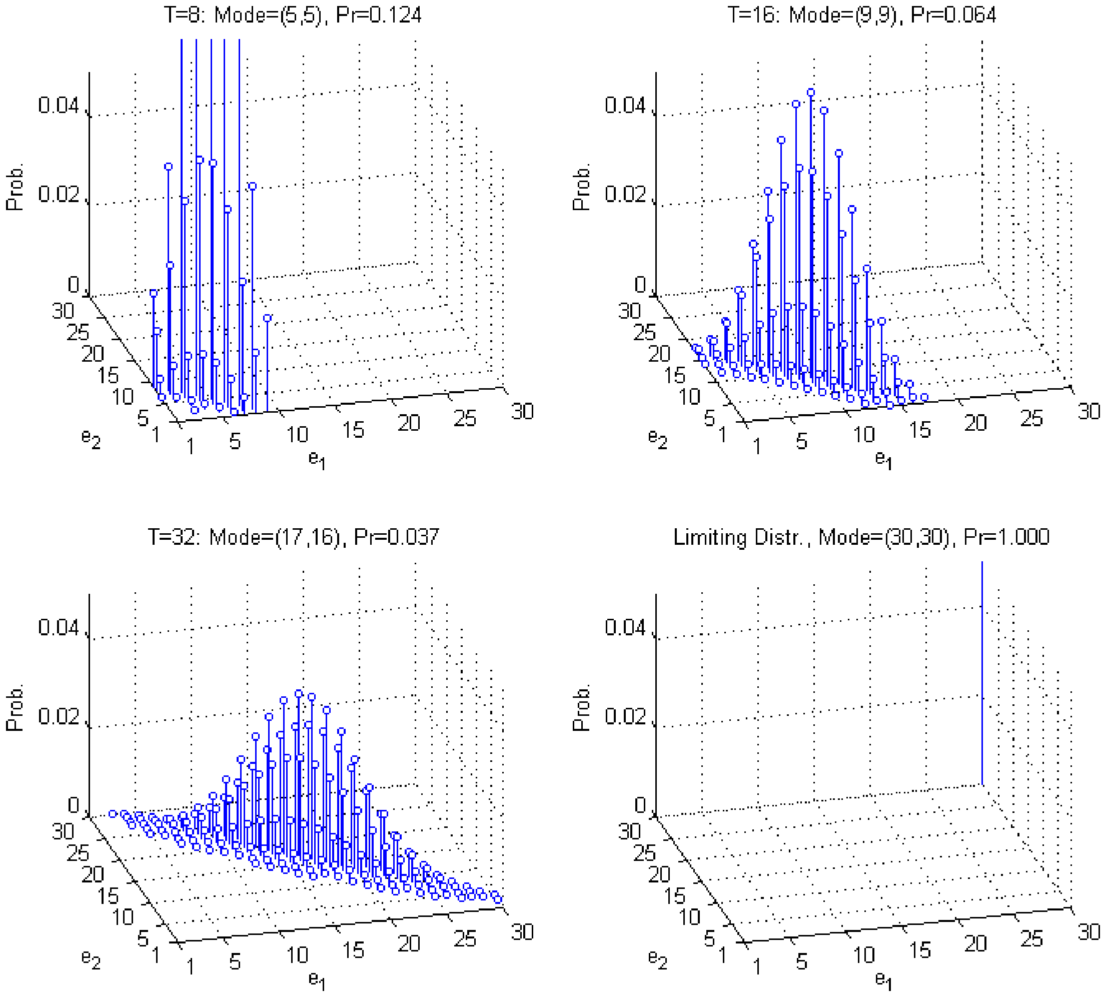
- Cabral-Riordan (1997): "An action is predatory if (1) a different action would increase the likelihood would remain viable and (2) a different action would be more profitable under the counterfactual hypothesis **that the rival's viability were unaffected**" (emphasis added).

- Imply different counterfactuals:
  - Ordover-Willig: compare actual price to price that would prevail in a world in which firms are assumed not to exit.
  - Cabral-Riordan: compare actual price to price that would prevail in a world in which the industry leader acted as if its price had no effect on the likelihood of its rival exiting.

