# OBJECTIVE RISK PREFERENCES IMPLIED BY ATTITUDES TOWARD AMBIGUITY

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# SEPARATING "OBJECTIVE RISK PREFERENCES"
# FROM BELIEFS, FROM STATE-DEPENDENCE, AND FROM AMBIGUITY

Say we only know an agent's subjective act preferences in some highly specific setting, that involves:

Completely Subjective Uncertainty $(\mathcal{S} = [\underline{s}, \overline{s}])$

State-Dependence

Perceived "Ambiguity"

Can we predict their behavior in any *other* subjective setting, where they may have:

*Different* Subjective Beliefs? $(\mathcal{T} = [\underline{t}, \overline{t}])$

*Different* State-Dependence?

*Different* perceived Ambiguity?

Can we predict their behavior in an "Idealized Casino" – that is, any setting where they may have:

*Completely Probabilistic* Beliefs?

*No* State-Dependence?

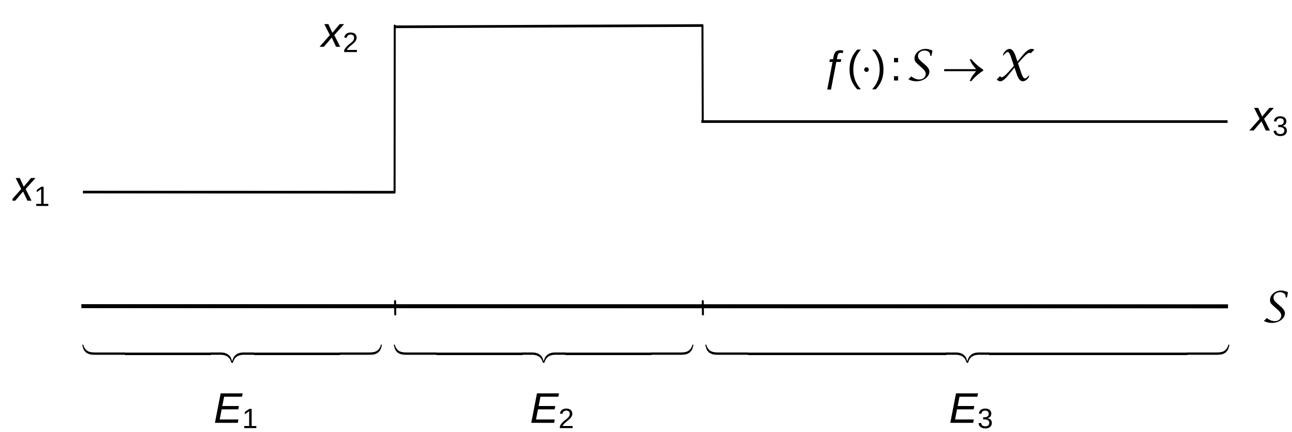
*No* perceived Ambiguity?

Answer: Even though an agent's *beliefs, state-dependence,* and *ambiguity* can differ across settings, their *attitudes toward objective risk* can always be recovered, and *will be the same* across settings.

# OBJECTIVE VERSUS SUBJECTIVE UNCERTAINTY

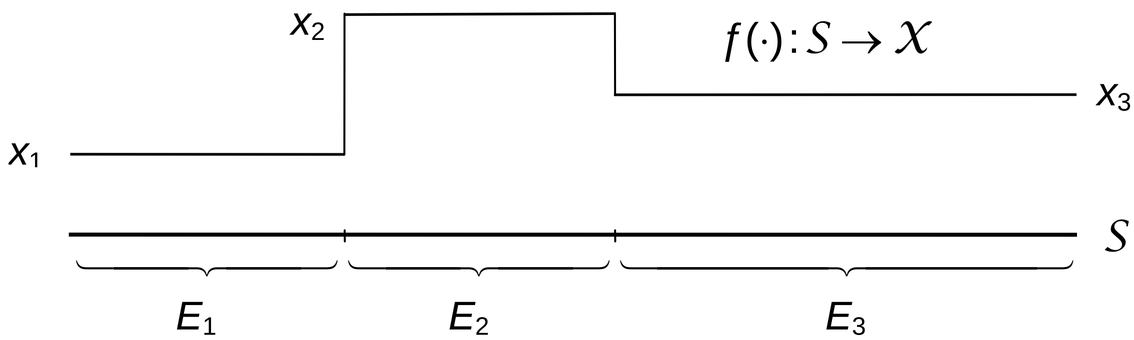
*Outcomes:* $\mathcal{X} = \{\ldots,x,\ldots\}$

| ***OBJECTIVE UNCERTAINTY:*** | | ***SUBJECTIVE UNCERTAINTY:*** | |
|---|---|---|---|
| *Probabilities*: | $p \in [0,1]$ | *States & Events*: | $\mathcal{S} = \{\ldots,s,\ldots\}$, $\mathcal{E} = \{\ldots,E,\ldots\}$ |
| *Lotteries*: | $\mathbf{P} = (x_1,p_1;\ldots;x_n,p_n)$ | *Acts*: | $f(\cdot) = [x_1 \text{ on } E_1 ; \ldots ; x_n \text{ on } E_n]$ |
| *Pref. Function*: | $V(\mathbf{P}) = V(x_1,p_1;\ldots;x_n,p_n)$ | *Pref. Function*: | $W(f(\cdot)) = W(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n)$ |
| *Exp. Utility form*: | $V_{EU}(\mathbf{P}) = U(x_1)\cdot p_1 + \ldots + U(x_n)\cdot p_n$ | *SEU form*: | $W_{SEU}(f(\cdot)) = U(x_1)\cdot\mu(E_1) + \ldots + U(x_n)\cdot\mu(E_n)$ |



# PASSING FROM SUBJECTIVE TO OBJECTIVE BETTING PREFERENCES

Consider an individual ranking bets under *completely subjective* uncertainty:




Say the individual satisfies the Savage Axioms, so they maximize

$$W_{SEU}(f(\cdot)) \equiv \int_{\mathcal{S}} U(f(s)) \cdot d\mu(s) \equiv \sqrt{x_1} \cdot \mu(E_1) + \ldots + \sqrt{x_n} \cdot \mu(E_n)$$

**Question**:

If this *same individual* were in a Las Vegas casino, how would they rank the following *objective bets:*

