Projection of a Medium

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Abstract

A "medium" is an axiomatized algebraic structure consisting of a set of transformations on a possibly infinite set of states. Its ability to capture a wide range of examples, mostly from combinatorics such as an arrangent of hyperplanes or the family of all semi-orders on a finite set, suggests that the concept may be ubiquitous (Falmagne and Ovchinnikov, 2002). A medium can also serve as the setting for a random walk in which the transformations formalize the effects on an individual of a random stream of unobservable "tokens" of information. A general class of stochastic models of this type is developed by Falmagne (1997), and an application to the 1992 presidential election is carried out by Regenwetter et al. (1999). The subject of this paper is a class of mappings called "projections", which partition the set of states of a medium into equivalence classes. We prove the necessary and sufficient conditions for the projection of a medium to produce a medium, and show how several projections of the same medium can be combined to recreate the original medium. We also explore the structural properties, such as closure and completeness, that are preserved under projection. Finally, we suggest a practical application of the projection in the context of the stochastic models developed by Falmagne (1997).