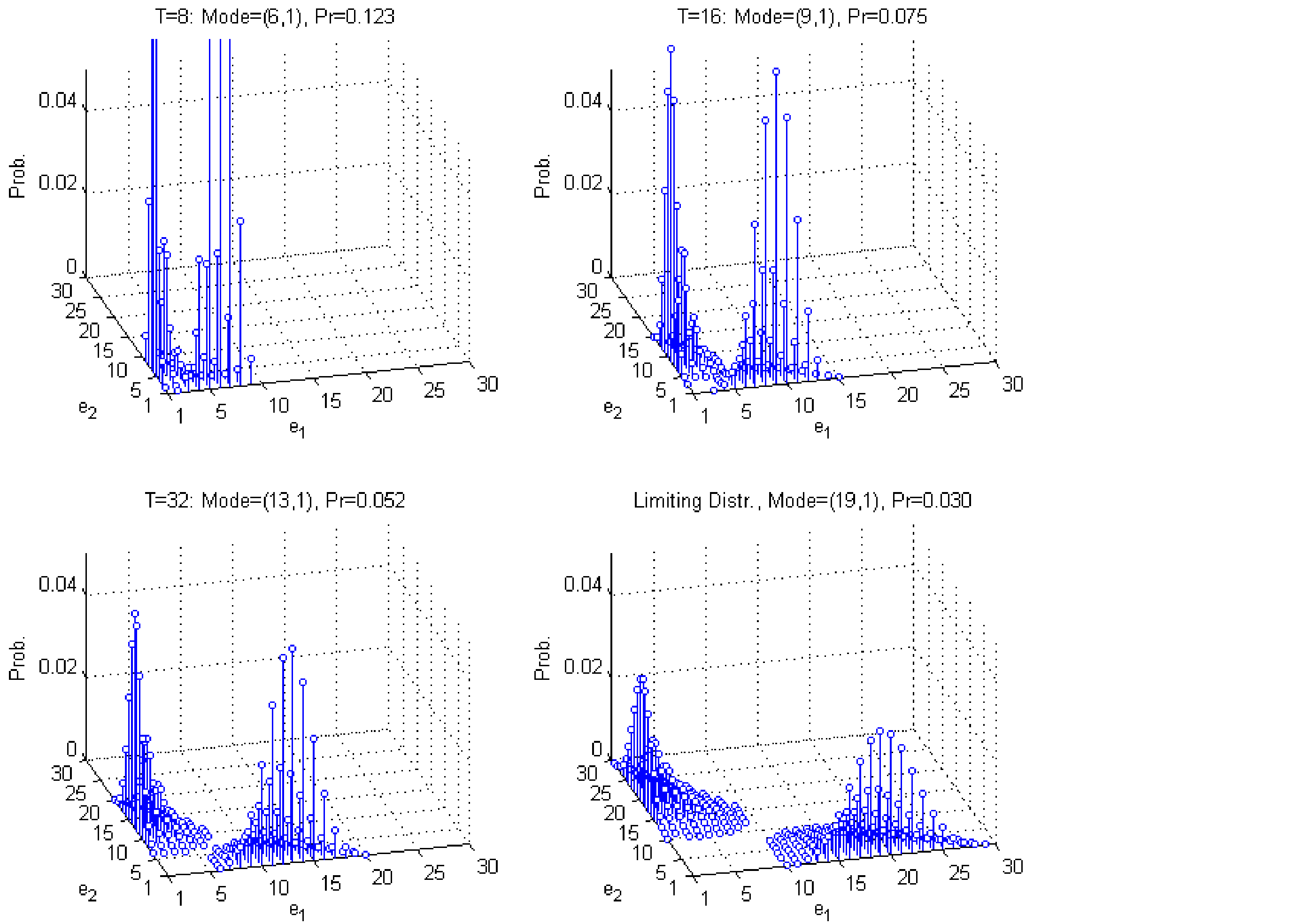
Pricing Behavior: Effect of Organizational Forgetting

