

# Dynamics of Human Behavior \*

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*Abstract:* I review network aspects of human dynamics that link macro-historical to micro-sociological and evolutionary processes. The ability to bond in communities of varying spatial scales is a special property of humans that happens through social networks. These networks have greater cohesion through invulnerability to disconnection without removal of  $k$  nodes. Menger's (1927) connectivity theorem shows that this property of  $k$ -cohesion mutually entails  $k$  node-independent paths between every pair of group members. Because of this property, i.e., by redundancy of communication, humans in such communities can utilize language and long-range communication to compensate for diminishing face-to-face interaction as groups grow large. For a given level  $k$  of cohesion, the maximally extensive  $e(k)$  group size is unbounded and scalable because, for each cohesive intensity level  $k$ , the maximal group size  $e(k)$  can expand indefinitely without the need to increase the average number of ties per member. Hence, the growth of human community size is scalable at a fixed cost in number of ties per person, unlike those species unable to take advantage of  $k$ -connectivity. Strong causal effects, using the  $k$  cohesion-level measure of empirical groups whose boundaries and extent are defined by  $e(k)$ , have been replicated and validated in various sociological and anthropological network studies. This allows me to examine the micro-macro linkages between scalable properties of  $k$ -cohesive groups and concomitant sociopolitical processes and how they relate to the social and historical dynamics of socially cohesive networks. Qualitative dynamics of major historical processes in human behavior, which are related for example to warfare and empire formation, are consistent with scale-up of sociopolitically  $k$ -cohesive groups. Such groups expand across metaethnic frontiers to evoke resistance that operates through scale-up of  $k$ -cohesive growth-by-opposition. Some current studies of such issues (Turchin 2003, 2005) use sufficient levels of aggregation to successfully assess dynamic interactions between macro-variables in sociopolitical processes (some of which involve political unit cohesion and scale). Others, such as the conflict studies of Lim, et al. (2007), use field-theory models of spatial interaction.

New hypotheses, questions, and results may help link scalable  $k$ -cohesive groups to human evolutionary modeling and to variables used in evolutionary models of cooperativity and of transitions in sociopolitical organization. These various kinds of empirical studies illustrate concepts and methods in dynamics and complex systems applicable to human behavior in the domains I review. The mainline arguments illustrated here are expanded by reviews of work on other causal process models that combine micro-analysis of sociopolitical and economic behavior in the context of institutions, networks, historical ethnography, and network economic experiments. I note new directions flourishing in causal modeling, including multifractality and agent behavior, that evince further need for development of historically longitudinal databases, advancement of methods for dynamical analyses, and use of multilevel modeling that incorporates network representation and conceptualization.

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## ARTICLE OUTLINE

|   |    |
|---|----|
| Glossary and Notation   | 1  |
| I. Definition of the Subject and Its Importance                               | 3  |
| II. Introduction  | 3  |
| III. Networks and Cohesion in HB Dynamics                                     | 4  |
| IV. Cooperation, Connectivity- $k$ , and “Critical Mass” in Collective Action | 8  |
| V. Transition Models with Thresholds  | 11 |
| VI. Aggregate (“sufficient unit”) Equation-based Modeling                     | 15 |
| VII. Institutions, Network Economics and Experiments: Testing Causality       | 17 |
| VIII. Future Directions   | 19 |
| IX. Bibliography  | 21 |

GLOSSARY AND NOTATION – items †marked in text on first occurrence

†**connectivity- $k$**  ( **$k$ -connected**,  **$k$ -cohesive**, †**structural cohesion**, †**cohesive.blocks**), refer to the Menger (1927) theorem for structure/traversal isomorphism in graph theory, as explained in the text, where  **$k$ -components** are the largest possible expansion (maximal group) that preserve structural  $k$ -cohesion. Computation is provided by **cohesive.blocks** in the **igraph** R package.

†**scale-free network**, where the probability that a node  $i$  in the network connects with  $k$  other nodes is inversely proportional to the number of  $k$ 's links (see: power law), more generally,  $p_i(k) \sim k^{-\lambda}$ , with  $\lambda=1$  for scale-free.

†**nonindependence** is characteristic of complex phenomena with built-in interdependencies, where distributions of attributes or relations should not be not directly subject to statistical inference using the null hypothesis of independence, as in structural measures sampled from networks, and autocorrelated time series or autocorrelated spatial distributions.

