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Uncovering Minority Power Under Majority Rule[1]

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Abstract

A new simulation technique enables us to compute the size, shape and location of the uncovered set, and, thus, to estimate the set of feasible outcomes in 'real world' legislative bodies (Bianco et. Al 2004, Bianco and Sened 2005). The uncovered set is a well known but under-exploited solution concept in the literature on spatial voting games and collective choice mechanisms. We explain this solution concept in non-technical terms, submit some theoretical observation to improve our theoretical and intuitive grasp of it and then use the new simulation technique to provide original findings on the role of minority groups in legislative political bodies governed by simple majority rule.

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1. Introduction

Minorities impose fundamental constraints and may exert considerable influence on legislative action (e.g. Strom, 1990; Laver and Shepsle, 1996, Sened, 1995, 1996; Kalandrakis, 2005). Yet, the exact nature of minority power under majority rule is only partially understood. This paper submits further insight towards a better understanding of the power of minority groups in shaping political outcomes under majority rule, using a new technique of estimating the set of feasible outcomes in a majority-rule legislative setting.

We operationalize feasibility using the *uncovered set* (McKelvey 1986; Miller 1980), a solution concept commonly interpreted to capture feasibility in real-world settings (e.g., Calvert 1985; Grofman et al 1987; Shepsle and Weingast 1984, Bianco and Sened, 2005). Until very recently, the uncovered set has not been applied to real-world settings, as it has defied general characterization. Bianco Jeliazkov and Sened (2004) developed a grid-search computational method for estimating the size, shape, and location of the uncovered set for any profile of Euclidean preferences on a two-dimensional space.[2] This paper uses this technique to highlight some important aspects of minority power under majoritarian legislative settings.

Structure of the paper: The next section reviews the known properties of the uncovered set. Section three reviews the literature on minority power in majoritarian legislative bodies. Section four contains the main argument of this paper and illustrates it with the use of two experimental examples. Section five uses simulations of the current Israeli Knesset to illustrate the argument in 'real time, real world' environment. Section Six concludes and suggest an agenda for further research.

[2] The technology was originally developed by Ivan Jeliazkov for Bianco et al. 2004. In this paper we use an improved C++ version developed by Haran Sened, Yanai Sened and Itai Sened for Bianco and Sened 2005.

2. The Uncovered Set: Theoretical Background and Relevance

Seminal works in formal theory suggest that stable equilibria rarely exist in multi-dimensional majority rule games (McKelvey, 1976, 1979; Schofield, 1978; McKelvey and Schofield, 1987), implying that outcomes are sensitive to agendas, voting rules and other institutional constraints (Shepsle, 1979, 1986). The so-called Chaos Theorems, (McKelvey, 1976; Schofield, 1978; McKelvey, 1979; McKelvey and Schofield, 1987) state that majority-based decision making, unchecked by institutions, can go 'from anywhere to anywhere,' rendering the ultimate outcome of legislative action, absent institutional constraints, indeterminate.

Further work refined these results, showing that if voters consider the ultimate consequences of their behavior, rather than choosing myopically between alternatives presented at each point, outcomes of social choice situations will lie in the uncovered set (McKelvey 1986; Miller 1980, Feld et al 1989, Miller et al 1987; Shepsle and Weingast 1984). Uncovered outcomes are not necessarily Condorcet winners – they need not be majority-preferred to all other outcomes.[3] Yet, regardless of what 'status quo point' a voting process begins at, when decision-makers vote using majority rule, there exists a simple two-step agenda that yields some point in the uncovered set as its final outcome (Shepsle and Weingast 1984). Thus, supporters of outcomes in the uncovered set can secure these outcomes using relatively simple agenda. Further work suggests that the uncovered set describes the set of feasible outcomes in many legislative and other majority rule decision-making environments.

A characterization of the feasible outcomes in a majority rule legislative setting is crucial for the understanding of how these institutions work. While the uncovered set is a very reasonable set to focus on in the search for an answer, it has defied analytical characterization to this day. Below we survey some of its known characteristics but, as we conclude this survey, they provide

little insight to the exact location, shape and size of the uncovered set, let alone its sensitivity to possible perturbations of the distribution of ideal points of legislators. Our technique for locating the uncovered set (Bianco et Al. 2004) was submitted to solve this problem. While it stops short of characterizing the uncovered set, analytically, it enables us to calculate the uncovered set with relative precision for any given distribution of legislators' ideal points in the policy space.

The following pages offer a plain-language presentation of the technical properties of the uncovered set. Bianco et. al (2004) supplement this survey with the customary mathematical notations and formal proofs.

Let N be the set of n voters or legislators. We assume n is odd. For any agent, $i \in N$, preferences are defined by an *ideal point* ρ_i. Let x,y,z be elements of the set X of all possible outcomes. A point x *beats* another point y by majority rule if it is closer than y to more than half of the ideal points.[4] A point x is *covered* by y if y beats x and any point that beats y also beats x. The uncovered set includes all points not covered by other points.

The attractiveness of the uncovered set as a solution concept lies in that if y covers x, then y dominates x, at least in a loose sense of the term, as an outcome of a majority-rule voting game (McKelvey, 1986; Ordeshook, 1986: 184-5). If y defeats x, any outcome that ties y defeats or ties x and any outcome that defeats y also defeats x, strategic legislators should eliminate covered points from voting agenda. Instead of promoting outcomes that are bound to be defeated, sophisticated legislators should promote uncovered policies that may survive the voting process (Cox, 1987). This logic suggests that the feasible set in a legislative process governed by majority rule may be restricted to the uncovered set.

[3] If a Condorcet winner exists, the uncovered set consists of that single outcome.

Five properties are known about the uncovered set.

1. The uncovered set is never empty (McKelvey, 1986: 290, Theorem 1).
2. The majority core is a point that beats all other points in X. If the core is not empty, the uncovered set collapses to the core (Miller, 1980: 74, Theorem 1; McKelvey, 1986: 285).
3. Assuming Euclidean preferences, a point x is *unanimously preferred* to a point y if x is closer than y to all ideal points. The Pareto set is the set of points such that there is no point that is unanimously preferred to any point in the Pareto set. The uncovered set is a subset of the Pareto set. (Miller, 1980: 80, Theorem 4; Shepsle and Weingast, 1984: 65, Proposition 3).
4. A *median hyperplane* is a hyperplane that passes through k ideal points, $k \geq 1$, so that there are at least $(n+1)/2 - k$ ideal points on each side of it. Thus, if n is odd, at least one ideal point must lie on any median hyperplane and an equal number of points is to be found on each side of it. Let Y be the smallest ball that intersects all median hyperplanes and Y4 be a ball centered on Y's center with a radius, 4r, equal to four times the radius, r, of the ball Y. The uncovered set is contained within Y4 (McKelvey 1986: 304). Y is referred to as the 'yolk.'[5]
5. ***Theorem 1*** (Bianco et Al. 2004)***:*** Any subset B of A is the uncovered set of A iff:
    1) Every point outside of B is covered by a point within B.
    2) No point within B is covered by a point inside B.

Unfortunately, these five known properties of the uncovered set do not establish the shape, location or size of the uncovered set. In particular, properties 3 – 5, the best analytic estimates for non-specific cases of the uncovered set, are very imprecise, rendering the uncovered set useless as a predictive tool. To appreciate the problem, consider Figure one, borrowed from Bianco et Al. (2004), which gives Poole-Rosenthal NOMINATE ideal points for legislators in the 106th U. S. House, the yolk, the 4r circle that the uncovered set lies within, and the uncovered

[4] Throughout the paper, we assume that preferences are Euclidian ("Type One").

computed by our procedure. In this figure, legislators' ideal points are blue dots. We calculate the yolk using a computational procedure whereby we draw candidate median lines through each ideal point at 1-degree intervals (that is, 360 lines through each ideal point), retaining the lines that have the same number of legislators on each side. The resulting median lines are the dark solid lines in figure one. We then add the yolk as the smallest possible circle that touches all the median lines (shaded circle) and add the 4r circle that contains the uncovered set (dashed circle). We then add the uncovered set as computed by our grid-search procedure.