- Key point: in any state (e_1,e_2), a firm can expect to augment its know-how only if probability of making the next sale exceeds the probability of forgetting
  - If firms have equal stocks of know-how, probability of making the next sale ≤ 0.5 .
  - But given a sufficiently large δ, there will be sufficiently large e such that probability of forgetting exceeds 0.5.
  - Implication: even though *one* firm could easily move to bottom of learning curve, it may be *extremely* difficult for *two* firms to move to the bottom of the learning curve in tandem.
- This gives rise to two offsetting effects on pricing behavior
  - **Investment-stifling effect:** *Why invest in accumulating know-how (via price cuts) when I know my know-how gains are likely to be transitory (and my rival's gains are transitory, too*)?
    - **Softens** price competition (relative to no-OF case) at **"top"** of learning curve (where cost reductions from accumulating more know-how are greatest).
    - Has minimal impact on pricing at "bottom" of learning curve (where cost reductions from accumulating more know-how are minimal, so it doesn't matter that much that gains are transitory).
  - **Preemption effect**: *Only one of us can remain at the bottom of the learning curve: it's going to be me!*
    - **Intensifies** price competition (relative to no-OF case) between symmetric firms at "bottom" of the learning curve.
    - This effect "cascades back" through state space, intensifying price competition when both firms are at top of the learning curve.

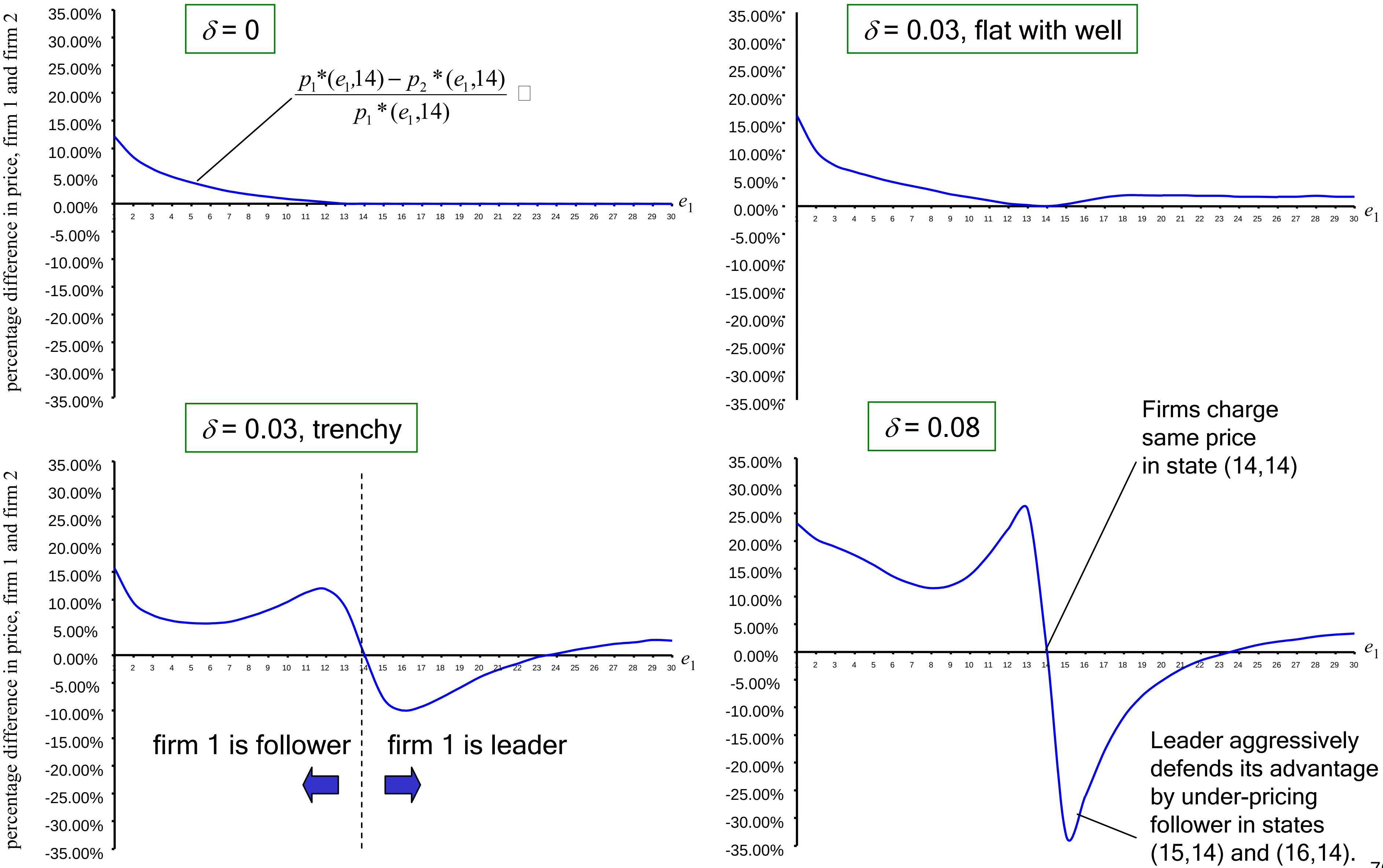
Transient and Limiting Distributions for $\rho = 0.85$ and $\delta = 0$