†**sufficient statistic**. a *sufficient statistic* for a statistical model is one that captures the information relevant to statistical inference within the context of the model, including the size and composition of the units of study. Let  $X_1, \dots, X_M$  be a random sample, governed by the density or probability mass function  $f(x|\theta)$ . The statistic  $T(x)$  is **sufficient** for  $\theta$  if the conditional distribution of  $x$ , given  $T(x)=t$ , is independent of  $\theta$ . Equivalently, the functional form of  $f_{\theta|x}(x)$  does not involve  $\theta$ , and the Fisher-Neyman Factorization Theorem may be used to help spot sufficient statistics. The Likelihood Ratio test can often be reduced to a sufficient statistic of the data for hypothesis testing. The Minimum Variance Unbiased Estimator of a parameter  $\theta$  can be characterized in parameter estimation by sufficient statistics and the Rao-Blackwell Theorem. See Scharf (1991) *Statistical Signal Processing* [125]. A **sufficient unit** is one for which a random sample of aggregate statistics are sufficient.

†**aggregate (“sufficient unit”) equation modeling** assumes that causality can be found with quantitative equation models that use **sufficient statistics**, which implies that the aggregate units studied have cohesive mass or entity for causal interactions to act on their aggregate characteristics. See Section VI.

†**NP-complete** (NPC) algorithms require an order of non-deterministic polynomial time (NP) but are exceptionally difficult: if a deterministic polynomial time solution can be found for any of them, it would provide a solution to every other problem in NP and empty out the class of NPC.

†**dictator game**, where the first player proposes a split of some endowment and the second, entirely passive, receives the remainder. Not formally a game at all (as the term is used in game theory, where every player's outcome must depend on the actions of others), it is used in decision theory to test the *homo economicus* model of individual behavior, where selfishness would dictate allocation entirely to oneself. Henrich, et al. [51] discovered in a 15-society cross cultural study that people do allocate a share of the endowment to others. Skyrms [57] gives the dynamics of an evolutionary game theory variant.

†**concentration indices** such as the Laakso-Taagepera Index  $1/\sum_i p_i$ , where  $p_i$  is an effective proportion weighting for each unit, are used for problems such as “what are the effective numbers of political parties self-weighted by their membership (for polities: by their population or area)”, e.g., U.S. party proportions { .49, .49, .02 } would have an effective number of 2.08 while France with 101 parties (each weighted by its number of members) might have effective party number of 22.1.

†**power law**, a Pareto distribution where probability  $p(x) \sim x^{-\alpha}$ , as for example: “multifractals have tails that follow a power law” (Mandelbrot and Hudson 2004:209 [107]) in how the frequency of similar units at different scales varies with the scale; see multifractal. Power laws tend to become ubiquitous when what is studied involves dimensional constraints. Power-law *growth* is expressed as  $N = K/(t_0-t)^k$  where  $K$  is an initial constant,  $t$  is calendrical time, and  $t_0$  is the calendrical singularity date at which  $K/(t_0-t) = K/0$ , where division by zero produces dynamical instability as  $K/(t_0-t) \rightarrow \infty$ .

†**fractal**: a pattern or object (e.g. geometrical) whose parts echo the whole, only scaled down, i.e., scale invariant; invariant at any scale of magnification or reduction. Fractal prices occur when positive and negative changes in prices (daily, weekly, monthly, yearly) follow a power law. “To improve almost any fractal model it is a good idea to replace it with a multifractal one” ([107]:209). A **multifractal** (with root and generator) is a composite pattern that begins with an initial root (e.g., a straight line) that is successively replaced with a generator (e.g., a zagged line) that replaces every instance of the initial element. See power law.

†**causality** is a relation holding between two variables such that manipulation of one of the variables (the potential cause) is reliably associated with variation in the other (the response), for some configuration of the values of other potential causes of the response. Estimation includes classical structural equations approaches [1], the treatment effects framework [2], the Directed Acyclic Graph (DAG) probabilistic approach [3], and the settable system probabilistic approach that unifies all three [4]. Another aspect of causation is probabilistic evaluation and decision theory, in which case the effect of evidence in revising beliefs about causation can be studied in a Bayesian framework [5][6]. Probability of causation is not causation of probability although there are probabilistic causative models.