As Figure one indicates, the 4r circle containing the uncovered set is quite large relative to the uncovered set computed by our procedure. Moreover, existing theory gives us no indication of how much of the 4Y ball is taken up by the uncovered set, or whether the set is centered or skewed to one side, up or down. The only other known bound on the uncovered set, i.e. that it lies within the Pareto set, here the convex hull of legislators' ideal points, supplies even less information.

Our technique for estimating the uncovered set treats the policy space as a collection of discrete potential outcomes rather than as a continuous space. This approach originates in a comment by the late Richard McKelvey (1986. 27): "…proposition 4.1 gives a potential "brute force" [iterative search] method for computing [the uncovered set] up to any desired degree of accuracy" (see also Miller 1980: 93).[6] In order to use McKelvey's intuition cited above, to characterize the UC we need to know two things: First, is the test in McKelvey's proposition 4.1 sufficient. Second, can we

[5] Feld et al. (1987: 138, Theorem 7) proved that at least for the two dimensional case, "the uncovered set is contained within a circle of radius 3.7r around the center of the yolk.

[6] DeDonder (2000) uses a similar approach to compare the predictions of the uncovered set with those of the bipartisan set and the minmax set in a model of purely redistributive taxation. DeDonder's focus is on whether any point is more or less likely to be in any of the three sets, given repeated sampling from a bivariate log-normal distribution of ideal points (p. 611). DeDonder does not report the exact simulation procedure he uses, but personal communication indicates that it is a grid-search procedure similar to ours. However, the grid used by De Donder is of a considerably lower resolution. More importantly, the use of repeated sampling from a bivariate log-normal distribution, leads him (2000: 625) to conclude that the uncovered set is both "selective (…selecting between 1% and 7% of the feasible options)…" and "not too sensitive to slight modifications of preference profiles." We show

approximate the UC by looking at grids with high enough resolutions. Theorem 1 took care of the first concern. Proposition 1 and 2 (due to Bianco et Al. 2004) resolve the second.

Proposition 1: If x is covered by a set with a non empty interior,[7] it will eventually appear as covered on a fine enough grid.

Proposition 2: If x is in the interior of the uncovered set, then on a fine enough grid x, or a point arbitrarily close to x, will appear as an element of the uncovered set provided by the grid procedure.

Together, Propositions 1 and 2 state that, at a high enough resolution, any point outside the uncovered set will disappear from the uncovered set produced by the grid procedure, and for every point in the uncovered set there will be a point as close to it as we want, in the uncovered set produced by the grid estimation procedure which yields Theorem 2.

Theorem 2 (Bianco et Al. 2004)***:*** Our grid procedure estimate of the uncovered set converges to the interior of the uncovered set. If the uncovered set has a non empty interior,[8] then the uncovered set estimated by an increasingly fine grids converges[9] to the true uncovered set.

Theorem 2 provides a theoretical asymptotic rationale to our grid procedure estimate of the uncovered set, stating that in the limit, the uncovered set delineated by the grid procedure will converge to the continuous uncovered set. It should be emphasized that in the discrete case, our procedure is not an approximation but actually computes the exact actual uncovered set.

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that both conclusions do not hold under more realistic specifications of legislators' preference profiles – the same is true for the common intuition that the uncovered set is small and centrally located.

[7] Thus, if the set that covers x has an empty interior, x may appear as uncovered even though it is covered. By the nature of the uncovered set this loss of generality does not pose a major problem because any x will almost always be covered by a set with a non empty interior if it is covered at all. It does however explain why our technique typically slightly over-estimates of the size of the uncovered set.

[8] See Bianco et Al. 2004, for a couple of technical qualification of this general statement of the Theorem. what we know of the uncovered set. In fact, we have every reason to believe that if the number of decision makers is odd, the uncovered set is connected in which case this problem never arises.

[9] Convergence is formally defined as follows: Let $V = \{V_1, V_2, ..., V\omega, ...\}$ be an infinite series of grids with $\lim_{w \to \infty} r(V_w) \to 0$ and $\forall w \in N : V_w \subseteq V_{w+1}$, $\forall x \in V_w$ such that the set that covers x has an interior $\exists \delta \in N : k > \delta \Rightarrow \ (x \notin UC(X) \Rightarrow x \notin UC(V_k))$ for any neighborhood of x, A(x) $\exists \delta > 0 : k > \delta \Rightarrow \exists y \in UC(V_k) \bigcap A(x)$, i.e., for any x in the UC(X) there exists a resolution that will depict a point as close to x as one would want as being in the uncovered set. Any point y not in the uncovered set, if it is

3. The Power of Minorities I: Minority Coalitions and Strategic Manipulation by Small Parties

Figure Two (Borrowed from Bianco et Al. 2004) is an application of our technique to a game with five legislators. Small diamonds are legislators' ideal points. Shaded areas are uncovered sets.

*** Figure Two About Here ***

When applied to Plott's (1967) equilibrium distribution of ideal points (top-left plot), our algorithm yields, as expected, the (0,0) point as the only point in the uncovered set.[10] The next five plots show how the uncovered set expands given changes in the location of a single legislator's ideal point. Figure Two suggests that the size and location of the uncovered set are extremely sensitive to the location of individual legislators' ideal points. Thus, the strategic representation of- or sincere changes in the preferences of minorities in legislatures can be very important determinants of the set of final outcomes in legislative politics. This observation lead us to start this new project to explore in more detail the power of minorities in legislative bodies, using this newly available tool to estimate the uncovered set of feasible outcomes in a legislature.

The potential of minority groups to influence legislative outcomes is not new. Strom (1990) brought to the forefront of the research agenda the role of minority governments in multiparty democracies. The importance of minority ruling coalition governments has since been extensively explored. Formal models emerged to explain this phenomena (Laver and Shepsle, 1996; Sened 1995, 1996). These models have since been reinvigorated and further generalized (e.g. Schofield and Sened, 2002). An extensive survey of these results and a very elegant general characterization of the conditions under which such minority coalition emerge and survive is found in Kalandrakis (2005). Most of these models share the same intuition. A large enough central party can *dominate* the legislative agenda because its declared ideal point

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covered by a set with a non empty interior, there exists a resolution that will eliminate it from the uncovered set obtained by the grid procedure (See technical appendices for details).

together with its electoral strength makes it a 'core' party (Sened, 1995, 1996, Schofield and Sened, 2002, 2005, 2006). Such a party has an obvious advantage in bargaining future coalition agreements. It has become common practice in the literature to assume that the coalition bargaining evolves around a trade off between government perquisites and policy payoffs (Laver and Schofield, 1990, Sened, 1996). Any party that deviates from its declared ideal policy point to endorse the policy position endorsed by a coalition pays a price that can only be offset by the government perquisites it guarantees itself in joining the coalition. A 'core' party does not have to pay this cost because it ends up implementing its own policy position as the coalition government coalition. Therefore, it has a serious advantage in bargaining process with other parties that do need to deviate from their declared ideal policy positions to join the coalition. It is not uncommon for parties in these environment to be so far apart that they cannot agree on an allocation of government spoils to satisfy all members who may join to oust the 'core' party from power. Under these conditions a 'core' party alone, or with some partners can govern as a minority coalition. Sened (1996) argues that this explains the long lived Rabin lead coalition that governed Israel between 1992-1995. Below, we argue that much of current Israeli politics can be explained as an effort by Ariel Sharon to capture a similar strategic position. Figure Three is a schematic presentation of an environment of this kind. It is easy to see that no majority coalition can defeat the policy position of the core party. If parties A,B,C and D are far enough from each other and care enough about policy positions, the 'core' party can count on these ideological distances to stand in the way of these four parties to coallessing against it and oust it from power. Thus the core party, while controlling only a minority of the seats in parliament, can still be a *dominant* party, govern on its own or at the head of a minority coalition and implement its ideal policy position unthreatened by the other parties in parliament.