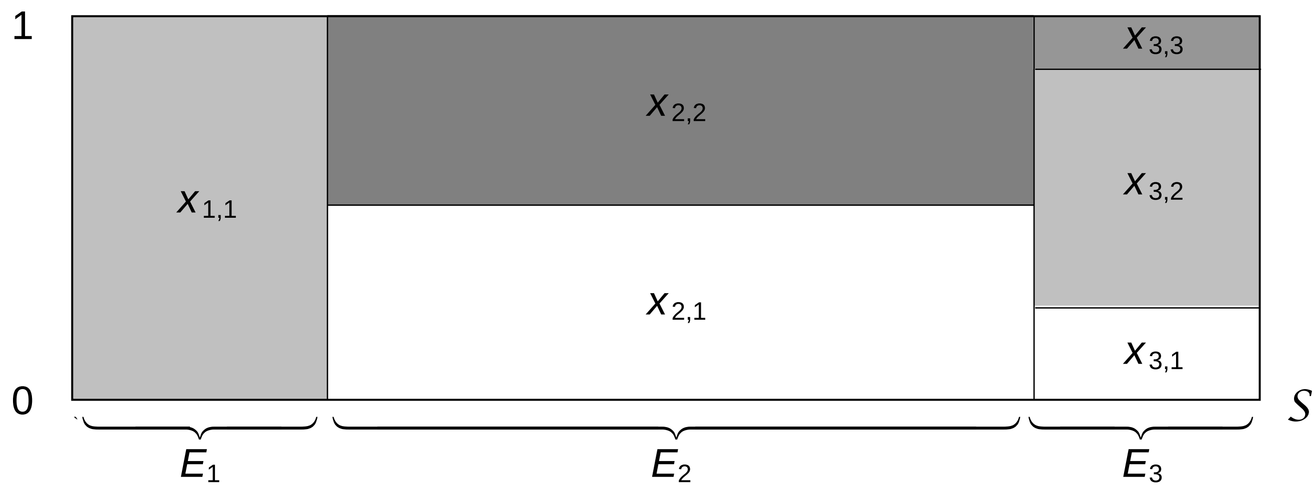
50:50 chance of \$100:\$0 versus \$25 with certainty

If your answer was "They'd be indifferent," did you just perform an act of

*Logical Deduction?* or *Scientific Induction?*

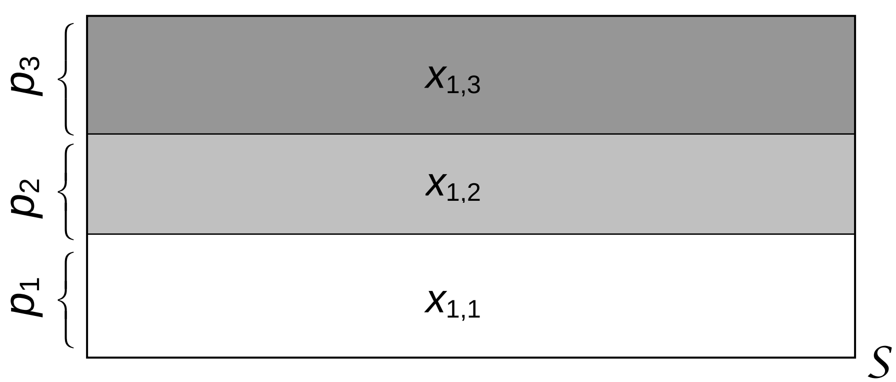
# REPRESENTING JOINT SUBJECTIVE×OBJECTIVE BETS

We can represent subjective×objective (“Anscombe-Aumann”) bets by:

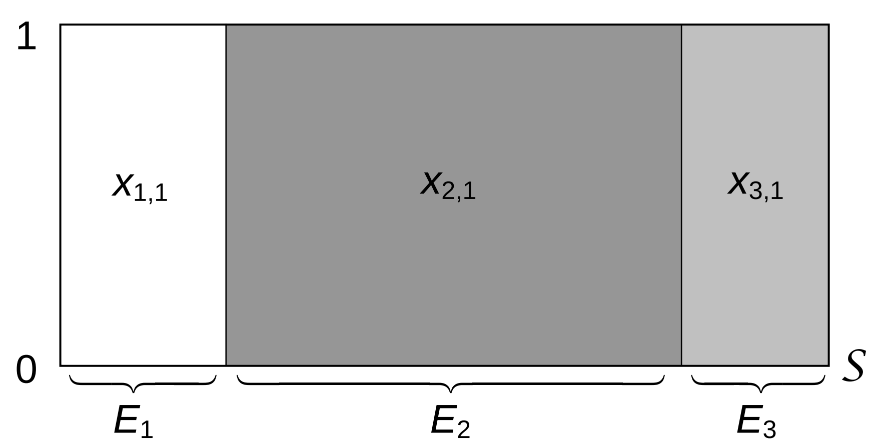



$$\alpha(\cdot) \;=\; [\;\ldots\;;\;(x_{i,1},p_{i,1};\ldots;x_{i,n_i},p_{i,n_i}) \text{ on } E_i\;;\;\ldots\;]$$

A typical purely objective act:




A typical purely subjective act:



# DO PREF'S OVER SUBJECTIVE ACTS AND OVER OBJECTIVE LOTTERIES

## *IMPLY ANYTHING ABOUT EACH OTHER* ?

Consider *Duncan's* preference function over joint subjective×objective bets:

$$W_{Duncan}(\ \ldots ; (x_{i,1},p_{i,1};\ldots;x_{i,n_i},p_{i,n_i}) \text{ on } E_i\ ; \ldots\ ) \equiv \sum_i \Big[\sum_j U(x_{i,j})\cdot\Big(G\big(\sum_{k=1}^{j} p_{i,k}\big) - G\big(\sum_{k=1}^{j-1} p_{i,k}\big)\Big)\Big]\cdot\mu(E_i)$$

Duncan has *expected utility* (*"Savage"*) preferences over *purely subjective acts:*

$$W_{Duncan}(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n) \equiv \sum_i \big[U(x_i)\cdot(G(1)-G(0))\big]\cdot\mu(E_i) \equiv \sum_i U(x_i)\cdot\mu(E_i)$$

but Duncan has *rank-dependent* (*"Quiggin"*) preferences over *purely objective lotteries:*

$$W_{Duncan}(x_1,p_1;\ldots;x_n,p_n) \equiv \sum_j U(x_j)\cdot\Big(G\big(\sum_{k=1}^{j} p_k\big) - G\big(\sum_{k=1}^{j-1} p_k\big)\Big)$$

Conversely, consider *Don's* preference function over joint subjective×objective bets:

$$W_{Don}(\ \ldots ; (x_{i,1},p_{i,1};\ldots;x_{i,n_i},p_{i,n_i}) \text{ on } E_i\ ; \ldots\ ) \equiv \sum_i \Big[\sum_j U(x_{i,j})\cdot p_{i,j}\Big]\cdot\Big[G\big(\sum_{k=1}^{i} \mu(E_k)\big) - G\big(\sum_{k=1}^{i-1} \mu(E_k)\big)\Big]$$

Don has *rank-dependent* (*"Choquet"*) preferences over *purely subjective acts:*