Transient and Limiting Distributions for $\rho = 0.85$ and $\delta = 0.08$



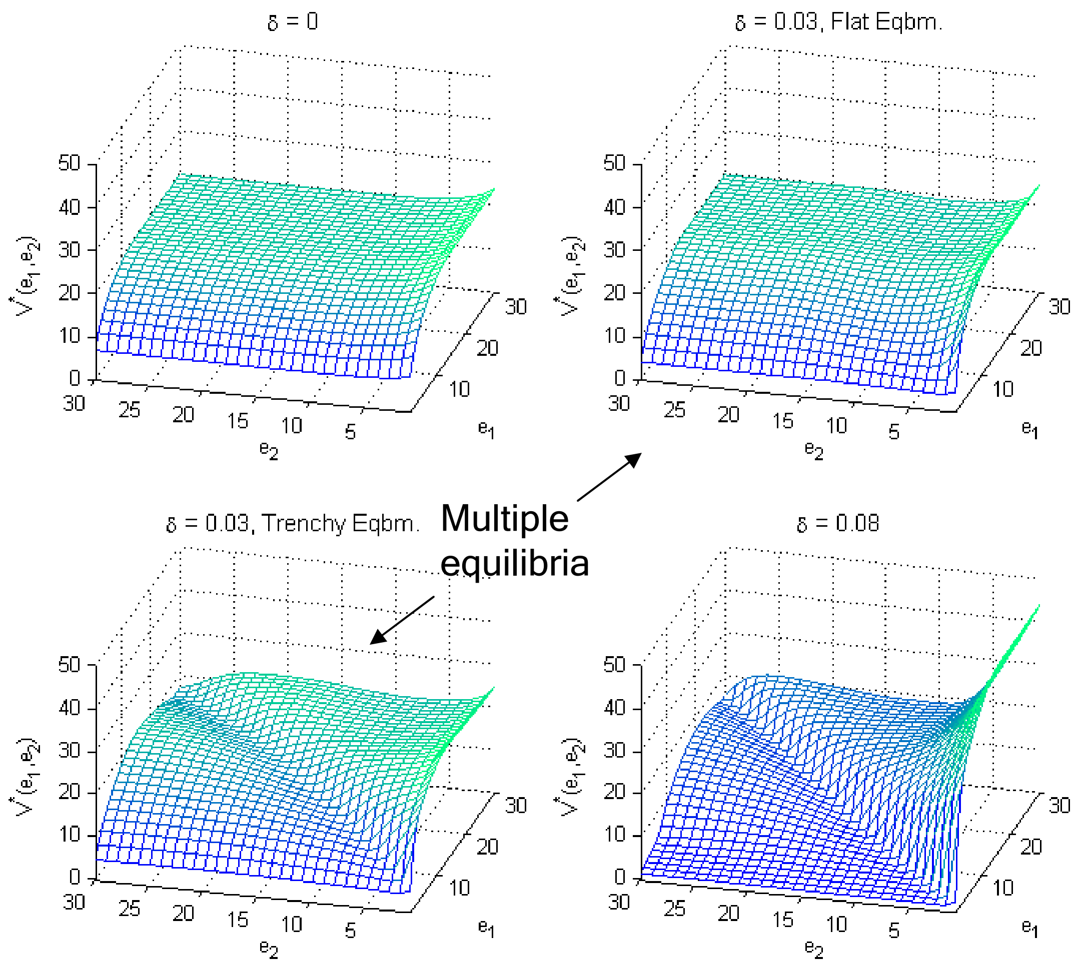
Equilibrium Price Differences: Firm 2 close to bottom of learning curve ($e_2 = 14$).