[10] As Plott (1967) proved, given this configuration, the uncovered set is also the core and the Condorcet winner.

Figure Three About Here

Schofield and Sened (2006) recently suggested another way in which minority parties may affect final outcomes in legislative politics. Figure Two showed how a single player can modify the set of feasible outcomes in a legislative body of five legislators by sincerely modifying or strategically misrepresenting its preferred policy position. Our Discussion of Figure Two did not represent any rationale for this player to actually take such an action. But consider the legislative body in Figure Four. Two large parties control 40% of the seats, each, and one small party controls the remainder 20%. The obvious post election coalition would match one of the large parties with the smaller party. Suppose the sincere ideal points of the three parties are represented as points A, B, and C in Figure Four. Suppose Party A and B cannot change or misrepresent their ideal points, say because these are vote maximizing positions. It is well known that the uncovered set of a triangular configuration like ACB is close to the entire triangle. Assume that all points in the uncovered set are equally probable. Then C should be indifferent between its sincere position at C in Figure Four and a strategic position at C". Since ACB, ACC" and BCC" are identical triangle, the expected utility for C of ACB and AC"B is identical, assuming as we do all along Euclidean preferences. Schofield and Sened (2006) show that by a fixed point argument, there must be some point C' on the arc C-C" that maximizes C's utility in terms of distances of expected outcomes, because there is some uncovered set, AC'B, that minimizes the expected distance of the points it from the sincere ideal point of C at C in Figure Four. Schofield and Sened (2006) offer this observation as an important centrifugal force that lead parties in multiparty democracies to declare, strategically, extreme ideal points relative to their true preferences and/or vote maximizing position. The

ensuing effect on the final set of expected outcomes, illustrated in Figure Four, is the artifact of strategic misrepresentation of preferences by a relatively small party.

Figure Four About Here

4. The Power of Minorities II: The Open Field of the Uncovered Set

Way too much of mathematical modeling in Political Science is still restricted to uni-dimensional spatial modeling. In uni-dimensional spatial decision making environments, the final outcome is either the ideal point of the median voter or some variation on this result owing to misperceptions, belief structures that fail to converge or institutional constraints. Regardless of the nature of variation on this theme, it is virtually impossible to analyze the role of minorities in legislative bodies, using uni-dimensional spatial modeling. The introduction of the technology that allows us to compute the uncovered set in two-dimensional environments is an important innovation in this respect. So far, we surveyed existing knowledge on the role of minorities in legislative politics. While the introduction of the uncovered set was a useful analytical tool to make sense of these earlier findings, they do not depend on the introduction of this technological innovation. We now turn to a generic *power* of minorities that was literally unobserved and, to the best of our knowledge barely discussed prior to the introduction of this technology.

Experimental work in progress by Bianco et al. (2005b) unveils a very clear dynamics in experimental spatial games with an empty core. Consider Figure Five and Six respectively. Each Figure provides an experimental setting for a spatial game of decision making. The first figure in each of these respective Figures, represent a typical sequence of proposals that ends with a final outcome. The lower figure in each Figure summarizes all of the proposals and final outcomes in the respective spatial setting in which the particular collective choice process,

governed by majority rule, took place.[11] A full report of these results is found in Bianco et al. 2005b. For our purposes here, two important observations are crucial. Firstly, both the prototype sequences of these two experiments and the summary of the sequence of proposals and final outcomes in the entire set of dozens of experiment, clearly illustrate the power of the uncovered set as a solutions concept. The exemplary sequences clearly illustrate the theoretical argument according to which decision makers in these environment tend to quickly move to motions in the uncovered set and then cycle within the uncovered set until for whatever reason (usually not because they run out of time) they agree on a final outcome in the uncovered set. The summary of the sequential proposals in the large number of experiments and the final outcomes in these experiments clearly support the theoretical expectation that both proposals and final outcomes in these setting will almost always fall in the uncovered set. This is probably less impressive in the first setting (Figure Five) where the uncovered set is relatively large. It is more impressive to observe in Figure Six where the uncovered set is relatively small.

Figures Five and Six about here.

Secondly, and more important for our discussion here, note the obvious respect that both the exemplary sequences and the summary data show to the minority group in the respective experiments. With few exceptions the final outcomes have a significant skew towards the minority. Actually in analysis not reported here but noted in Bianco et al. 2005b, we show that the uncovered set of the majority group in these experiments is not a very good predictor of the final outcomes. In other words, winning coalitions show remarkable respect to the ability of the minority to pull them over and closer to the ideal points of minority players. Since these are

[11] We are grateful to our coauthors, Gary Miller, Michael Lynch and James Holloway for allowing us to use these figures in this paper while we are completing our work in progress on our experimental results. The details of the experimental setting and the full report of the results are found in Bianco et al., 2005b. A detailed report of the experimental setting and a detailed report of the results can be obtained from Michael Lynch upon request.

controlled experiments, we have a record of the coalitions that approve the final outcomes. With very few exceptions, it is the obvious minimum winning coalition at each experiment. Nevertheless, the final outcomes are clearly skewed towards the couple of players in the minority in a significant way. The two dimensional setting and the estimate of the uncovered set clearly illustrate, in this way, the effect of the minority couple of players on the exact location of the final outcome. In previous work (Bianco and Sened 2005) we emphasized one side of the coin: it is not exactly clear how much power the majority caucus in the U.S. House of Representatives really has to dominate the outcome most favorable to its members or leadership. Here we emphasize the other side of the same coin: Experiments guided by our estimate of the uncovered set clearly demonstrate that minorities have considerable power to tilt if not significantly skew final outcomes in their direction.

5. The Predictive Power of the Uncovered Set: Explaining Current Israeli Politics

All of the work presented up to now is subject to an important caveat: does the uncovered set have any predictive power? Even if our technique allows us to locate the uncovered set given real-world preference data, this innovation is useless if the set's intuitive appeal is not matched by its ability to capture real-world outcomes. The controlled experiments discussed above take us a step further towards predictability, but they are what they are: controlled experiments where students are carefully introduced to the logic of decision making and, all be it to a lesser degree, know what is expected of them. In this section we use existing estimates of political positions of parties in the Israeli Parliament, the Knesset, to explain current political maneuvering by Prime Minister Ariel Sharon and make clear prediction of what may happen after the March 2006 election based on our estimate of different possible uncovered sets that may results from different anticipated electoral outcomes.

Figure Seven provides a schematic presentation of the current Knesset based on Party positions estimates by Schofield and Sened (2005, 2006). To make the picture somewhat clearer, a number of parties were omitted from the analysis with very little effect on the configuration of the party structure of the Israeli Knesset and the uncovered set that we estimated for this paper. Figure Seven elucidates a puzzle of current Israeli Politics: why did Sharon become so conciliatory towards the Palestinians? Sharon's long time friends and members in his party literally exonerated him from the party following his consistent endorsement of positions considerably to the left of the policy he and other members of the elite of the Likud party endorsed prior to the 1999 and then the 2003 that brought Sharon to power. Figure Seven provides a rare insight on a very probable explanation of this otherwise enigmatic phenomenon. The entire set of feasible outcomes, based on the structure[12] of the current Israeli Parliament is considerably to the left of the Likud, declared and publicized, ideal point. The party governance argument might suggest that Sharon could have endorsed points much closer to Likud. The power of the uncovered set as a solution concept lies in the recognition that different points in the uncovered set are equally likely. Actually some recent research suggests that points at the center of the uncovered set are more likely than points at the boundaries Bianco et al. 2005a,b), which goes a step further to explain Sharon's 'defection' (term used by his own party members to describe his recent policy stands) to the left.

