On Market Risk Premia

Material in Pages 13 - 37 of
Private Information and Diverse Beliefs: How Different?

by

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Definition of The Risk Premium

- **Actual Premium**

$$\pi_{t+1} = \frac{p_{t+1} + D_{t+1} - R_t p_t}{p_t}.$$

- **m = probability induced by the data's empirical distribution**
- **Easy to show that m is unique and stationary probability.**
- **Long Run Premium =** $E^m[\frac{p_{t+1} + D_{t+1}}{p_t}] - \bar{R}$

- **"The" Premium is a conditional expectations under m**

$$E_t^m[\pi_{t+1}] = \frac{1}{p_t} E_t^m[p_{t+1} + D_{t+1} - R_t p_t]$$

<u>Problem</u>: What are the factors determining the Premium

Some Answers: Macroeconomic Variables

Fama and Bliss (1987) – past yields
Cambpell and Shiller (1991) – Shocks to the bond market hence past yields
Bernanke and Kuttner (2003) – Federal Reserve Policy shocks
Cocharane Piazzesi (2005) – Past yields
Piazzesi and Swanson (2004) – past yields and recessions forecasters such as Non Farm Payroll.

- **We study problem in context of heterogenous beliefs**
- **Endogenous Uncertainty - Kurz (1974) = component of volatility and risk induced by market beliefs**

Our Interest: <u>Effects of market beliefs on the risk premium</u>

Literature on Heterogenous Beliefs

Harrison and Kreps (1978)
Varian (1985), (1989)
Harris and Raviv (1993)
Detemple and Murthy (1994)
Kurz (1994), (1997a)
Kurz and Beltratti (1997)
Kurz and Motolese (2001)
Kurz Jin and Motolese (2005a) (2005b)
Motolese (2001)
Nielsen (1996)
Wu and Guo (2003), (2004).

An Infinite Horizon Model

Assumptions

- **Large number of agents**
- **A single commodity -- "consumption"**
- **Riskless technology producing R > 1 at t+1 with 1 unit of input at t**
- **Dividend process $\{D_t$, t = 1, 2, ...$\}$ is non-stationary with unknown probability Π.**
- **Under m $\{D_t$, t = 1, 2, ...$\}$ is Markov with unconditional mean μ and transition**

$$d_{t+1} = \lambda_d d_t + \rho^d_{t+1} \quad , \quad \rho^d_{t+1} \sim N(0\,,\sigma^2_d) \quad \text{where} \quad d_t = D_t - \mu .$$

- **m induces a marginal probability measure m on $(D^\infty, \mathscr{F})$ hence**

$$E^m_t[d_{t+1} | d_t] = \lambda_d d_t .$$

Notation

- **θ^i_t = date t stock purchases of agent i. Aggregate supply = 1**
- **B_t = amount invested in the riskless asset**
- **p_t = price of the stock. Think of it as the S&P 500**

An Infinite Horizon Model (Cont.)

(1) $$\text{Maximize} \max_{(\theta^i, B^i)} E_t^i\left[\sum_{k=t}^{\infty} -\beta^{k-1} e^{-\left(\frac{c_k^i}{\tau}\right)}\right]$$

Subject to **(i) Budget Constraint** $c_t^i = \theta_{t-1}^i [p_t + d_t + \mu] + B_{t-1}^i R - \theta_t^i p_t - B_t^i$,

(ii) Initial values (θ_0^i, B_0^i)

<u>Exponential utility is common in study of asset pricing. Examples</u>

Singleton (1987)
Brown and Jennings (1989)
Grundy and McNichols (1989)
Wang (1994)
He and Wang (1995)
Duffie (2002)
Dai and Singleton (2002)
Allen, Morris and Shin (2005) and many others

An Infinite Horizon Model (Cont.)

Assume for a moment: Agents believe $p_{t+1} + d_{t+1}$ is conditionally normal.

