What Next?
Or
Things I wish I knew that Mathematical Economics can maybe provide answers to?

Introduction

- An applied problem - Combinatoric Auctions
  - Allocate K heterogeneous items to N bidders
  - Utility depends on subsets: v(S, e) - p(S)
  - What is the “best” auction design?
- Theory (of mechanism design)
  - Form a prior on e’s,
  - Apply the revelation principle
  - Choose a direct mechanism to maximize expected revenue subject to IC and VP

Problems

- Problems:
  - What is the right equilibrium concept?
  - Computational constraints invalidate the revelation principle - what is the space of designs?
  - Multi-dimensional type space -an unsolved problem
- What next?
  - Go to lab and try things - using what principles?
  - If items are substitutes, general equilibrium theory is a good predictor of outcomes in ascending bid auctions
  - If items are complements, there is little theory to help.

Thoughts

- Why do we know it is substitutes vs complements that is important?
  - Not through a collection of examples but …
  - Convexity is the principle discovered with Mathematical Economics
- What happens if we use markets in non-convex situations?
  - Theory is silent. (Maybe next?)
  - Conjecture: dynamics matter

Dynamics

- Early General Equilibrium dynamics have been well studied, but …
- What is the "right" model?
- Laboratory data are now available
  - Replicable, known fundamentals,…
- Time to revisit competitive market dynamics

Equilibrium

- Environment
  - Endowment w^i, trade d^i, consumption $x^i = w^i + d^i$, $x^i \geq 0$
  - Utility: $u^i(x^i, \theta^i)$, q.c., $du^i/dx^i_n >> 0$, $\nabla^2 u^i$ cnts.
- Excess demands

  $e^i(p, w^i, \theta^i) = \text{argmax } u^i(w^i + d^i, \theta^i)$ subject to $pd^i = 0$

  $E(p, w, \theta) = \sum e^i(p, w^i, \theta)$
- Equilibrium: (p^*, d^*) such that

  $E(p^*, w, \theta) = 0$

  $d^{i*} = e^i(p^*, w^i, \theta^i)$

Finding Equilibrium -Theory of price formation

- Walras, Arrow, Hurwicz,…
- Tatonnement Dynamics are:

  $dp/dt = E(p,w, \theta)$

  No trade unless $E(p^*) = 0$.
- Fact: $dp/dt \rightarrow p^*$ sometimes
  - Gross substitutes, ….
  - Scarf, Gale examples
- Observations:
  - Really only a theory about prices: d follows p

Finding Equilibrium -Theory of trading

- Marshall, Hahn, Negishi, Smale, Uzawa, …
- Non-Tatonnement Dynamics are:

  $dd^i/dt = g^i\,(p, w^i + d^i, e^i(p, w^i + d^i, \theta^i))$

  $dp/dt = E(p,w, \theta)$ when $x = x^*$.
- Under "no-speculation" hypothesis that

  $(\nabla u^i)(dd^i/dt) > 0$ when $\nabla u^i \neq p$

$d \rightarrow$ Pareto-optimal allocation d^*

- Observation:
  – Really only a theory about trading: p follows d

Finding Equilibrium - Experiments with “markets”

- Each “day” begins with allocation of endowments.
  - Traders submit bids (offers) = (P, Q) at each commodity “post”
  - Price feedback, new bids
    - The Continuous Double Auction
      - Trades occur along path (non-tatonnement)
    - The Call Market
      - Trades occurs only after price adjustment (tatonnement)
- The “day” ends and utility is paid.
- Usually repeated across “days”

Specific Situation

- Three markets (short sales allowed in the one risk free asset)
- Three equally likely states with payout

Security	State X	State Y	State Z
A	170	370	150
B	160	190	250
NOTES	100	100	100

Specific Situation

- Endowment of risky assets and cash refreshed each period
  - E.g., 5 of A, 4 of B, and 400 cash
  - May vary across subject
  - Loan repayment of, say, 1900 at end of each period - (provides leverage!)
- **Let them trade, then draw state, then pay $, then restart**
- **Subjects did not know market portfolio. So can't use CAPM to predict prices.**

Experiment "Factoids"

Plott et. al.

- Scarf environments, CDA

- For <u>inter-day</u> trading,
  - it appears that $\Delta p = aE(p,w, \theta)$ fits the data
  - where p = average price in a day.
- For <u>intra-day</u> trading,
  - neither $\Delta p = aE(p,w, \theta)$ nor $\Delta p = aE(p,w+d, \theta)$ seem to fit the data.

Experiment "Factoids"

Bossaerts et. al.

- CAPM environment, CDA and CM
- $x = (r,s)$, $u^i(x^i, \theta^i) = \mu r^i - (a^i/2)r^{i\prime}\Omega r^i - s^i$,

- For intra-day trading, with $p = (q,1)$,

  $dq/dt = b\Omega E(q,w+d,\theta)$ fits the data and

  $dr^i/dt = k^i\,[\nabla u^i - \sum k^j \nabla u^j]$ fits the data

where $\nabla u^i = \nabla u^i(w^i+d^i, \theta^i)$ and $\sum k^j = 1$.