Figure Seven About Here

In November of 2005, Amir Peretz, a life time union activist, won the primaries in the Israeli Labor Party and became the new head of the party.[13] Many observers saw in this election

[12] Schofield and Sened (2006) use the term 'structure of parliament' to refer to the actual positions of the parties in parliament weighted by the number of seats each party got. It turns out that these two variable are sufficient to calculate the set of feasible outcomes as our work there and our work here clearly demonstrate.

[13] Schofield and Sened (2006) dedicate two chapters in their forthcoming book to explain that parties who fail to compete over voters, because of what they call 'low relative salience, (vis-à-vis Sharon and the Likud Party in this case) eventually fall into the hands of the core activists. They use the case of the Conservative Party after Thacher as their example. The Labor 'falling' into Peretz hands is another exemplary case in point.

a critical change in the Israeli Political map. Figure Eight of the uncovered set after the ascendance to power of Amir Peretz, taking into account by a 10 point shift to the left on the security dimension, suggests otherwise. In research claiming to illustrate the power of minorities in legislative bodies governed by majority rule, it is important to emphasize the limit of this power. In this case the overall structure of the Israeli Parliament in terms of positions of the other parties and their relative weights in Knesset seats, makes the change of heart of the Labor Party of little if any consequence to Israeli Politics, at least in terms of expected final outcomes.

Figure Eight About Here

But one thing did change. In a high profile move Sharon left the Likud Party and signaled a strong move to the left by joining the former head of Labor and the author of the Oslo accords, Shimon Peres, and a couple other senior Labor Party members, to form the new party Kadima ('Forwrard'). This move, positions Sharon at the center of the political map at (50,50) as shown in Figure Nine. The Figure shows how this move considerably diminishes the expected manipulability of final outcomes as the size of the uncovered set is clearly reduced. Sharon's true preferences at this point are anybody's guess, but in terms of governablilty, he certainly consolidated his position and ability to control future policies after the March 2006 election.

Figure Nine about here

But, in a much less publicized move, Sharon took his political maneuvering a step further enlisting the support of Uriel Reichman, founder of the Shinui party, to his new Kadima Party. It is hard to explain this move. Professor Reichman is a very notable figure in the Israeli Society, but he never held an elected office and his marginal electoral effect is expected to be rather small. Given Reichman's opportunity cost associated with joining the new party (Uriel Reichman is currently the President of IDC, the largest and by far most successful private university in Israel)

we should not expect Sharon to have enlisted Reichman's support for free. The gain in this move is clearly illustrated in Figure Ten. By enlisting the support of Uriel Reichman, Sharon ensures a strong anti-clerical move south in the policy space. This move lands him the position of a structurally stable core position at the center of Israeli Politics. It literally moves him very close to the position previously held by the Labor Party when headed by the slain Itzhak Rabin following the 1992 elections in Israel. This position will allow Sharon to do literally whatever he sees fit, unchallenged by any other party in the Israeli Parliament (Sened, 1996). Once again, a party that is currently expected to earn about 30% of the votes in the 2006 election has, thus, secured itself a *dominant* position in the Israeli Knesset. Dominant, of course, as long as Sharon remains faithful to the position he currently seems to have signaled the voters he holds.

Once again, it is important to emphasize the limits of the argument. Sharon 'captures' the core in the 2006 election if Kadima secures at least 25% of the votes. Below this threshold, the power of the central position is diminished and Sharon finds himself to the left instead of to the right of an uncovered set almost as large as he had to deal with during his current tenure as prime minister. This eventuality is illustrated in Figure Eleven.

6. Discussion

Our analysis of the size, shape, and location of the uncovered set in abstract social choice situations, lab experiments and the contemporary, otherwise hard to make sense of, Israeli Political arena, yields two important tentative insights.

Firstly, and most importantly in the context of this paper, the uncovered set proves itself to be a unique tool in the analysis of the power of minorities to affect and determine final outcomes in legislative political bodies ruled by majority rule. The power of minority coalitions (Strom, 1990) and the ability of relatively small parties to manipulate the set of final outcomes in such legislative

institutions (Schofield and Sened, 2006) was previously noticed. The main contribution of this paper is the illustration that minorities are left with much maneuverability, that they obviously use to their advantage, within any existing uncovered set of feasible outcomes, provided that this set does not collapse to a core. A relatively rare occurrence in multiparty democracies.

Secondly, our analysis suggests that the uncovered set has considerable explanatory and predictive power. In experiments it seems to perform considerably better than any other solution concept currently in use (Bianco et al. 2005a, 2005b). Its explanatory power was illustrated previously in the context of the contemporary and former U.S. House of Representatives (Bianco and Sened 2005). In this paper we showed its relevance in the analysis of multiparty legislative bodies by using the uncovered set to articulate the underlying logic of current Israeli Politics.

The main challenge ahead of us is to develop precise measures of relative predictive success to support this research program. The deviation of the empirical evidence presented in this paper from traditional quantitative data analysis, presents interesting challenges in terms of precise measures of success and statistical significance that need to be developed to land further credibility and strength to this analysis.

In terms of substance the use of this new technology to estimate uncovered sets in real life environments opens up a large number of research arena, not the least of which is the further exploration of the power of minorities under majority rule.

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Figure One.
Ideal Points, Median Lines, and Yolk, 106th U. S. House