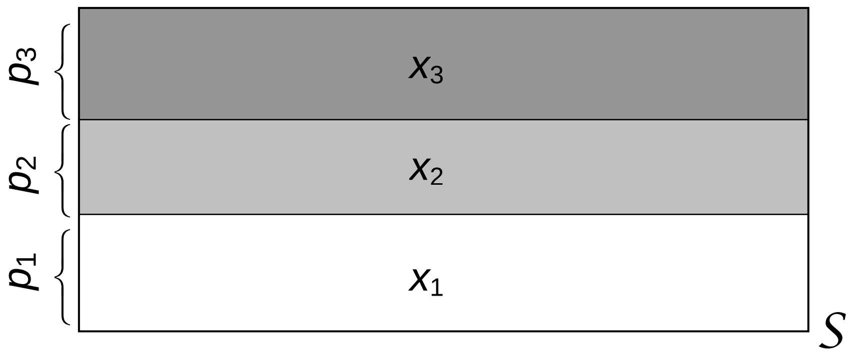
$$W_{Don}(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n) \equiv \sum_i U(x_i)\cdot\Big[G\big(\sum_{k=1}^{i} \mu(E_k)\big) - G\big(\sum_{k=1}^{i-1} \mu(E_k)\big)\Big]$$

but Don has *expected utility* preferences over *purely objective lotteries:*

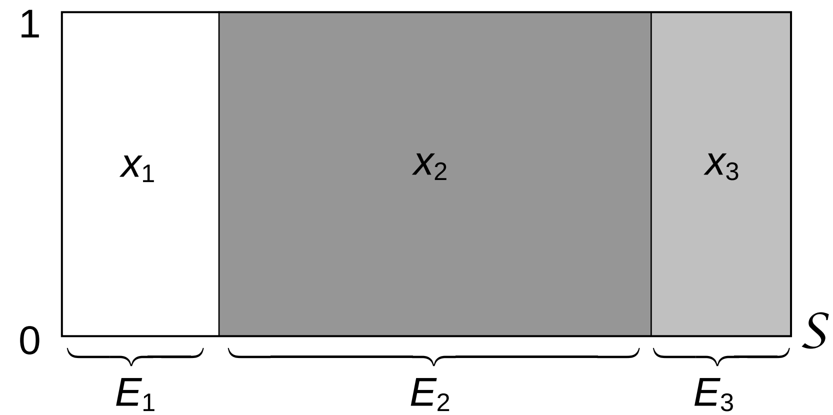
$$W_{Don}(x_1,p_1;\ldots;x_n,p_n) \equiv \sum_j U(x_j)\cdot p_j$$

# THE FAMILY OF SUBJECTIVE ACTS vs THE FAMILY OF OBJECTIVE LOTTERIES

$\mathcal{Obj}$ = {objective lotteries}
= {*horizontally-striped* bets}

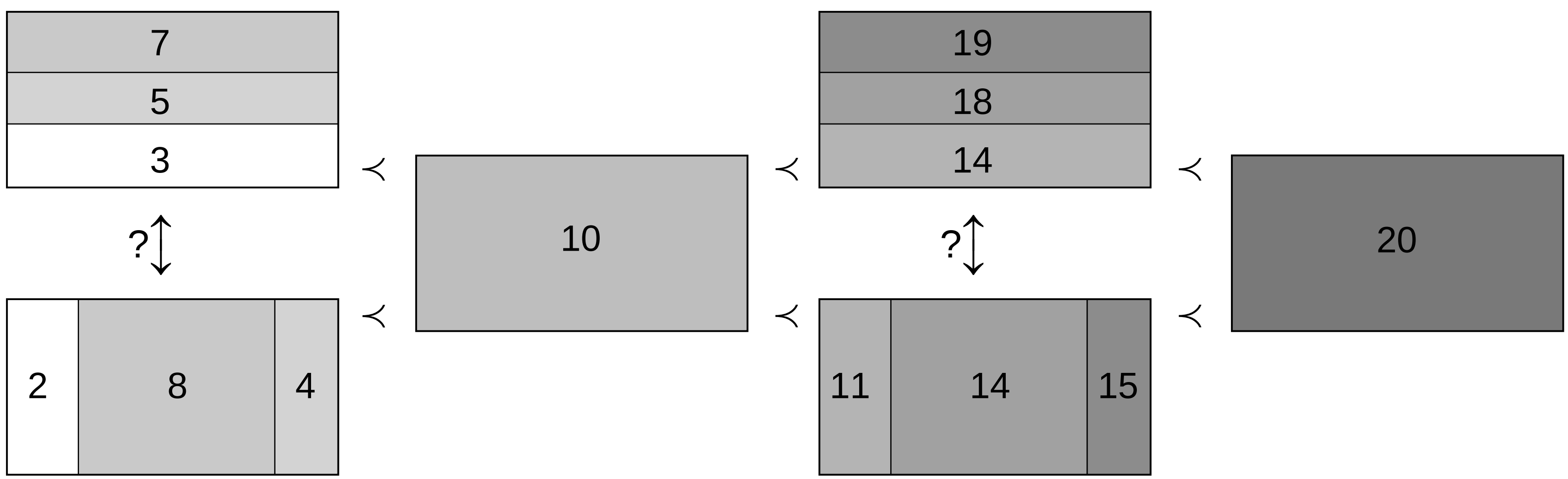



$\mathcal{Sub}$ = {purely subjective acts}
= {*vertically-striped* bets}




The families $\mathcal{Obj}$ and $\mathcal{Sub}$ have only the *constant bets* in common.

Does "Dominance" imply anything about preferences across these two families?  Not much:



# EVENT–SMOOTHNESS

**DEF**: A preference function $W(\cdot)$ over purely subjective acts $f(\cdot) = [x_1 \text{ on } E_1;...;x_n \text{ on } E_n]$ on $\mathcal{S}$ is said to be ***event-additive*** if there exists a family of signed *evaluation-measures* $\{\Phi_x(\cdot) \,|\, x \in \mathcal{X}\}$ on $\mathcal{S}$ such that

$$W\big(x_1 \text{ on } E_1;...;x_n \text{ on } E_n\big) \;=\; \Phi_{x_1}(E_1) + \ldots + \Phi_{x_n}(E_n) \quad \text{or equivalently} \quad W\big(f(\cdot)\big) = \textstyle\sum_{x\in\mathcal{X}} \Phi_x\big(f^{-1}(x)\big)$$