Hence demand functions

$$\theta_t^i(p_t) = \frac{\tau}{R\sigma_\varepsilon^2}[E_t^i(p_{t+1} + d_{t+1} + \mu - Rp_t]. \tag{2}$$

$\sigma_\varepsilon^2 = Var^i[p_{t+1} + d_{t+1} + \mu - Rp_t | H_t]$ **assumed constant, the same for all agents.**

We later clarify the exact value of σ_ε^2

We now must be explicit about the belief of agent i, which is the main issue.

The Structure of Beliefs: Preliminaries

- **Under true Π, $\{D_t, t = 1, 2, \ldots\}$ is non-stationary**
- **Observation: $m \neq \Pi$ – central to our approach.**
- **All know $d_{t+1} = \lambda_d d_t + \rho^d_{t+1}$, $\rho^d_{t+1} \sim N(0, \sigma^2_d)$ may not be the truth**

FACTS

(i) Subjective modeling contribute more than 50% to forecasts

(ii) Agents use m only as a reference from which to deviate

(iii) Vast data on market forecast distribution of most variables.

Two Central Points On PI vs. HB

(i) <u>With PI, data on market forecasts of exogenous variables lead an agent to update his own belief.</u> Such forecasts contain new information!

(ii) <u>With HB all know they have the same information. Others' forecast data do not lead an agent to update belief about exogenous variables as these</u> *reflect others' "opinions" or subjective models.* Use forecasts of "others" only to forecast endogenous variables.

Individual Belief As A State Variable

- **Individual are *"anonymous"***
- **i's belief state g_t^i pins down perceived transition of state variables**
- **The distribution of g_t^i is observed.**
- **<u>The Problem</u>: the dynamics of g_t^j?**

<u>1. The information structure</u>

(i) Quantitative $\{d_t, t = 1, 2, \ldots\}$

(ii) Qualitative $(C_{t1}, C_{t2}, \ldots, C_{tK_t})$ statements unique to date t, do not repeat.

- **Subjective map: for each subset of indices A_k**

$$C_{tA_k} \Rightarrow \Psi^i_{tA_k}$$

If only C_{tA_k} are realized, i forecasts of $(d_{t+1} - \lambda_d d_t) = \Psi^i_{tA_k}$

- **Subjective interpretation of the public information $(C_{t1}, C_{t2}, \ldots, C_{tK})$**

$$\Psi^i_t = \sum_{k=1}^{K_t!} \pi^i_{tA_k} \Psi^i_{tA_k}.$$

- **$(C_{t1}, C_{t2}, \ldots, C_{tK_t})$ offer alternate subjective forecast $(d_{t+1} - \lambda_d d_t) = \Psi^i_t$**

Individual Belief As A State Variable

2. A Bayesian Motivation

- i believes d_t has true transition with unobserved mean b_t

$$d_{t+1} - \lambda_d d_t = b_t + \rho^d_{t+1} \quad , \quad \rho^d_{t+1} \sim N(0, \frac{1}{\beta})$$

- At t = 1 a *prior belief* about b_t $\quad b_t \sim N(\varphi, \frac{1}{\hat{\alpha}})$

After observing $d_{t+1} - \lambda_d d_t$ i updates to $E^i_{t+1}(b_t | d_{t+1})$. Needs a belief on b_{t+1}.
How to go from $E^i_{t+1}(b_t | d_{t+1})$ to $E^i_{t+1}(b_{t+1} | d_{t+1})$?

Without new information must use $E^i_{t+1}(b_t | d_{t+1})$.

$(C_{(t+1)1}, C_{(t+1)2}, \ldots C_{(t+1)K_{t+1}})$ imply a subjective estimate Ψ^i_{t+1}

<u>Assumption (*)</u>: i uses a subjective probability μ to form date t+1 prior

(3) $$E^i_{t+1}(b_{t+1} | d_{t+1}, \Psi^i_{t+1}) = \mu E^i_{t+1}(b_t | d_{t+1}) + (1-\mu)\Psi^i_{t+1} \qquad 0 < \mu < 1.$$