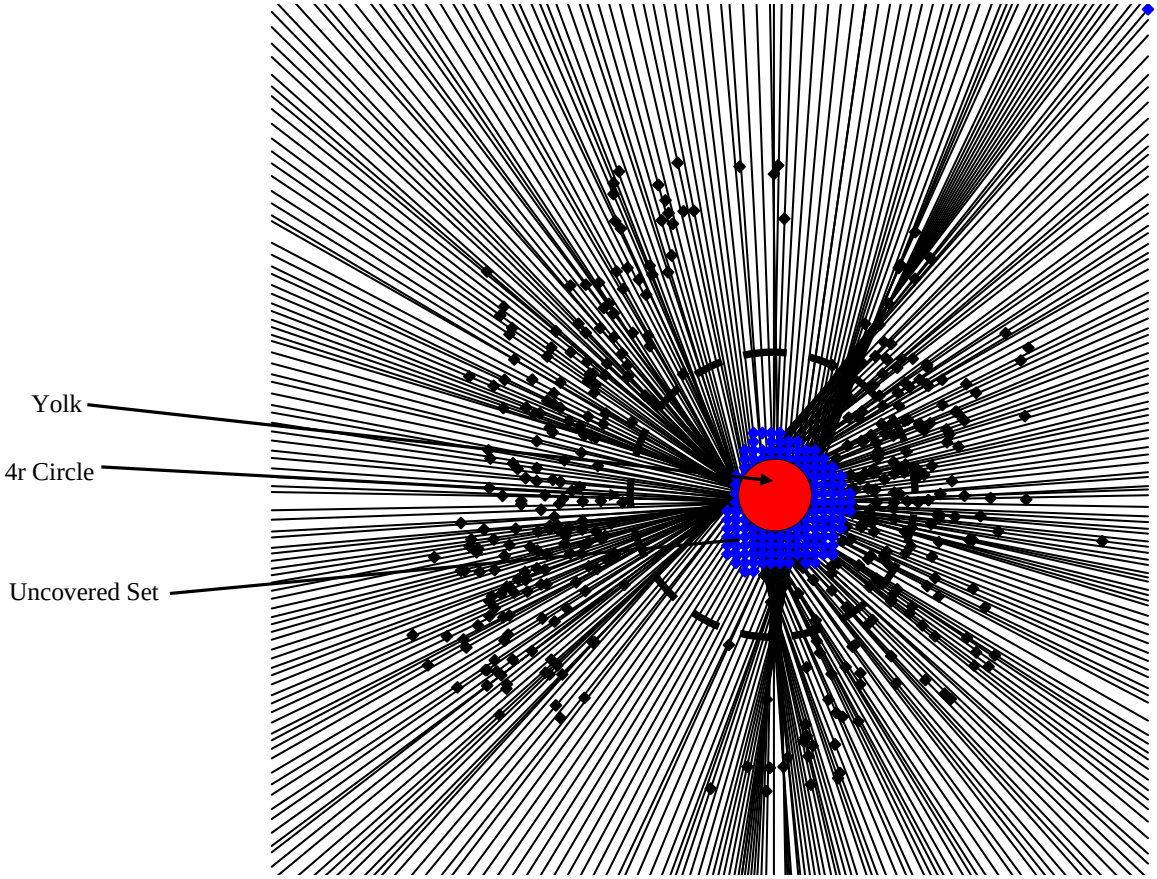


Figure Two:
The Uncovered Set and the Plott Equilibrium Condition

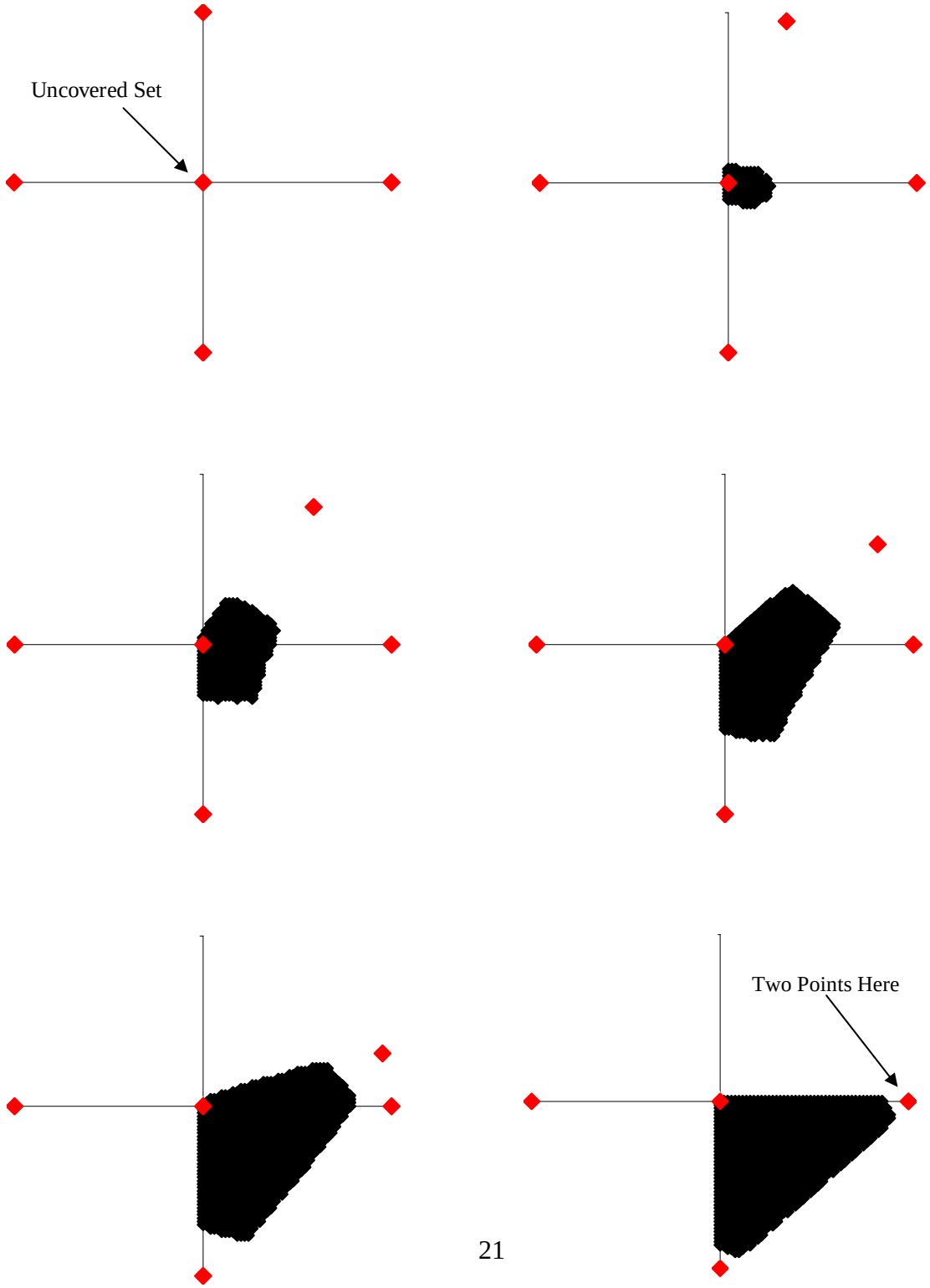


Figure Three: A Core *Dominant Party* can Govern as a Minority Coalition Government Even if it Controls only 35% of the Seats in Parliament

