On interval-families, transitive orientations and Gallai arc-equivalence in graphs

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Let G be an undirected graph and K a clique of G. We deØne a K-Ølament as a subset F of K, such that for each x 2 K nF, all arcs xy with y 2 F are equivalent, in the sense of the transitive closure of the ° relation deØned for any graph by Gallai in his characterization of comparability graphs. When G is a comparability graph, the set F[K] of all K-Ølaments consists of those subsets of K which are intervals of each transitive orientation of G. In addition, the set Lin(F[K]) of all linear orders L on K such that each K-Ølament is an interval of L, coincides with the set of restrictions to K of all transitive orientations of G. We deduce that there exist some pairwise disjoint maximal cliques, say $K_1; : : : ; K_p$, such that the set of transitive orientations of G is in bijection with the cartesian product of sets $Lin(F[K_i])$. Furthermore, it turns out that a set-system (X; S) is the collection F[X] of X-Ølaments of a clique X of some graph G iÆS is a partitive set family. In this case, the tree decomposition of S coincides with the trace on X of the Gallai modular decomposition of G. Finally, we consider C_5-free graphs that satisfy some injectivity condition concerning the map x ! N(x), and we prove that in this case, the recognition of comparability graphs can be reduced to the problem of deciding if there exists some linear order for which a speciØc family of clique-Ølaments is a collection of intervals. This last result generalizes a characterization obtained by Moore for 2-dimensional posets of height 2.