*Subjective Expected Utility* and *State-Dependent Expected Utility* are both event-additive:

$$W_{SEU}\big(x_1 \text{ on } E_1;...;x_n \text{ on } E_n\big) \;=\; U(x_1)\cdot\mu(E_1) \,+ \ldots +\, U(x_n)\cdot\mu(E_n) \qquad \text{thus} \qquad \Phi_x(E) \,\equiv\, U(x)\cdot\mu(E)$$

$$W_{SDEU}\big(x_1 \text{ on } E_1;...;x_n \text{ on } E_n\big) \;=\; \int_{E_1} U(x_1|s)\cdot d\mu(s) + \ldots + \int_{E_n} U(x_n|s)\cdot d\mu(s) \qquad \text{thus} \qquad \Phi_x(E) \,\equiv\, \int_E U(x|s)\cdot d\mu(s)$$

**DEF**: A preference function $W(\cdot)$ over purely subjective acts $f(\cdot) = [x_1 \text{ on } E_1;...;x_n \text{ on } E_n]$ on $\mathcal{S}$ is said to be ***event-smooth*** at act $f_0(\cdot)$ if there exists a family of signed *local evaluation-measures* $\{\Phi_x(\cdot;f_0) \,|\, x \in \mathcal{X}\}$ on $\mathcal{S}$ such that

$$W(f(\cdot)) - W(f_0(\cdot)) \;=\; \sum_{x\in\mathcal{X}} \Phi_x\big(f^{-1}(x);f_0\big) - \sum_{x\in\mathcal{X}} \Phi_x\big(f_0^{-1}(x);f_0\big) \;+\; o\big(\delta(f(\cdot),f_0(\cdot))\big)$$

where the *distance function between acts* is defined by $\delta\big(f(\cdot),f_0(\cdot)\big) \equiv \lambda\big\{s \in \mathcal{S} \,|\, f(s) \neq f_0(s)\big\}$.

We also assume that $W(\cdot)$'s *local phi-densities* $\{\phi_x(\cdot;f) \,|\, x \in \mathcal{X}\}$ vary continuously in $s$ and $f(\cdot)$.

# A MORE SCIENTIFIC APPROACH
# TO THE "OBJECTIVE" vs. "SUBJECTIVE" DISTINCTION

Let's replace the Pascal / Savage / Anscombe-Aumann *exogenous classification* of "objectively uncertain settings" vs. "subjectively uncertain settings" by more scientific approach.

Such an approach must be *defined in terms of observed behavior,* but it must also have a *theoretical underpinning*.

This means we must ask two questions:

What *observable properties of decision makers' attitudes toward an event* should lead us to define it as "objective" ?

Can we *theoretically predict* when a given event will, or will not, be "objective" ?

And a final question:

Is it *processes* or *events* that are "objective" or "subjective" ?

# SIX PROPERTIES OF "PURELY OBJECTIVE" EVENTS

1. ***Unanimous, Outcome-Invariant Revealed Likelihoods:*** All individuals exhibit identical likelihood beliefs over purely objective events, given by their objective probabilities.

2. ***Independence from Arbitrary Subjective Events:*** Likelihood beliefs over purely objective events are independent of the realization of any *fixed subjective event*.

3. ***Probabilistic Sophistication over Objective Lotteries:*** Preferences over objectively uncertain lotteries depend only on their outcomes and corresponding probabilities.

4. ***Reduction of Objective×Subjective Uncertainty:*** Agents evaluate any *objective mixture of subj. acts*

$$\alpha \bullet [x_1 \text{ on } E_1;...;x_n \text{ on } E_n] \;\oplus\; (1-\alpha)\bullet[x_1^* \text{ on } E_1^*;...;x_{n^*}^* \text{ on } E_{n^*}^*]$$

   according to its implied map $[\,\ldots;(x_i,\alpha;x_j^*,1-\alpha) \text{ on } E_i \cap E_j^*;\ldots]$ from events to objective lotteries.

The above properties hold for *all individuals*. We also have two *more specialized* properties:

5. ***Under Probabilistic Sophistication, Two-Way Independence of Objective & Subjective Events:*** For a probabilistically sophisticated agent with subjective probability measure $\mu(\cdot)$, subjective event prob's are *independent of the realization of objective events,* and vice versa.

6. ***Under Expected Utility, Linearity in Objective Probabilities and in Objective Mixtures:*** Both state-independent and state-dependent expected utility are *linear in objective probabilities*, and both are *linear in objective mixtures of subjective acts*.

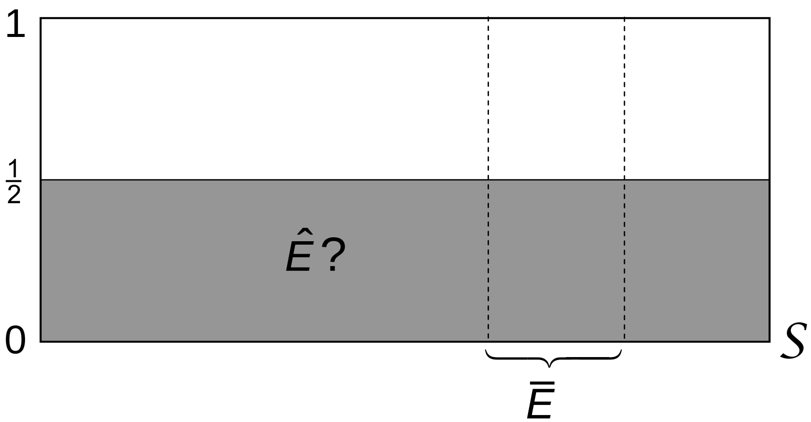
***<u>None</u> of these six properties hold for <u>general subjective events</u>.***

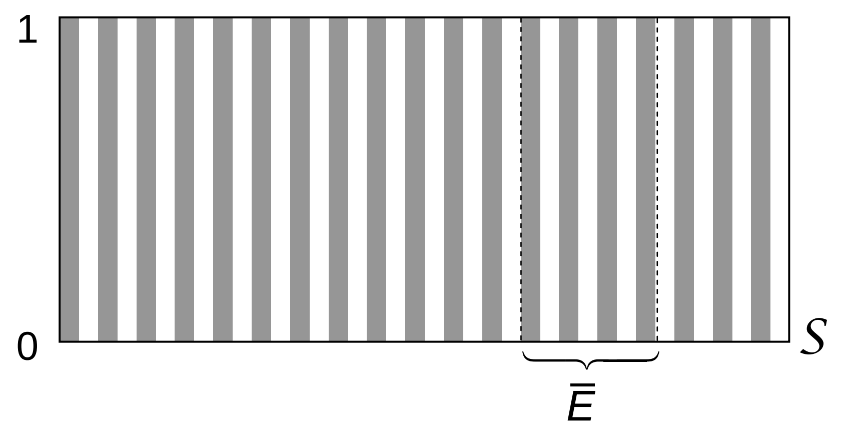
# AM I DESCRIBING AN *OBJECTIVE* OR A *SUBJECTIVE* EVENT?