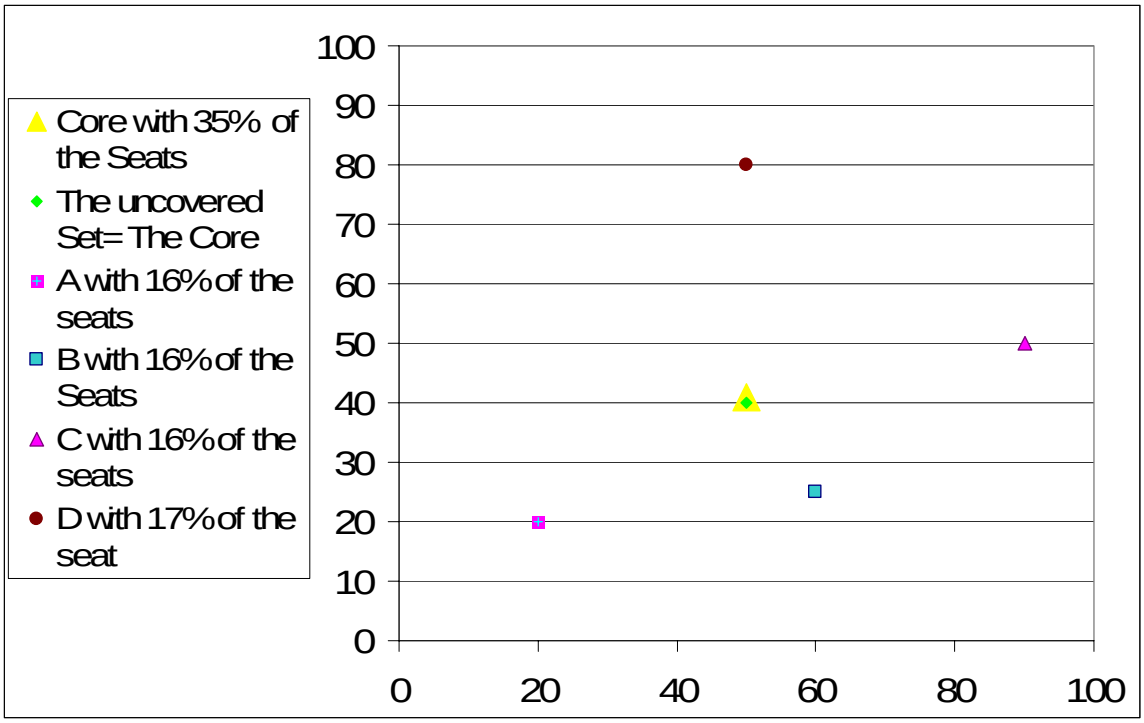


Figure Four: Strategic Maneuvering of the Uncovered Set by a Minor Party

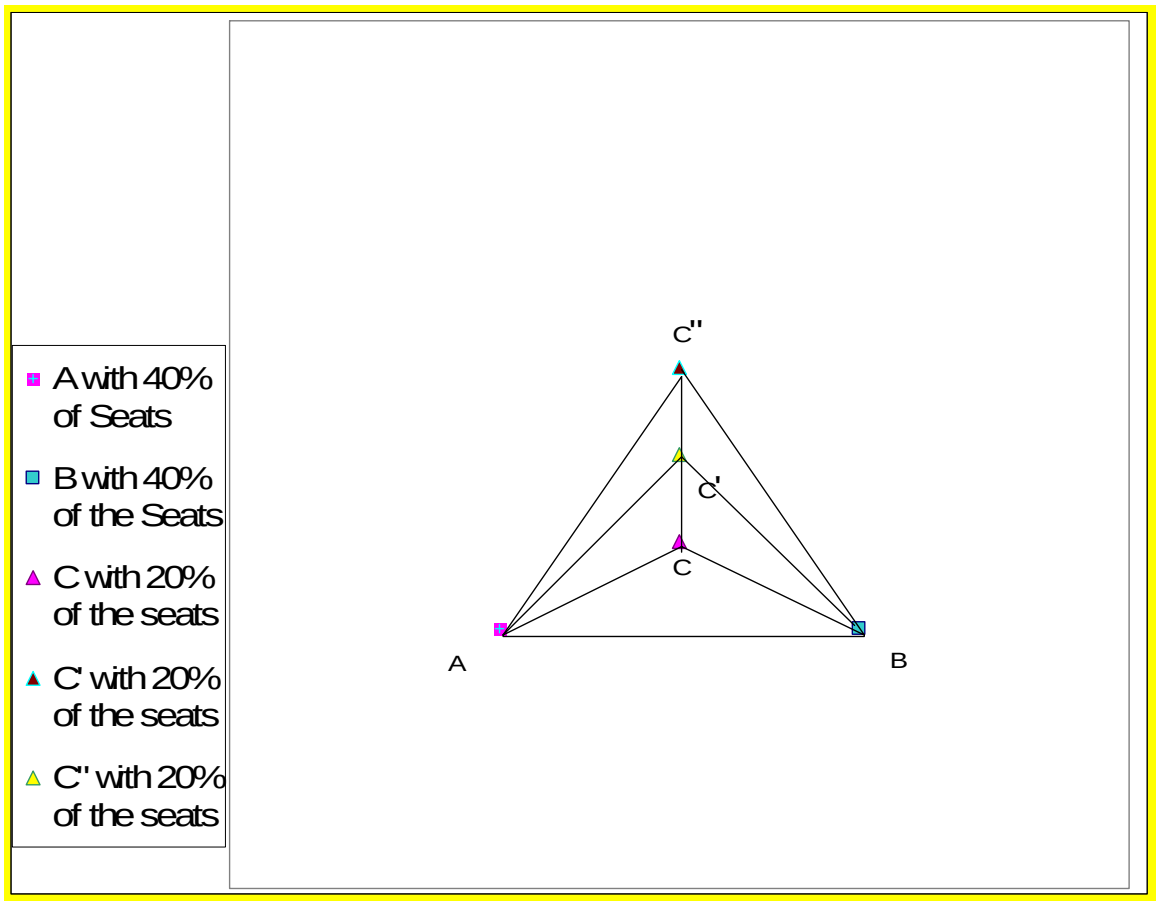
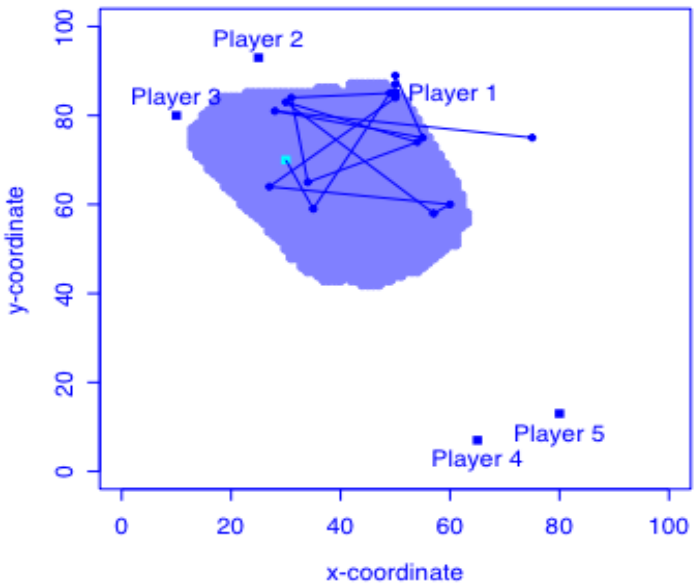
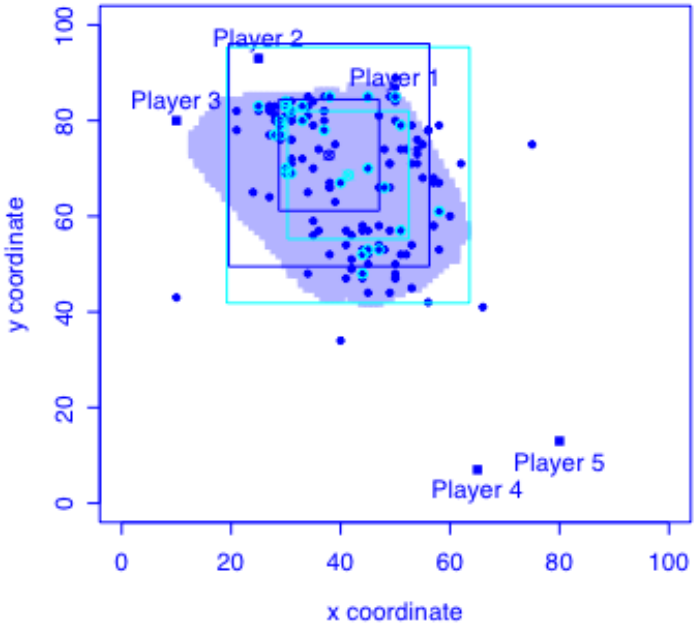


Figure 5a. Configuration 1 - Winning Proposals from A Prototype Negotiation Session



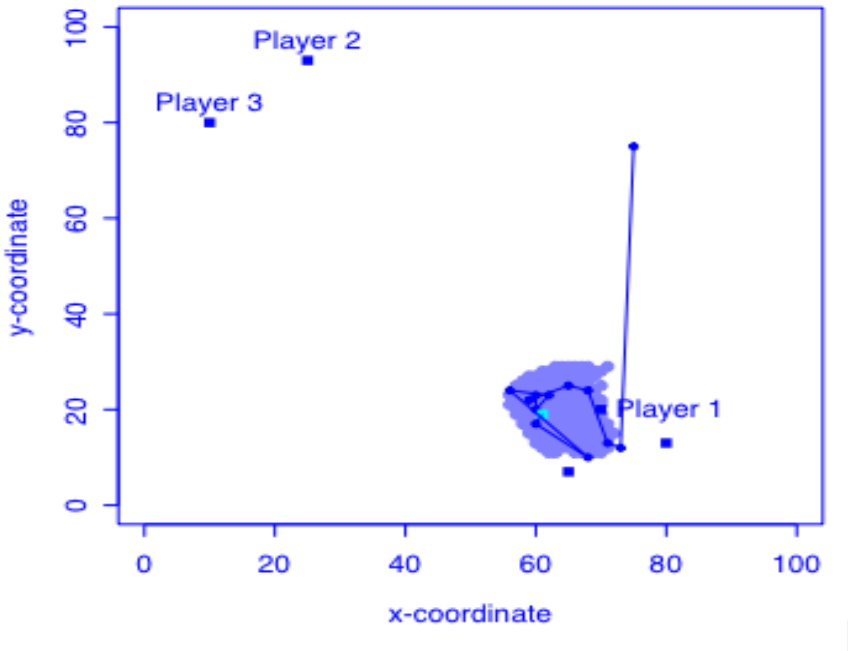
Note: This figure shows the winning proposals for Group I from the September 24, 1:00 p.m. session. Blue square dots are ideal points. The grey field is the estimated uncovered set. Blue dots are winning proposals. The turquoise dot is the final outcomes of the session. The blue lines show the progression of the winning proposals.