- Can we explain these?

Search for principles: "Local" Equilibrium Theory

- Local demand, at x given p:

  Max $du^i(x^i+d^i, \theta^i)/dt] = [du^i(x^i+d^i, \theta^i)/dd^i](dd^i/dt)$

  Subject to $p(dd^i/dt) = 0$ and $(dd^i/dt) \in F$.

- Local equilibrium is p(x) such that $\sum dd^i/dt = 0$.
  - If F is open around 0, then

    $du^i/dt = \nabla u^i(x^i, \theta^i)(dd^i/dt) > 0$ unless $\nabla u^i = kp$ and so $d(t) \rightarrow$ Pareto- optimal allocation.

- Example: Champsaur and Cornet (1990):
  - F is $dd^i/dt \geq -\delta$.
  - Size of trade is independent of $\nabla u^i(x^i, \theta^i)$

Another Example

Ledyard (1975)

- Assume there is a numeraire: d_n
- Let $\nabla u^i(x^i, \theta^i)/(du^i/dx^i_n) = (p^i,1)$
- Let $d = (r, s)$, $p = (q,1)$
- And let F be $||dr/dt|| \leq 1$.
  - No constraint on ds^i/dt so no "income effects"
- Then $dr^i/dt = c^i(p^i - q)$ and $ds^i/dt = -q(dr^i/dt)$.
- Note: $du/dt = c^i(p^i - q)(p^i - q) > 0$ if $p^i \neq q$.

Alternative Explanation

- i sends bids (asks) per local willingness to pay.
- Each i communicates m^i_j to the market post j.

$m^i = c^i p^i + (1- c^i)q$ where $0 \le c^i \le 1$

– "demand reduction" or "risk aversion,"

$dr^i/dt = m^i - q = c^i(p^i - q)$, $ds^i/dt = - q\,dr^i/dt$

– Trade is responsive to larger bids and lower asks

- Equilibrium analysis

If $q = (1/I)\sum m^i$, then $\sum dr^i/dt = 0$.

So q is a local equilibrium price.

And $q = \sum v^i p^i$, where $v^i = c^i/\sum c^j$.

Price Dynamics:
For the CAPM world

- In the CAPM world, the local equilibrium is

  $q = \mu - \sum v^i a^i \Omega r^i$

  – If $c^i = (1/a^i)$, then $q = \mu - (1/b)\Omega w$, the global equilibrium price.

- Dynamics: $dq/dt = (1/I)\sum dm^i/dt$

  $dq/dt = - \Omega^2 \sum h^i e^i(q, x^i, \theta^I)$

  – Strange: "Excess demand" => price decrease,

  – But $x \rightarrow$ Pareto- optimal allocation.

  – Explanation: transactions change gradients proportionally to Ω, the Hessian of u, which pulls prices

- This is not consistent with observations.

A Delayed Local Process

Friedman (1979), Bossaerts (2003)

- Prices respond to local excess demands

  $$dq/dt = \rho \sum m^i$$

- In the CAPM world, this means

  $$dq/dt = \rho \Omega \sum (c^i a^i) e^i(q, x^i, \theta^i)$$

- We also want $dr^i/dt = k^i(p^i - q)$ so $du^i/dt > 0$.
- But then $\sum dr^i/dt \neq 0$

  unless q is a local equilibrium

Trading Dynamics in CAPM

- Suppose, as suggested by the data, that

  $$dr^i/dt = c^i[p^i - \sum(c^k/\sum c^j)p^k] = c^i[p^i - p']$$

  – Note: $p' = q^*$, the local equilibrium price.
- Now $\sum dr^i/dt = 0$ but

  $du^i/dt = (p^i - q)c^i[p^i - p']$ may not be > 0 when $(p^i - q) \neq 0$.
- But if $||q - p'||$ is small, then $du^i/dt > 0$.

We Need Different Time Scales

- Remember $dq/dt = \rho\sum m^i = - \rho(\sum c^i)(q - q^*)$
- So $q(t) = q^* + (q(0) - q^*) \exp[- \rho(\sum c^i)t]$

  and $\|q=p'\| = \|q(0) - p'\| \exp[- \rho(\sum c^i)t]$
- If prices adjust first and “fast enough” relative to trading then this all hangs together.
- Open: The speed of price adjustment is increasing in N (for a fixed ρ). What are the implications for “thin” markets?

Summary of Model

- Local Demand or willingness to pay (linear) is

  $r^i = c^i (p^i - q)$

- Prices adjust per Walras locally

  $dq/dt = \rho \sum m^i$

- Trading adjusts per Marshall locally

  $dr^i/dt = c^i(p^i - p'), \quad \text{where } p' = (1/\sum c^i)\sum c^i p^i$

- Prices adjust fast relative to trading adjustments.
- Result: In the CAPM world,

  $x(t) \rightarrow$ Pareto- optimal allocation.

Summary

- Without income effects, there is a consistent model that "fits the facts"
  - Local Walrasian price adjustment
  - Local Marshallian quantity adjustment
  - Requires different time scales
    - Prices move faster
- To do
  - income effects