## I. DEFINITION OF THE SUBJECT

Dynamics of Human Behavior (abbreviations DHB, HB, HD) deals with the effects of multiple causal forces in human behavior, including network interactions, groups, social movements, and historical transitions, among many other concerns. Description of movement and change distinguishes kinematics from statics, while dynamics considers causes of movement and change. Pearl (2000) [3] summarizes issues of †causality with two fundamental questions: (1) What empirical evidence is required for legitimate inference of cause-effect relationships? (2) Given that we are willing to accept causal information about a phenomenon, what inferences can we draw from such information, and how? Policy issues entail beliefs about causation and open a second framework for evaluating beliefs about causality [5][6]. HB dynamics is a field replete with new discoveries—and applications of methods derived from problems and principles that apply across disciplines. Insights transfer across disciplinary boundaries. This is because research strategies for studying causality in a hierarchy of sciences are typically not a reductionism of one level to another but involve recognition of emergence at different levels. Common principles that apply are often shared but with different detailed applications more finely tuned to irreducible aspects of concurrent phenomena. Mathematics and physical principles apply at various levels in the scientific disciplines, but principles discovered in the human and evolutionary sciences are increasingly found to apply and generalize as well.

DHB takes into account the distinctive behaviors of humans and the range of their sociopsychocultural variations. Focusing on causes, HB dynamics may refer, for different levels of social entities, to spatial and temporal, local and long-distance interactions, growth and decline, oscillations, changes in distributional properties, and synchronous or time-lagged causality in dynamical evolution. Examples of precursors in DHB include Ibn-Khaldun's (c.1379) dynamical characterizations of the oscillations of Muslim and Berber political dynasties and charismatic tribal initiatives [7]. Ibn-Khaldun's work was an extraordinary early precursor of the empirical study of oscillatory sociopolitical dynamics (as contrasted with beliefs in cycles of renewal, for example, derived from experience with cycles in nature) and is incorporated into contemporary DHB modeling. Similarly, Richardson's *Statistics of deadly quarrels* (1960) searched for causality of war and posed behavioral dynamic equation-based decision models with basins of attraction for stability, disarmament, or the arms race [8]. Schelling's (1960) "focal point" solution in the study of strategic behavior and bargaining ("each person's expectation of what the other expects him to expect to be expected to do") advanced the game theoretic policy sciences while his *Micromotives and Macrobehavior* [9] was seminal for modeling complex causal feedbacks. Interest in lower-level processes and how they link to higher levels motivates much of HB dynamical modeling. This is the case as well in biological modeling, as in SFI researcher David Krakauer's statement of research on "the evolutionary history of information processing mechanisms in biology, with an emphasis on robust information transmission, signaling dynamics and their role in constructing novel, higher level features. The research spans several levels of organization finding analogous processes in genetics, cell biology, microbiology and in organismal behavior" [10].

## II. INTRODUCTION

HB dynamics is grounded within an evolutionary framework and interacts well with research in biology and primate and human ethology. Fundamental problems in new and old approaches to HB dynamics include general approaches to identify and model (1) units of analysis, (2) interaction equations and structures, and (3) levels of analysis, with (4) †sufficient statistics. Many problems concerned with the "units" of investigation, organized into systems, are multifractal, and are explored through detailed study of social organization, biological reproduction, evolutionary phylogeny, and developmental ontology. A focus on networks recognizes the fluidity of dynamical interactions in living systems (i.e., recognizing the limits of hard-unit and hard-wired modeling). Network analysis also links to hydrodynamics, nonlinear synchronization, percolation, and other physical processes as well as models derived from the study of graphs and lattices. Generalizations of entropy measures may also provide approaches for testing general principles in physics that are more useful than mechanics, solid-state physics, or conventional models of entropy. While many principles of complexity sciences will apply across many disciplines, how they apply varies with subject matter.

Formal approaches to HB dynamics—where *formal* means theories have been stated in a formalized language, usually mathematical, that does not allow for variable readings [11][28][29] — require construction on the basis of careful descriptive, qualitative, and quantitative research about human behavior and institutions such as are independently carried out in the disciplines (history, sociology, economics, psychology, cognitive science, political science, linguistics, and anthropology, including ethnography, archaeology and other domains) as well as in cross-disciplinary fields including those of complexity sciences.