<u>Theorem 1:</u> **Suppose $\Psi_t^i \sim N(0, \frac{1}{\gamma})$ and Assumption (*) holds. Then for large t the posterior $E_{t+1}^i(b_{t+1}|d_{t+1}, \Psi_{t+1}^i)$ is a Markov state variable such that if we let $g_t^i = E_t^i(b_t|d_t, \Psi_t^i)$ and $\mu = \lambda_Z$ then (3) implies**

(4) $$g_{t+1}^i = \lambda_Z g_t^i + \rho_{t+1}^{ig} \quad , \quad \rho_{t+1}^{ig} \sim N(0, \sigma_g^2).$$

- **Individual beliefs are correlated via ρ_t^{ig}.**

3. Implied Individual Perception

(5) $$d_{t+1}^i = \lambda_d d_t + \lambda_d^g g_t^i + \rho_t^{id} \quad , \quad \rho_t^{id} \sim N(0, \hat{\sigma}_d^2).$$

Hence as before we have data to measure

$$E^i[d_{t+1}^i|H_t, g_t^i] - E^m[d_{t+1}|H_t] = \lambda_d^g g_t^i.$$

Definition: $Z_t = \frac{1}{N}\sum_{i=1}^{N} g_t^i$ **– market state of belief. N is large.**

The Role of the Market Belief State Variable

- Z_t is observed.
- Z_t is non stationary with empirical distribution

(6) $$Z_{t+1} = \lambda_Z Z_t + \rho^Z_{t+1}.$$

- We thus *expand the empirical distribution* to $\{(d_{t+1}, Z_{t+1}), t = 1, 2, \ldots\}$.

(7a) $$d_{t+1} = \lambda_d d_t + \rho^d_{t+1}$$

(7b) $$Z_{t+1} = \lambda_Z Z_t + \rho^Z_{t+1}$$

$$\begin{pmatrix} \rho^d_{t+1} \\ \rho^Z_{t+1} \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_d^2, & 0, \\ 0, & \sigma_Z^2 \end{bmatrix} = \Sigma \right), \quad \text{i.i.d.}$$

- Individual i's *perception model* (together with (4))

(8a) $$d^i_{t+1} = \lambda_d d_t + \lambda^g_d g^i_t + \rho^{id}_{t+1}$$

(8b) $$Z^i_{t+1} = \lambda_Z Z_t + \lambda^g_Z g^i_t + \rho^{iZ}_{t+1}$$

$$\begin{pmatrix} \rho^{id}_{t+1} \\ \rho^{iZ}_{t+1} \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \hat{\sigma}_d^2, & \hat{\sigma}_{Zd} \\ \hat{\sigma}_{Zd}, & \hat{\sigma}_Z^2 \end{bmatrix} = \Sigma^i \right)$$

$\lambda^g_d \geq 0$ and $\lambda^g_Z \geq 0$ orient model: When $g^i_t > 0$, i believes t+1 dividend and market belief will be above normal .

The Role of the Market Belief State Variable

(8a)-(8b) means

$$E_t^{\,i}\begin{pmatrix} d_{t+1} \\ Z_{t+1} \end{pmatrix} - E_t^{\,m}\begin{pmatrix} d_{t+1} \\ Z_{t+1} \end{pmatrix} = \begin{pmatrix} \lambda_d^g g_t^{\,i} \\ \lambda_Z^g g_t^{\,i} \end{pmatrix}.$$

Note: i's belief Q^i is a probability on $((D \times Z \times G^i)^\infty, \mathscr{F})$

Define the <u>market belief</u> by $\bar{Q} = \frac{1}{N}\sum_{i=1}^{N} Q_t^{\,i}$

The <u>market expectations operator</u> $\bar{E}_t(X_{t+1}) = \frac{1}{N}\sum_{i=1}^{N} E_t^{\,i}(X_{t+1})$

<u>Theorem 2</u>: The market belief $\bar{Q}$ is not a proper probability and the market expectations operator violates iterated expectations: $\bar{E}_t(d_{t+2}) \neq \bar{E}_t\bar{E}_{t+1}(d_{t+2})$.