Figure 5b. Configuration 1 - Outcomes, Mean, and Standard Deviations



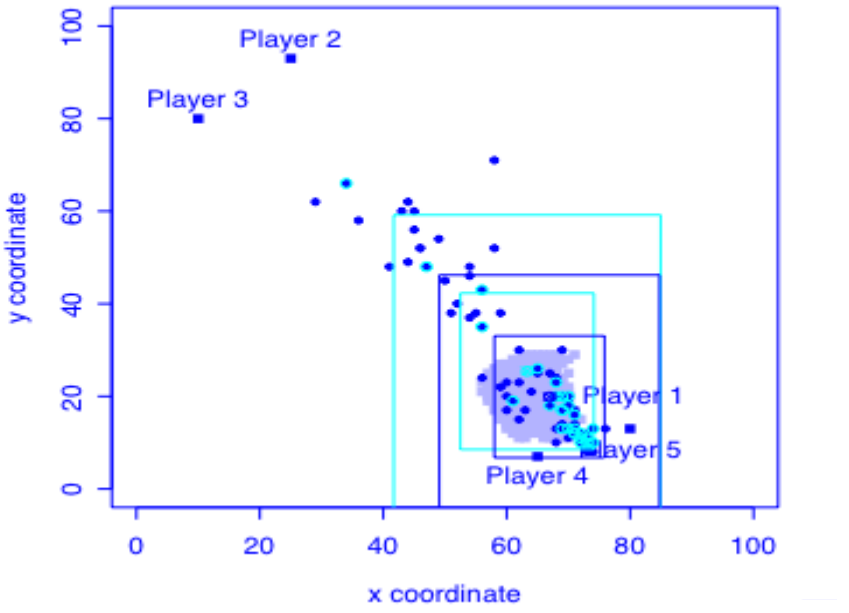
Note: Blue square dots are ideal points. The grey field is the estimated uncovered set. The blue dots are winning proposals. The turquoise circles are final outcomes. The blue rectangles contain winning proposals one and two standard deviations from the mean outcome. The turquoise rectangles contain final outcomes one and two standard deviations from the mean outcome.

Figure 6a. Configuration 2 – A Prototype of a Sequence of Proposals in a Single Experiment.



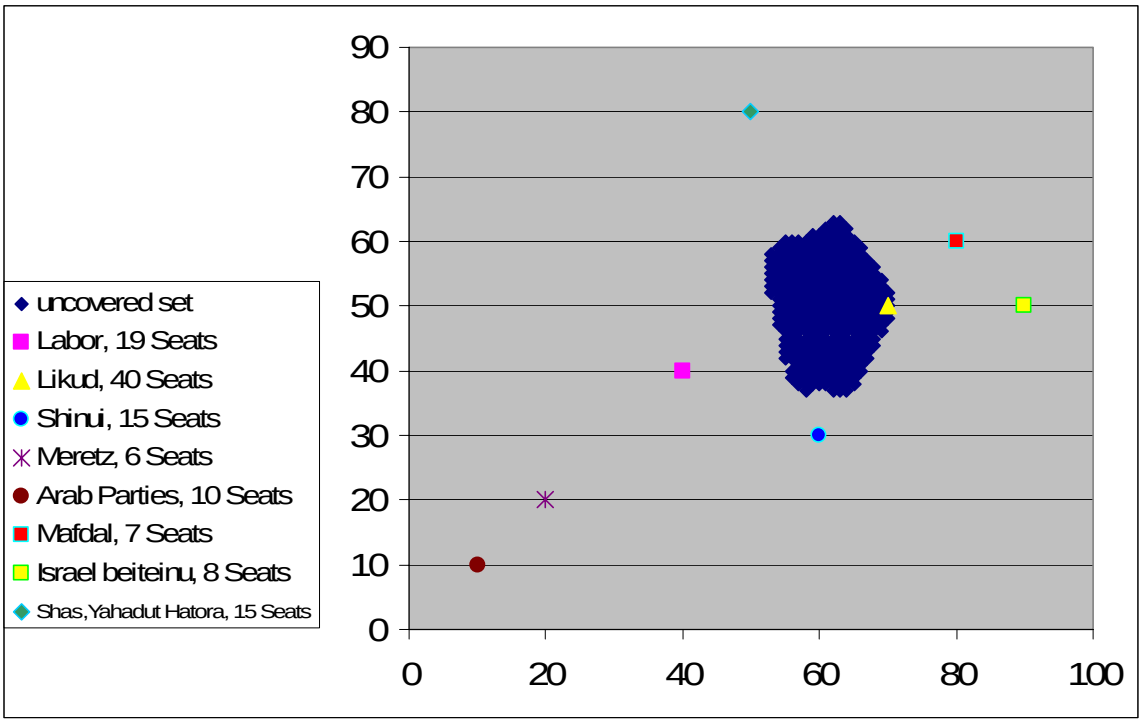
Note: This figure shows the winning proposals for Group 2 from the September 17, 2:30 p.m. session. Blue square dots are ideal points. The light blue field is the estimated uncovered set. Dark blue dots are winning proposals. The light turquoise dot is the final outcomes of the session. The blue lines show the progression of the winning proposals.

Figure 6b. Configuration 2 - Outcomes, Mean, and Standard Deviations



Note: Blue square dots are ideal points. The grey field is the estimated uncovered set. The blue round dots are winning proposals. The turquoise circles are final outcomes. The blue rectangles contain winning proposals one and two standard deviations from the mean outcome. The turquoise rectangles contain final outcomes one and two standard deviations from the mean outcome.

Figure 7: Current Politics in the Israeli Knesset Parliament



Note: The X coordinate represent position on the Security (resolution of the Israeli-Palestinian Conflict) dimension. The Y dimension represents positions on the Religious Dimension (to what extent Israel should adopt/abide by Jewish Law).