My event $\hat{E}$ has the following properties:

- Everyone (even Ellsberg) has a revealed likelihood for $\hat{E}$
- Everyone has the same revealed likelihood for $\hat{E}$
- Everyone is indifferent between $[\$100 \text{ on } \hat{E}; \$0 \text{ on } {\sim}\hat{E}]$ and $[\$0 \text{ on } \hat{E}; \$100 \text{ on } {\sim}\hat{E}]$
- For any *fixed subjective event* $\bar{E}$, everyone is indifferent between

$$[\$100 \text{ on } \hat{E}\cap\bar{E};\ \$0 \text{ otherwise}] \text{ and } [\$100 \text{ on } {\sim}\hat{E}\cap\bar{E};\ \$0 \text{ otherwise}]$$

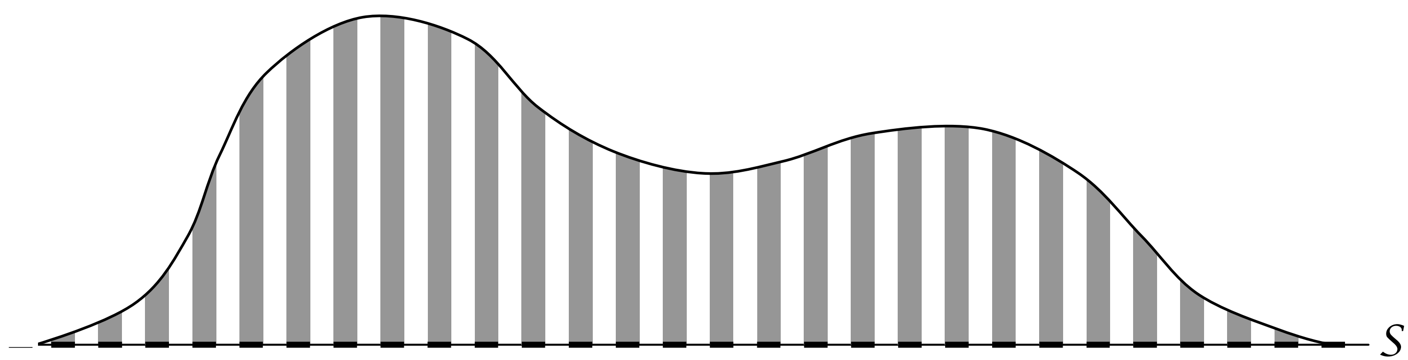



**Poincaré's Theorem**:

For any bounded smooth density $\nu(\cdot)$ on $[\underline{s},\bar{s}]$

$$\lim_{m\to\infty}\int_{E_m}\nu(s)\cdot ds = \frac{1}{2}$$

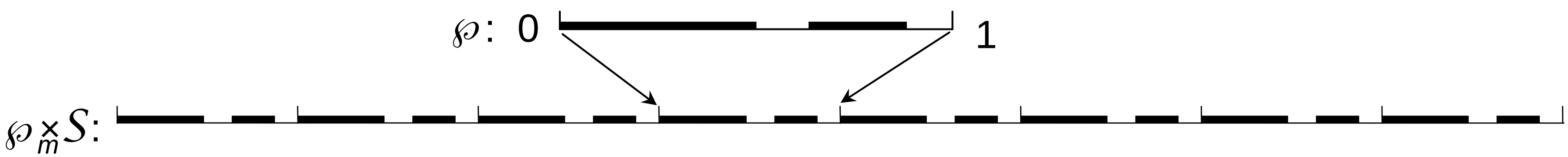


# PURELY SUBJECTIVE EVENTS WITH (ALMOST) PURELY OBJECTIVE PROPERTIES

Dividing each interval into *thirds* would approximate *objective events of probability* 1/3, and so forth…

Divide the state space $S = [\underline{s}, \bar{s}]$ into *m* equal-length intervals $I_i = \left[\, \underline{s} + \frac{i \cdot \lambda_S}{m}, \underline{s} + \frac{(i+1) \cdot \lambda_S}{m} \right)$

For any *finite interval union* $\wp \subseteq [0,1]$, define the ***almost-objective event*** $\wp \underset{m}{\times} S \;\equiv\; \bigcup_{i=0}^{m-1} \left\{ \underline{s} + \frac{(i+\omega) \cdot \lambda_S}{m} \,\middle|\, \omega \in \wp \right\}$




For each *m*, consider the mapping $\wp \to \wp \underset{m}{\times} S$ from subsets $\wp \subseteq [0,1]$ to subjective events $\wp \underset{m}{\times} S$ :

Disjointness is preserved: $\wp \cap \wp' = \varnothing \;\;\Rightarrow\;\; (\wp \underset{m}{\times} S) \cap (\wp' \underset{m}{\times} S) = \varnothing$

Unions & Intersections are preserved: $(\wp \cup \wp') \underset{m}{\times} S = (\wp \underset{m}{\times} S) \cup (\wp' \underset{m}{\times} S)$ and $(\wp \cap \wp') \underset{m}{\times} S = (\wp \underset{m}{\times} S) \cap (\wp' \underset{m}{\times} S)$

Partitions are preserved: $\{\wp_1, \ldots, \wp_n\}$ partitions $[0,1] \;\;\Rightarrow\;\; \{\wp_1 \underset{m}{\times} S, \ldots, \wp_n \underset{m}{\times} S\}$ partitions $S$

But most important:

For an *arbitrary continuous-density probability measure* $\mu(\cdot)$ on $S$: $\lim_{m \to \infty} \mu(\wp \underset{m}{\times} S) \equiv \lambda(\wp)$

# ALMOST-OBJECTIVE SUBEVENTS, ACTS AND MIXTURES

### *Almost-Objective Subevents*

Given arbitrary subjective event $E \subseteq \mathcal{S}$, define : $\quad \wp \underset{m}{\times} E \;=\; (\wp \underset{m}{\times} \mathcal{S}) \cap E$

Unions, Intersections and Complements are preserved : $\quad (\wp \cup \wp') \underset{m}{\times} E \;=\; (\wp \underset{m}{\times} E) \cup (\wp' \underset{m}{\times} E)$, etc.