- **Market Belief is neither a proper probability nor Rational.**

Equilibrium Asset Prices

Stability Conditions: $R = 1 + r > 1 , \quad 0 < \lambda_d < 1 \quad , \quad 0 < \lambda_Z + \lambda_Z^g < 1 \; .$

This is natural since $\bar{E}_t [Z_{t+1}] = (\lambda_Z + \lambda_Z^g) Z_t .$

Theorem 3: **For the model with HB and under the stability conditions, there is a unique equilibrium price function taking the form** $p_t = a d_t + b Z_t - c S$ **with parameters**

$$a = \frac{\lambda_d}{R - \lambda_d} > 0 \; .$$

$$b = \frac{\lambda_g^d}{R - (\lambda_Z + \lambda_Z^g)} [1 + \frac{\lambda_d}{R - \lambda_d}] > 0$$

$$c = \frac{\sigma_\varepsilon^2 R}{\tau r} > 0 \; .$$

Note: Price map confirms earlier conjecture the price is conditionally Normal

Equilibrium Risk Premia

1. **Analytic Results**

Realized Premium $$\pi_{t+1} = \frac{p_{t+1} + d_{t+1} + \mu - Rp_t}{p_t}$$

Agent i's Perceived Premium $$\frac{1}{p_t} E_t^i(p_{t+1} + d_{t+1} + \mu - Rp_t)$$

The Market Perceived Premium $$\frac{1}{p_t} \bar{E}_t(p_{t+1} + d_{t+1} + \mu - Rp_t)$$

The standard Risk Premium $$\frac{1}{p_t} E_t^m[p_{t+1} + d_{t+1} + \mu - Rp_t]$$

What is the relationship among these in equilibrium?

Relationship Among Premia

(9a) $$\bar{E}_t(p_{t+1} + d_{t+1} + \mu - Rp_t) = R\frac{\sigma_\varepsilon^2}{\tau}$$

(9b) $$E_t^i[p_{t+1} + d_{t+1} + \mu - Rp_t] = R\frac{\sigma_\varepsilon^2}{\tau} + [(a+1)\lambda_d^g + b\lambda_Z^g](g_t^i - Z_t).$$

- $(a+1)\lambda_d^g + b\lambda_Z^g > 0$ by model orientation

(9c) $$E_t^m[p_{t+1} + d_{t+1} + \mu - Rp_t] = R\frac{\sigma_\varepsilon^2}{\tau} - (\lambda_d^g + \lambda_Z^g)Z_t$$

- $\lambda_d^g + \lambda_Z^g > 0$; premium declines with market belief
- (9c) is key result: negative when $Z_t > 0$ and positive when $Z_t < 0$.
- Result remains true for percentage premium since

$$E_t^m\left[\frac{p_{t+1} + d_{t+1} + \mu - Rp_t}{p_t}\right] = \frac{R\frac{\sigma_\varepsilon^2}{\tau} - (\lambda_d^g + \lambda_Z^g)Z_t}{ad_t + bZ_t - p_0}$$

Endogenous Uncertainty and Risk Premium: Decomposition

(I) Volatility = Its effect on mean premium via $R\frac{\sigma_\varepsilon^2}{\tau}$.

- $$\sigma_\varepsilon^2 = Var_t(p_{t+1} + d_{t+1})$$
  $$= Var[(a+1)\rho_{t+1}^{id} + b\,\rho_{t+1}^{iZ}] = ((a+1),b)^T \Sigma^i ((a+1),b)$$
- **It increases volatility of returns since b > 0**

(II) Risk Perception reflected in $-(\lambda_d^g + \lambda_Z^g) Z_t$

- **Sign is of great interest**
- **When $Z_t > 0$ the market views the long position as less risky and a lower risk premium is awarded to it.**
- **Think of Z the same as Non Farm Payroll**

2. Empirical Test

- **Results in Kurz and Motolese (2005)**
- **Estimate premia in futures, bond and the stock markets**
- **Dependent variable = realized excess holding returns of: Fed Funds futures, 3 month T Bills, 6 month T Bills and the S&P500.**
- **Data restrict to holding period from 1 - 6 months for Fed Funds futures, from 1 - 12 months for T Bills and one quarter for the S&P500.**
-

Bond and Federal Funds Futures:1980:1 to 2003:10.