Figure 8: Israeli Politics After Amir Perestz Won the Labor Party From Shimon Peres

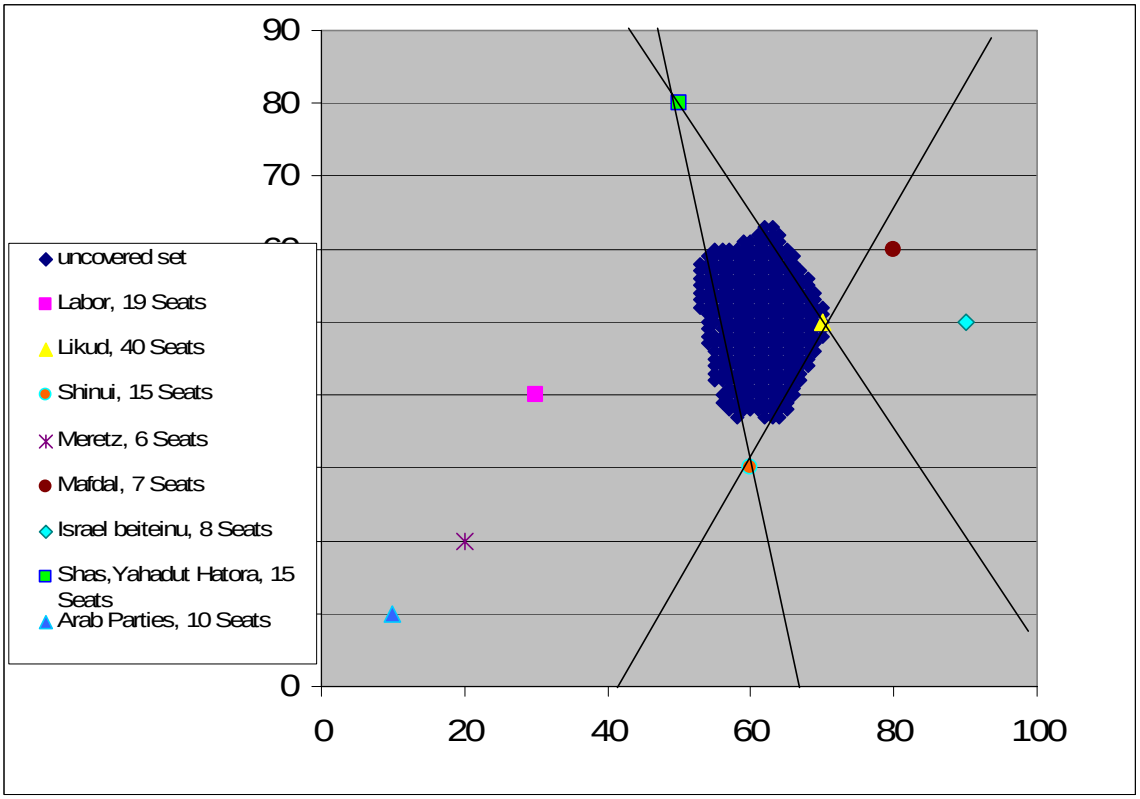


Figure 9: The Creation of Kadima as a new Center Party in the Israeli Knesset

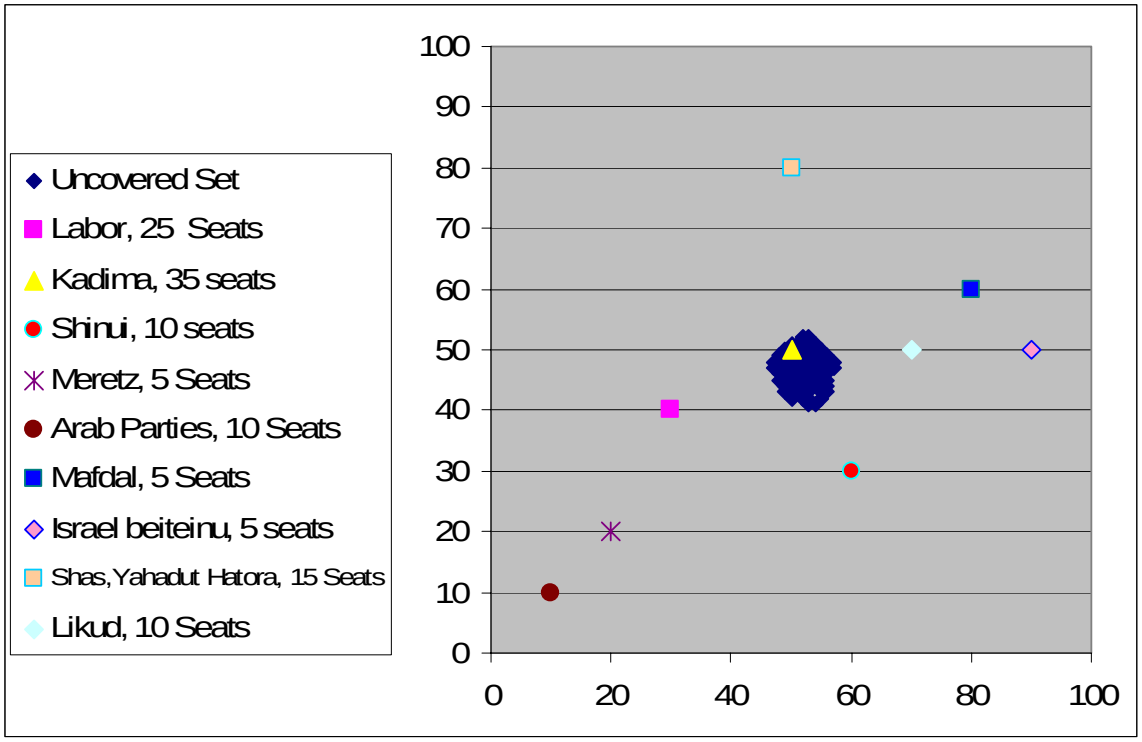


Figure 10: Kadima Lead by Ariel Sharon Captures the Core of Israeli Politics

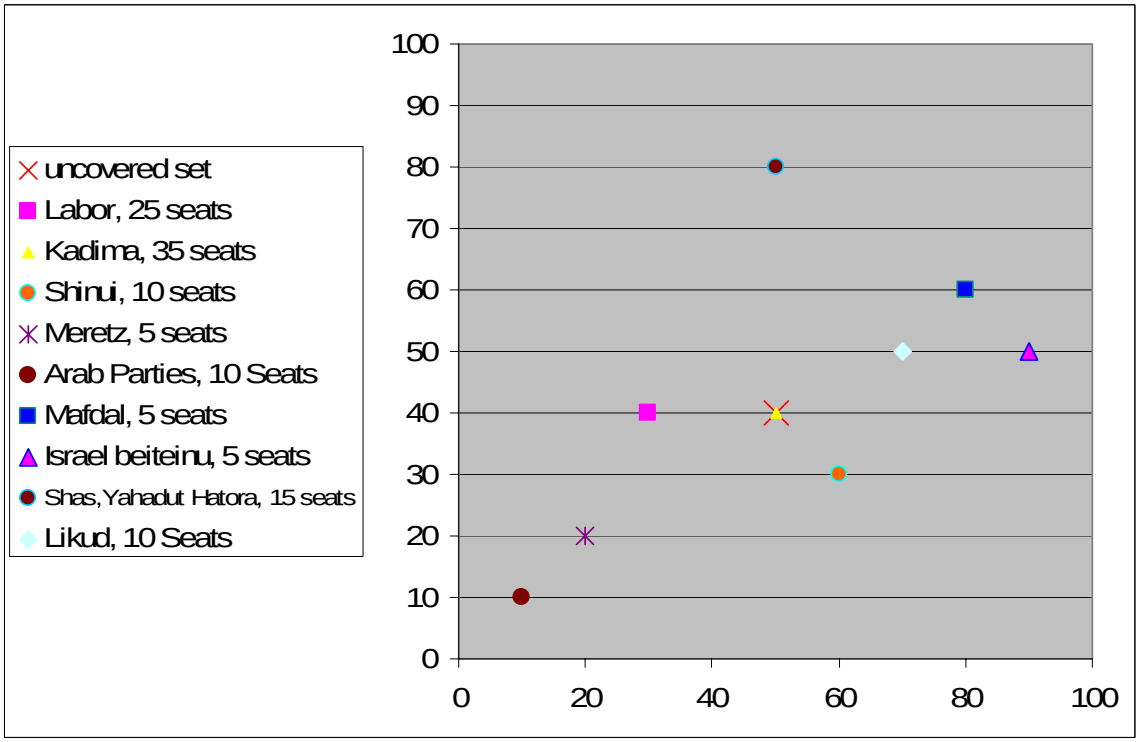


Figure 11: What If? Kadima and Sharon Fail to 'Capture the Core' of Israeli Politics