The twenty-two minimal dichotomy decompositions of the equilateral distance on five points.

Bernard Fichet
Laboratoire de Biomathematiques
University of Marseille, France
Bernard.Fichet@medecine.univ-mrs.fr

Given a finite set I with cardinality n, a distance d on I is of L_1-type if (I, d) is isometrically embeddable in some $\mathbb{R}^N$ endowed with the l_1-norm. It is known that this condition is fulfilled if and only if d may be written as :
$d = \sum_J \alpha_J d_J, \alpha_J \geq 0, (J, J^c)$ cut (or bipartition) of I, where d_J is the dichotomy associated with J.
Geometrically, this formula shows that the set D_1 of semi-distances of L_1-type is a polyhedral cone. All dichotomy decompositions characterising d as above, form a polytope in the dual space, the vertices of which are the minimal dichotomy decompositions. These minimal elements are strongly connected with some dimensionality problems, generally of very high complexity.
We exhibit here the twenty-two minimal dichotomy decompositions of the distance d_1 of the regular simplex for $n = 5$. A precise investigation of this family allows us to settle a conjecture on the axes defined by an L_1-norm principal component analysis of (I, d_1), and to recover a result concerning the minimum L_1-dimensionality of (I, d_1).