- **forecast data of Blue Chip Financial Forecasts.**
- **In theory g_t^i and Z_t are for one asset and one period**
- **Use forecasts of interest rates to construct beliefs as in theory**
- **Use $Z_t^{(k,h)}$ for maturity k and holding period h**
- **$Z_t^{(k,h)} > 0$ means mean market believes interest rate on maturity k will be lower than normal at t + h.**
- **Orientation of $Z_t^{(k,h)}$ is as in the theory.**

Variables on Distribution of Beliefs

$\sigma_t^{(k,h)}$ – Cross sectional variances of individual beliefs

$SZ_t^{(6-F,h)} = Z_t^{(6,h)} - Z_t^{(F,h)}$ -- beliefs about the slope of yield curve

Traditional Variables: Recession, Monetary Policy and Past Yields

NFP_{t-1} –year over year growth rate of Non Farm Payroll at t-1

CPI_{t-1} – rate of inflation at t-1

F_t^{Cum} – measured cumulative intensity of monetary policy

R_t^{Fj} , $j = 1,2,3$ – three principal components of past interest rates

Results

Table 1A: Predictability of Excess Returns, Fed Funds

	Constant	NFP_{t-1}	CPI_{t-1}	F_t^{Cum}	R_t^{F1}	R_t^{F2}	R_t^{F3}	$\sigma_t^{(F,h)}$	$Z_t^{(F,h)}$	$SZ_t^{(6-F,h)}$	R^2
h=4	**0.503*** **(0.231)**	**-0.177*** **(0.040)**	**0.087*** **(0.036)**	**-0.007** **(0.033)**	**0.234*** **(0.107)**	**-0.013** **(0.046)**	**-0.098** **(0.057)**	**-0.945*** **(0.493)**	**-0.573*** **(0.126)**	**- 0.871*** **(0.273)**	**0.289**
h=6	**0.633*** **(0.312)**	**-0.232*** **(0.052)**	**0.169*** **(0.047)**	**-0.005** **(0.042)**	**0.373*** **(0.141)**	**0.052** **(0.068)**	**0.039** **(0.108)**	**-0.988*** **(0.488)**	**-0.930*** **(0.188)**	**-1.661*** **(0.482)**	**0.436**

Table 2: Predictability of Excess Returns, 3 Months Treasury Bills

	Constant	NFP_{t-1}	CPI_{t-1}	F_t^{Cum}	R_t^{F1}	R_t^{F2}	R_t^{F3}	$\sigma_t^{(3,h)}$	$Z_t^{(3,h)}$	$SZ_t^{(6-F,h)}$	R^2
h=6	**0.820*** **(0.174)**	**-0.185*** **(0.026)**	**0.078*** **(0.025)**	**0.006** **(0.021)**	**0.360*** **(0.072)**	**0.032** **(0.036)**	**-0.044** **(0.042)**	**-0.820*** **(0.189)**	**-0.516*** **(0.095)**	**-0.636*** **(0.153)**	**0.447**
h=10	**1.272*** **(0.133)**	**-0.168*** **(0.025)**	**0.067*** **(0.020)**	**0.027** **(0.015)**	**0.561*** **(0.063)**	**-0.016** **(0.030)**	**-0.013** **(0.025)**	**-0.887*** **(0.120)**	**0.437*** **(0.063)**	**- 0.413*** **(0.149)**	**0.663**

Table 4C: Predictability and Time Variability of Excess Returns, 6 Months Treasury Bills