For arbitrary continuous-density measure $\mu(\cdot)$ : $\quad \lim_{m \to \infty} \mu(\wp \underset{m}{\times} E) \;=\; \lambda(\wp) \cdot \mu(E)$

### *Almost-Objective Acts*

Given outcomes $x_1,...,x_n$ and partition $\{\wp_1,...,\wp_n\}$ of $[0,1]$, define the *almost-objective act*

$$[\, x_1 \text{ on } \wp_1 \underset{m}{\times} \mathcal{S} \,; \ldots ; x_n \text{ on } \wp_n \underset{m}{\times} \mathcal{S} \,]$$

### *Almost-Objective Mixtures of Purely Subjective Acts*

For subjective acts $f_1(\cdot),...,f_n(\cdot)$ and partition $\{\wp_1,...,\wp_n\}$ of $[0,1]$, define the *almost-objective mixture*

$$[\, f_1(\cdot) \text{ on } \wp_1 \underset{m}{\times} \mathcal{S} ; \ldots ; f_n(\cdot) \text{ on } \wp_n \underset{m}{\times} \mathcal{S} \,]$$

For each outcome $x$, its event is given by $\; E_x \;\equiv\; \wp_1 \underset{m}{\times} f_1^{-1}(x) \cup \ldots \cup \wp_n \underset{m}{\times} f_n^{-1}(x)$

### *Almost-Objective Mixture Path between Two Subjective Acts*

For subj. acts $f(\cdot)$ and $f^*(\cdot)$, define the path $\; f_\alpha^m(\cdot) \;\equiv\; \left[\, f^*(\cdot) \text{ on } [0,\alpha] \underset{m}{\times} \mathcal{S} ; f(\cdot) \text{ on } (\alpha,1] \underset{m}{\times} \mathcal{S} \,\right]$ for $\alpha \in [0,1]$

# UNANIMOUS ATTITUDES TOWARD ALMOST-OBJ. EVENTS, ACTS & MIXTURES

*As $m \to \infty$, all event-smooth $W(\cdot)$ satisfy*

1. ***Unanimous, Outcome-Invariant Revealed Likelihoods***: For all disjoint finite-interval unions $\wp, \wp' \subseteq [0,1]$ with $\lambda(\wp) > (=)\ \lambda(\wp')$, the *almost-objective events* $\wp \underset{m}{\times} S$ and $\wp' \underset{m}{\times} S$ satisfy

$$\lim_{m\to\infty} W\begin{pmatrix} x^* & \text{on } \wp \underset{m}{\times} S \\ x & \text{on } \wp' \underset{m}{\times} S \\ f(\cdot) & \text{elsewhere} \end{pmatrix} > (=) \lim_{m\to\infty} W\begin{pmatrix} x & \text{on } \wp \underset{m}{\times} S \\ x^* & \text{on } \wp' \underset{m}{\times} S \\ f(\cdot) & \text{elsewhere} \end{pmatrix} \qquad \begin{matrix} \text{all } x^* \succ x \\ \text{all } f(\cdot) \end{matrix}$$

2. ***Independence from Arbitrary Subjective Events***: For all disjoint $\wp, \wp' \subseteq [0,1]$ with $\lambda(\wp) > (=)\ \lambda(\wp')$ and subjective events $E$, the *almost-objective events* $\wp \underset{m}{\times} S$ and $\wp' \underset{m}{\times} S$ satisfy

$$\lim_{m\to\infty} W\begin{pmatrix} x^* & \text{on } (\wp \underset{m}{\times} S)\cap E \\ x & \text{on } (\wp' \underset{m}{\times} S)\cap E \\ f(\cdot) & \text{elsewhere} \end{pmatrix} > (=) \lim_{m\to\infty} W\begin{pmatrix} x & \text{on } (\wp \underset{m}{\times} S)\cap E \\ x^* & \text{on } (\wp' \underset{m}{\times} S)\cap E \\ f(\cdot) & \text{elsewhere} \end{pmatrix} \qquad \begin{matrix} \text{all } x^* \succ x \\ \text{all } f(\cdot) \end{matrix}$$

3. ***Probabilistic Sophistication over Almost-Objective Acts***: For each $W(\cdot)$ there exists a preference function $V_W(\cdot)$ over objective lotteries such that every *almost-objective act* satisfies

$$\lim_{m\to\infty} W\big(x_1 \text{ on } \wp_1 \underset{m}{\times} S; \ldots; x_n \text{ on } \wp_n \underset{m}{\times} S\big) \equiv V_W\big(x_1, \lambda(\wp_1); \ldots; x_n, \lambda(\wp_n)\big)$$

4. ***Reduction of Almost-Objective × Subjective Uncertainty***: If two *almost-objective mixtures* $[f_1(\cdot)$ on $\wp_1 \underset{m}{\times} S; \ldots; f_n(\cdot)$ on $\wp_n \underset{m}{\times} S]$ and $[\hat{f}_1(\cdot)$ on $\hat{\wp}_1 \underset{m}{\times} S; \ldots; \hat{f}_{\hat{n}}(\cdot)$ on $\hat{\wp}_{\hat{n}} \underset{m}{\times} S]$ imply probabilistically equivalent almost-objective subacts over each event in the refinement of $\{f_1(\cdot), \ldots, f_n(\cdot), \hat{f}_1(\cdot), \ldots, \hat{f}_{\hat{n}}(\cdot)\}$, then

$$\lim_{m\to\infty} W\big(f_1(\cdot) \text{ on } \wp_1 \underset{m}{\times} S; \ldots; f_n(\cdot) \text{ on } \wp_n \underset{m}{\times} S\big) = \lim_{m\to\infty} W\big(\hat{f}_1(\cdot) \text{ on } \hat{\wp}_1 \underset{m}{\times} S; \ldots; \hat{f}_{\hat{n}}(\cdot) \text{ on } \hat{\wp}_{\hat{n}} \underset{m}{\times} S\big)$$

# SPECIALIZED ATTITUDES TOWARD ALMOST-OBJ. EVENTS, ACTS & MIXTURES

As $m \to \infty$, all continuous-density $W_{PS}(\cdot)$ satisfy 5,
and all continuous-density $W_{SEU}(\cdot)$ & $W_{SDEU}(\cdot)$ satisfy 6

**5. *Under Probabilistic Sophistication, Two-Way Independence of Almost-Obj. & Subjective Events:***

Given a pair of *almost-objective events* $\wp \underset{m}{\times} \mathcal{S}$ and $\wp' \underset{m}{\times} \mathcal{S}$ and a pair of *subjective events* $E$ and $E'$, if either $\{\wp, \wp'\}$ or $\{E, E'\}$ are disjoint, then

$$\lambda(\wp)\cdot\mu(E) \geq \lambda(\wp')\cdot\mu(E') \quad \Rightarrow \quad \lim_{m\to\infty} W_{PS}\begin{pmatrix} x^* & \text{on } (\wp \underset{m}{\times} \mathcal{S})\cap E \\ x & \text{on } (\wp' \underset{m}{\times} \mathcal{S})\cap E' \\ f(\cdot) & \text{elsewhere} \end{pmatrix} \geq \lim_{m\to\infty} W_{PS}\begin{pmatrix} x & \text{on } (\wp \underset{m}{\times} \mathcal{S})\cap E \\ x^* & \text{on } (\wp' \underset{m}{\times} \mathcal{S})\cap E' \\ f(\cdot) & \text{elsewhere} \end{pmatrix} \qquad \begin{matrix} \text{all } x^* \succ x \\ \text{all } f(\cdot) \end{matrix}$$