A Selection Story I:
Value Function for $\rho = 0.85$ and $\delta \in \{0, 0.03, 0.08\}$

Flat equilibrium

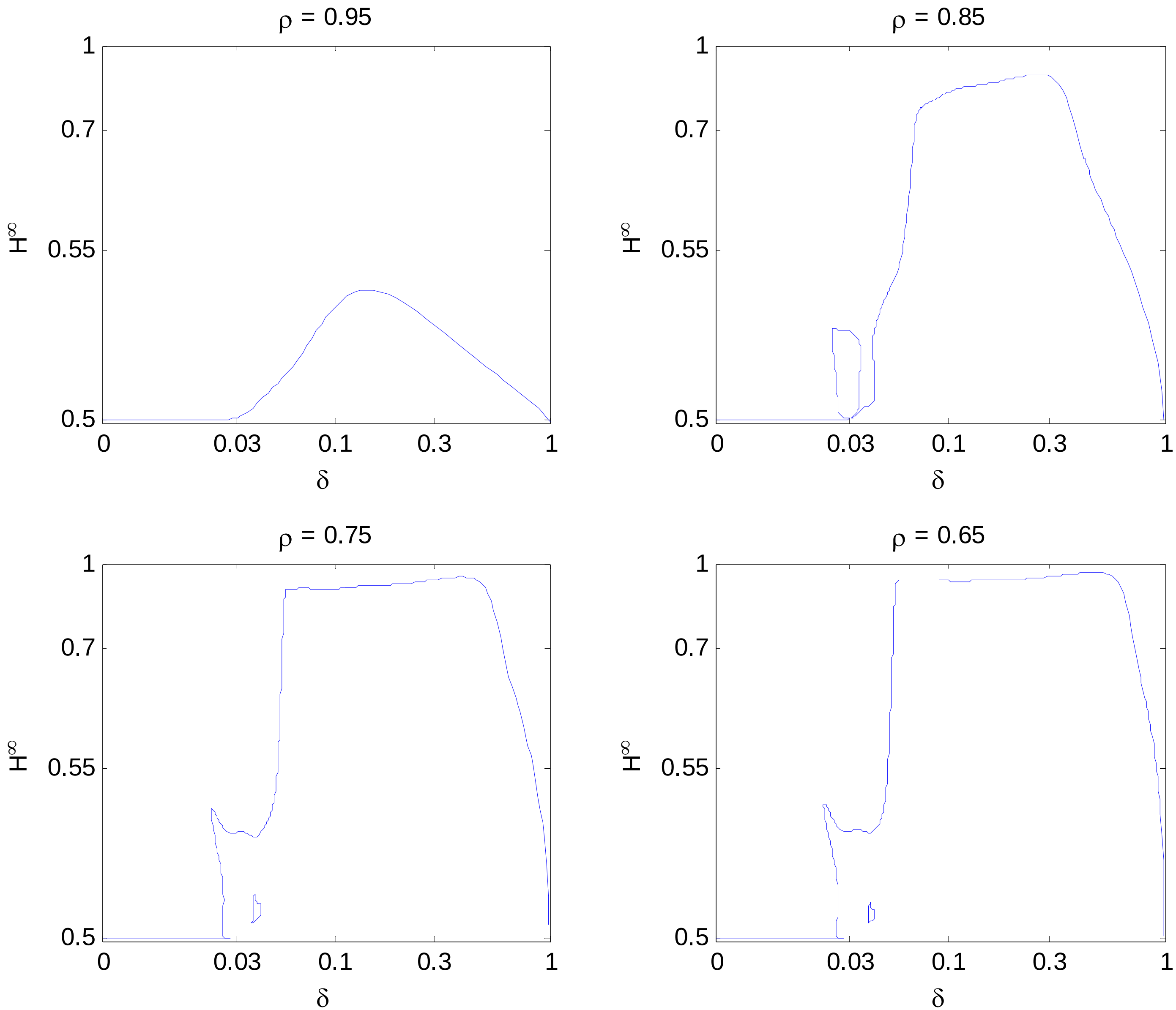
Flat equilibrium with "well



Trenchy equilibrium

Extra-trenchy equilibrium

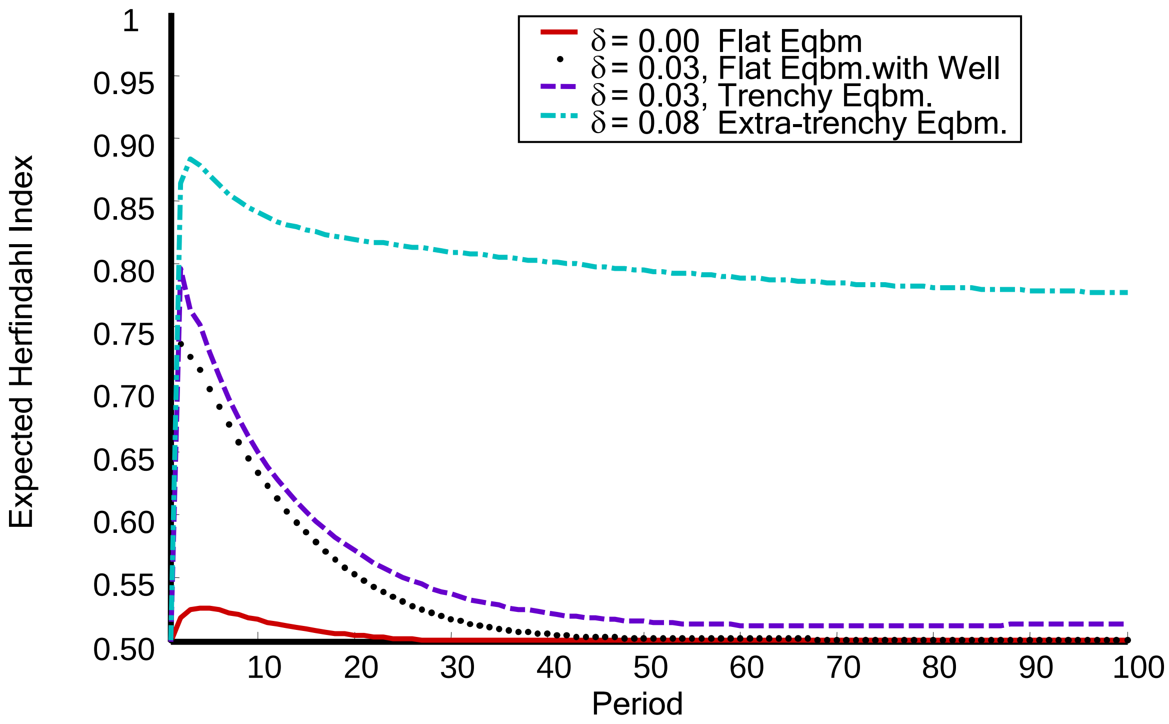
Equilibrium Correspondence (Expected Long-Run Herfindahl)



Increasing Dominance (ID) and Increasing Increasing Dominance (IID)

- Cabral and Riordan, 1994; Athey and Schmutzler, 2002) emphasized the properties of increasing dominance (ID) and increasing increasing dominance (IID)
  - ID: leader (firm that is further down the learning curve) sets lower price than follower.
  - IID: leader's incentive to under-price follower grows as its lead grows.
- Cabral and Riordan's main result, which applies to our model too, is that if there is no OF ($\delta = 0$) and the discount factor is close to unity ($\beta \approx 1$), then IID (and perforce ID) holds. But the equilibrium is flat and symmetry holds in the long run.
- ID and IID are not necessary for market dominance. Consider our fourth illustrative equilibrium:
  - $\rho = 0.85$ and $\delta = 0.08$
  - Extra-trenchy equilibrium
  - If the leader has a big enough lead, then in some states it coasts and charges a higher price than the follower. Not even dominance holds, even though asymmetry in both the short and long run is extreme.

Overview of Equilibria: Time Path of Expected Herfindahl Index for ρ=0.85 and $\delta \in$ {0, 0.03, 0.08}



- Approach:
  - Compute transient distribution for period T, $\pi^T(e_1,e_2)$, from equilibrium pricing function $p^*(e_1,e_2)$
  - Compute expected value of Herfindahl index H^T based on this distribution:

$$H^T = \sum_{e_1=1}^{M} \sum_{e_2=1}^{M} \frac{D^*(e_1,e_2)^2 + D^*(e_2,e_1)^2}{[D^*(e_1,e_2) + D^*(e_2,e_1)]^2} \pi^T(e_1,e_2), \text{ where } D^*(e_1,e_2) = D_1(p^*(e_1,e_2), p^*(e_2,e_1)).$$

Ongoing Robustness Checks

- Attractiveness of the outside alternative
- Product differentiation
- Linear vs. logit demand
- Change in the rate at which the product is demanded
- Constant rate of forgetting
- Alternative transition probabilities