	Constant	NFP_{t-1}	CPI_{t-1}	F_t^{Cum}	R_t^{F1}	R_t^{F2}	R_t^{F3}	$\sigma_t^{(6,h)}$	$Z_t^{(6,h)}$	$SZ_t^{(6-F,h)}$	R^2
h=6	**1.964*** **(0.370)**	**-0.388*** **(0.063)**	**0.175*** **(0.050)**	**0.012** **(0.047)**	**0.828*** **(0.158)**	**0.045** **(0.081)**	**-0.052** **(0.087)**	**-2.508*** **(0.441)**	**- 1.434*** **(0.195)**	**-1.384*** **(0.348)**	**0.519**
h=10	**2.717*** **(0.269)**	**-0.387*** **(0.056)**	**0.092*** **(0.038)**	**0.072*** **(0.032)**	**1.141*** **(0.133)**	**-0.100** **(0.063)**	**-0.014** **(0.052)**	**-1.867*** **(0.258)**	**- 0.906*** **(0.129)**	**- 0.733*** **(0.279)**	**0.677**

Sum: Risk premium declines with market belief and with diversity of beliefs.

GRAPHS

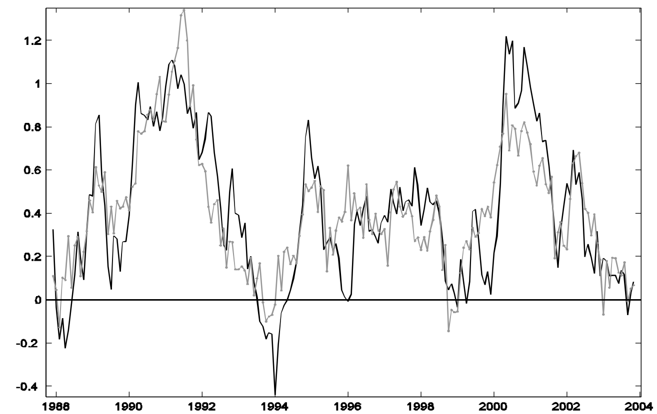

Figure 2 Excess Returns on 3 Months T-Bills 10 months ahead. The gray line (green in color) represents the fitted values from regression.

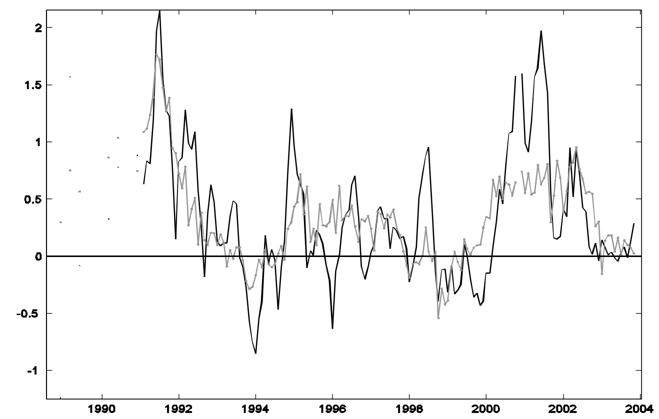

Figure 1 Excess Returns on Fed Funds Futures contract 6 months ahead. The gray line (green in color) represents the fitted values from regression.

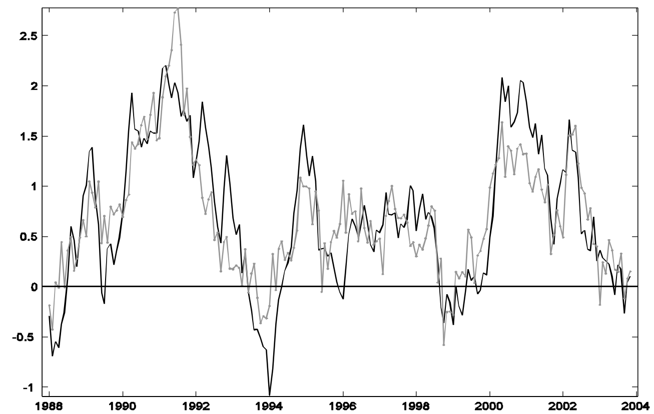

Figure 3 Excess Returns on 6 Months T-Bills 10 months ahead. The gray line (green in color) represents the fitted values from regression).