**6. *Under Expected Utility* (*SEU or SDEU*), *Linearity in Almost-Objective Probabilities & Mixtures:***

Preferences over *almost-objective acts* satisfy :

$$\lim_{m\to\infty} W_{S(D)EU}\big(x_1 \text{ on } \wp_1 \underset{m}{\times} \mathcal{S};\ldots;x_n \text{ on } \wp_n \underset{m}{\times} \mathcal{S}\big) \quad \equiv \quad \sum_{i=1}^{n} \lambda(\wp_i)\cdot W_{S(D)EU}\big(x_i \text{ on } \mathcal{S}\big)$$

Preferences over *almost-objective mixtures of purely subjective acts* satisfy :

$$\lim_{m\to\infty} W_{S(D)EU}\big(f_1(\cdot) \text{ on } \wp_1 \underset{m}{\times} \mathcal{S};\ldots;f_n(\cdot) \text{ on } \wp_n \underset{m}{\times} \mathcal{S}\big) \quad \equiv \quad \sum_{i=1}^{n} \lambda(\wp_i)\cdot W_{S(D)EU}\big(f_i(\cdot)\big)$$

So preferences are *linear* along the *almost-objective mixture path* between two subjective acts:

$$\lim_{m\to\infty} W_{S(D)EU}\big(f^*(\cdot) \text{ on } [0,\alpha] \underset{m}{\times} \mathcal{S};\; f(\cdot) \text{ on } (\alpha,1] \underset{m}{\times} \mathcal{S}\big) \quad \equiv \quad \alpha\cdot W_{S(D)EU}\big(f^*(\cdot)\big) + (1-\alpha)\cdot W_{S(D)EU}\big(f(\cdot)\big)$$

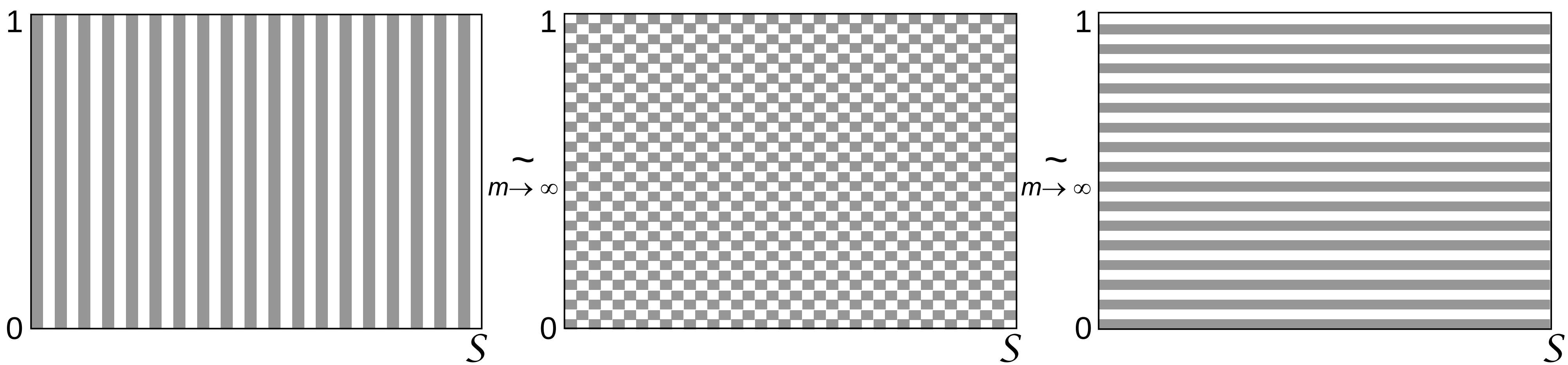
# DERIVING RISK PREFERENCES OVER PURELY OBJECTIVE LOTTERIES FROM PREFERENCES OVER PURELY SUBJECTIVE ACTS

Each event-smooth $W(\cdot)$ over purely subjective acts has an risk preference function $V_W(\cdot)$ such that

$$\lim_{m\to\infty} W\big(x_1 \text{ on } \wp_1 \underset{m}{\times} \mathcal{S};\ldots;x_n \text{ on } \wp_n \underset{m}{\times} \mathcal{S}\big) \;\equiv\; V_W\big(x_1,\lambda(\wp_1);\ldots;x_n,\lambda(\wp_n)\big)$$

What's to guarantee that our individual will use *the same risk preference function* when evaluating *purely objective lotteries*?

Consider the almost-objective bet $\big[\$100 \text{ on } [0,\tfrac{1}{2}]\underset{m}{\times}\mathcal{S};\$0 \text{ on } (\tfrac{1}{2},1]\underset{m}{\times}\mathcal{S}\big]$. As $m \to \infty$, $W(\cdot)$ converges to indifference between the following three bets:

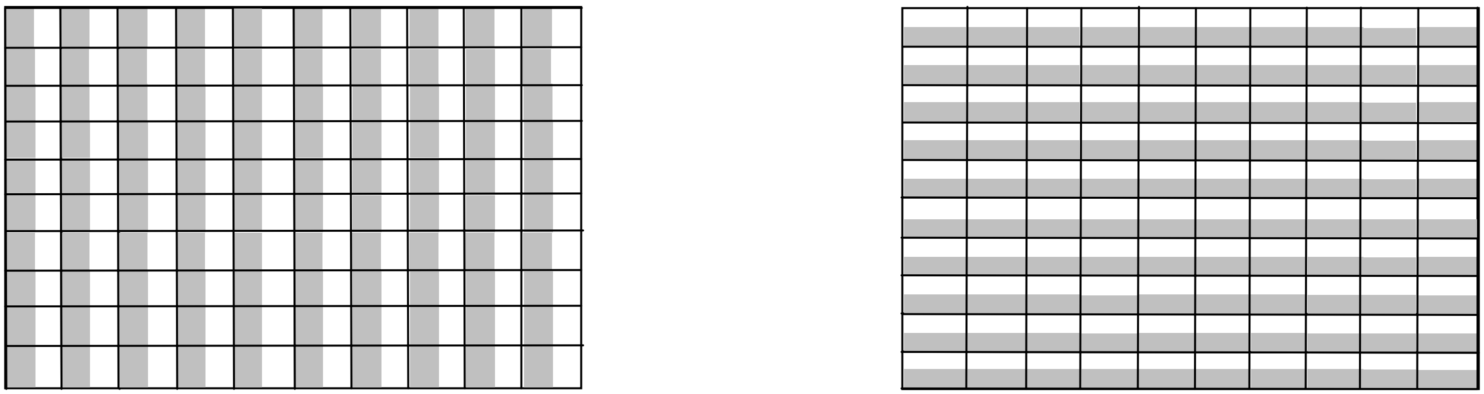



the purely subjective bet $\big[\$100 \text{ on } [0,\tfrac{1}{2}]\underset{m}{\times}\mathcal{S};\$0 \text{ on } (\tfrac{1}{2},1]\underset{m}{\times}\mathcal{S}\big]$

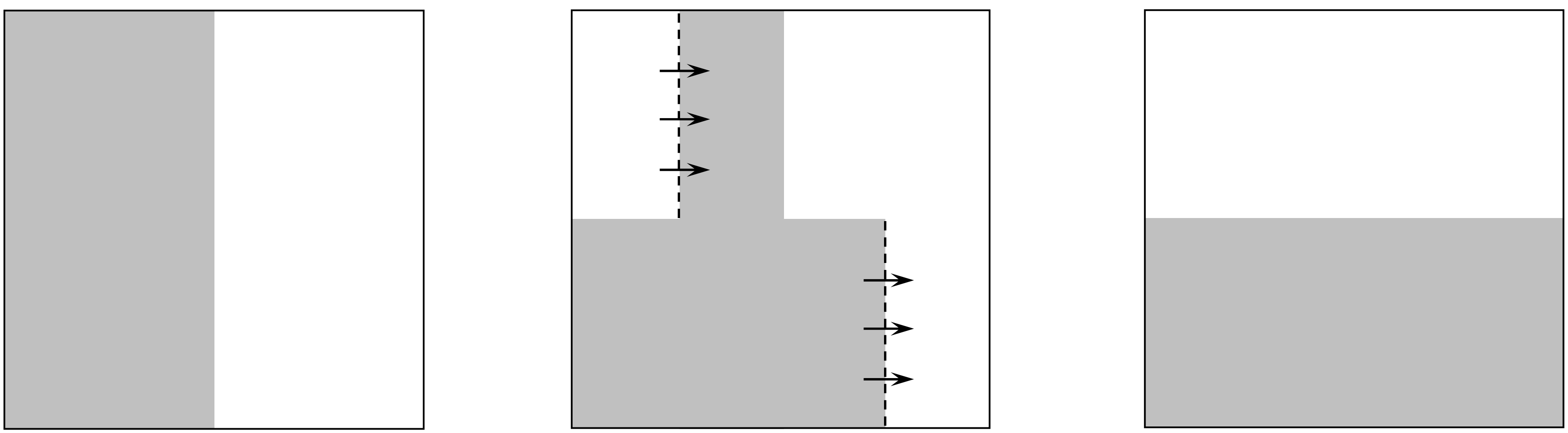
a mixed objective ×subjective bet

the purely objective lottery $\big(\$100,\tfrac{1}{2};\$0,\tfrac{1}{2}\big)$

# PROVING THE SUBJECTIVE → OBJECTIVE RESULT



In each of those $(m/2)^2$ small squares, we smoothly change from a vertical stripe to a horizontal stripe by



Throughout this global path, smoothness implies that as $m \to \infty$, the differential effects of the *upper shrinking vertical boundary* and the *lower expanding vertical boundary* will cancel.

# SUMMARY (AND IMPLICATIONS FOR MODELING UNCERTAIN CHOICE)

For event-smooth preferences over a given subjective state space $\mathcal{S}$:

- There exists a *special class of purely subjective events* in $\mathcal{S}$ – termed *"almost-objective events"* – which all event-smooth agents treat *virtually as if they were purely objective events*. That is, all event-smooth agents have *probabilistically sophisticated betting pref's* over such events, and all event-smooth agents have *identical revealed likelihoods* for these events.
- Each event-smooth $W(\cdot)$ thus implies an *objective risk preference function* $V_W(\cdot)$ for bets on almost-objective events, derived solely from the agent's preferences over *purely subjective acts.*

For event-smooth pref's on an objective×subjective space $[0,1]\times\mathcal{S}$, or a subjective×subjective space $\mathcal{S}\times\mathcal{T}$:

- Each agent is *indifferent* between any *almost-objective bet* (which is a purely subjective act) and its probabilistically equivalent *purely objective lottery*.
- Thus, an agent's risk preference function $V_W(\cdot)$, derived from *pref's over purely subjective acts,* serves to *completely encode their objective risk preferences*.
- An agent's derived risk pref. function $V_W(\cdot)$ *will be the same for every subjective space* $\mathcal{S},\mathcal{T}$, etc.

That is, under event-smoothness, everything about an agent's pref's over *purely objective lotteries* is encoded in their preferences over a rich enough space of *purely subjective acts.*

Thus – *and whether we like it or not* – any probabilistic or non-probabilistic model of choice under purely subjective uncertainty ("ambiguity") is *also* a model of choice over purely objective lotteries.

## IMPLIED OBJECTIVE RISK PREFERENCES OF SOME COMMON MODELS OF PREFERENCES UNDER SUBJECTIVE UNCERTAINTY

| | $W\left(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n\right)$ | $V_W\left(x_1,p_1;\ldots;x_n,p_n\right)$ |
|---|---|---|
| Subjective Expected Utility | $\sum_{i=1}^{n} U(x_i)\cdot\mu(E_i)$ | $\sum_{i=1}^{n} U(x_i)\cdot p_i$ |
| State-Dependent Expected Utility | $\sum_{i=1}^{n}\int_{E_i} U(x_i\|s)\cdot d\mu(s)$ | $\sum_{i=1}^{n}\left[\int_{\mathcal{S}} U(x_i\|s)\cdot d\mu(s)\right]\cdot p_i$ |
| Probabilistically Sophisticated | $V\left(x_1,\mu(E_1);\ldots;x_n,\mu(E_n)\right)$ | $V\left(x_1,p_1;\ldots;x_n,p_n\right)$ |
| Maxmin Expected Utility | $\min_{\mu_1(\cdot),\ldots,\mu_K(\cdot)} \sum_{i=1}^{n} U(x_i)\cdot\mu_k(E_i)$ | $\sum_{i=1}^{n} U(x_i)\cdot p_i$ |
| Choquet Expected Utility | $\sum_{i=1}^{n} U(x_i)\cdot\left[C(E_1\cap\ldots\cap E_i)-C(E_1\cap\ldots\cap E_{i-1})\right]$ | $\sum_{i=1}^{n} U(x_i)\cdot\left[G(p_1+\ldots+p_i)-G(p_1+\ldots+p_{i-1})\right]$ |
| Klibanoff-Marinacci-Mukerji Preferences | $\int_{\pi(\cdot)\in\Delta}\phi\left(\sum_{i=1}^{n} U(x_i)\cdot\pi(E_i)\right)\cdot d\mu(\pi(\cdot))$ | $\sum_{i=1}^{n} U(x_i)\cdot p_i$ |

# REFERENCES


Machina, M. (2004). "Almost-Objective Uncertainty," *Economic Theory*, 24, 1-54.

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