Rationality of Beliefs: Can It All Be Rationalized

- **Not REE**
- **Reject Arrow-Debreu's equating individual states with Maket States**

Without Rationality restrictions theory has internal contradictions:

Example:

$$E_t^{i}(p_{t+1} + d_{t+1} + \mu - Rp_t) = (a+1)(\lambda_d d_t + \lambda_d^g g_t^{i}) + b(\lambda_Z Z_t + \lambda_Z^g g_t^{i}) + \mu - c - Rp_t$$

while under m

$$E_t^{m}[p_{t+1} + d_{t+1} + \mu - Rp_t] = (a+1)(\lambda_d d_t) + b(\lambda_Z)Z_t) + \mu - c - Rp_t$$

Rationality would insist the time average of g_t^{i} is zero. True under the Bayesian procedure outlined.

Rational Beliefs (RB), Kurz (1994), (1997a)

Definition: A belief is said to be an RB *if it is a probability model which, if simulated, reproduces the empirical distribution known from the data*.

- **An RB cannot be rejected by the data**
- **In the perception models (4) and (14a) -(14b) the covariance matrix and two parameters $(\lambda_Q^g, \lambda_Z^g)$ are defined by the agent's belief**
- **For (4), (14a) -(14b) Rationality of Belief requires**
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The empirical distribution of $$\begin{pmatrix} \lambda_d^g g_t{}^i + \rho_t^{id} \\ \lambda_Z^g g_t{}^i + \rho_{t+1}^{iZ} \end{pmatrix}$$

Equals the distribution of $$\begin{pmatrix} \rho_t^d \\ \rho_{t+1}^Z \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_d^2, & 0, \\ 0, & \sigma_Z^2 \end{bmatrix} \right), \text{ i.i.d.}$$

What Does These Amount To?

Five implied rationality conditions:

$$(1)\ \frac{(\lambda_d^g)^2\sigma_g^2}{1-\lambda_Z^2} + \hat{\sigma}_d^2 = \sigma_d^2 \quad (2)\ \frac{(\lambda_Z^g)^2\sigma_g^2}{1-\lambda_Z^2} + \hat{\sigma}_Z^2 = \sigma_Z^2 \quad (3)\ \frac{\lambda_d^g\lambda_Z^g\sigma_g^2}{1-\lambda_Z^2} + \hat{\sigma}_{Zd} = 0$$

$$(4)\ \frac{(\lambda_d^g)^2\lambda_Z\sigma_g^2}{1-\lambda_Z^2} + \mathrm{Cov}(\hat{\rho}_t^{id}, \hat{\rho}_{t-1}^{id}) = 0 \quad (5)\ \frac{(\lambda_Z^g)^2\lambda_Z\sigma_g^2}{1-\lambda_Z^2} + \mathrm{Cov}(\hat{\rho}_t^{iZ}, \hat{\rho}_{t-1}^{iZ}) = 0.$$

(1)-(3) pin down the covariance matrix in (14a)-(14b).
(4)-(5) pin down the serial correlation of the two terms $(\hat{\rho}_t^{id}, \hat{\rho}_t^{iZ})$.

Further Restrictions on the "Free" $(\lambda_d^g, \lambda_Z^g)$

(I) $\hat{\sigma}_d^2 > 0$, $\hat{\sigma}_Z^2 > 0$ **restrict** $(\lambda_d^g, \lambda_Z^g)$**:**

$$|\lambda_d^g| < \frac{\sigma_d}{\sigma_g}\sqrt{1-\lambda_Z^2} \qquad |\lambda_Z^g| < \frac{\sigma_Z}{\sigma_g}\sqrt{1-\lambda_Z^2}.$$

(II) The covariance matrix in (14a)-(14b) is positive definite. This implies

$$\frac{1-\lambda_Z^2}{\sigma_g^2} > \frac{(\lambda_Z^g)^2}{\sigma_Z^2} + \frac{(\lambda_d^g)^2}{\sigma_d^2}.$$

The "free" $(\lambda_d^g, \lambda_Z^g)$ **are restricted to a narrow range but sufficient to generate volatility in order of magnitude seen in the data.**