The modeling of human behavior is still in its infancy and there are likely to be widespread advances in many different areas in coming years. The examples here show a range of concepts and practices but are not intended to cover all of the definitive techniques for modeling human behavior. Among the formal and complexity science approaches in HB dynamics, some of the examples include network modeling, †aggregate equation-based modeling, and simulation modeling (equation or agent-based, or both), and how these deal with problems of †non-independence. Network modeling depends on finding means of bounding and measuring fields of interaction where particular kinds of units and their causal interrelations can be specified. “Sufficient unit” modeling looks for aggregates at particular scales that represent relative closures of systems in which causality from internal dynamics can be studied for certain types of relatively well-bounded units that occur within limited ranges of scale. Briefly exemplified are institutional studies of the evolution of market systems extended by experiments in network economics. Not covered are generalized “open system” entropy maximization [120], fields such as fractal dynamics that have challenged fundamental economic axioms and start with the notion that “units” of behavior operate with memory compressed through repetitions of structure that are not dependent on scale. The topics and examples presented form an overall outline about †structural  $k$ -cohesion and resistance as measurable social forces in human behavior; what enhances or limits scalability in cohesion; what produces and inhibits resistance; and the multiple ways that these two social forces, very different from physical forces, interact dynamically.

### III. NETWORKS AND COHESION IN HB DYNAMICS

The interconnected theme of these illustrative examples will vary from basic measurement to exploratory models to findings built on the mathematics of universality classes, focusing on two features of human ethology that make for unusual dynamics. One is an open-field bonding ability, like chimpanzees, gorillas, and orangutans, which involves recognition of community organized by weak rather than strong ties [47][132]. Humans are additionally equipped with a huge range of social and cultural abilities that derive from our use of symbols, which can widen community and cohesion and enable scalable networks of trust through strong ties as well [60]. These emergents can alter the scale and especially the dynamics of human social organizations. One foundational base for a theory of such emergents are the scalable cohesive groups whose boundaries are identified with the concept of *structural  $k$ -cohesion* in sociology [12][19], with new parallels recently discovered in the signaling properties of human and biological networks [13]. Another is the role of  *$k$ -cohesive resistance* in human ethology. Taken together, the scalability of cohesive human groups, which allow the scale-up of group sizes that contribute greatly to political expansion and warfare, and the role of decentralized cohesive resistance in pushing back political aggression, exhibit some of the properties of laws of momentum and of proportional reaction, not atypical of complex systems with complex interiors.

To understand the potential for such regularities in phenomena as irregular as human sociopolitical histories (ones that were not lost on the pre-Einsteinian Henry Adams [133]), the concepts underlying indefinite extensibility of scalably emergent cohesive human groups need to be carefully drawn. Rather than harking back to Ibn Khaldun, they draw on Menger’s 1927 theorem [14] for graphs or networks, which is now in use in sociology [12][17][19][134] and anthropology [92] even if rarely used in physics or chemistry, although applications are beginning in graph-theoretic formalization of biological signaling network models [13]. In a network of connected elements, a maximal group (one that cannot be expanded further without losing the property) with structural cohesion  $k$  is one that (a) cannot be disconnected without removal of at least  $k$  elements, and which, as proved by Menger [14], is equivalent to its having (b) at least  $k$  element-disjoint paths between every pair of elements. Property (a) provides *external resistance* to complete disruption (i.e., removing fewer than  $k$  elements leaves the structurally  $k$ -cohesive group connected), and property (b)

proves the existence of a measure  $k$  of *internal cohesive traversal* through concomitant existence of at least  $k$  redundant paths of transmission or potential communication between every pair of elements. Neither the internal nor the external *cohesive* properties can be surpassed by extending its boundary to include others, whereby each structural cohesion  $k$ -group has a unique social boundary. Perfect *scalability* occurs for the numeric size of the *intensive* variable  $k$  by any scale-up *extensive* multiplier  $m$  because while a structurally  $\dagger k$ -cohesive group of size  $n$  requires only  $k < n$  links per element, the same is true at size  $n \cdot m$ . Note that while dying or migrating might be due to external forces or attractions that remove people from groups, sometimes group members themselves decide to leave, or are expelled. This raises the point that cohesion models and measures are appropriate where the inter-element or interpersonal ties are positive, not antagonistic or negative, by restriction on what should be included in such a model.

Broad problem areas of HB dynamics can be understood from the pairing of (1) the indefinite extensibility of scalably emergent structurally cohesive groups (which have an indefinite supportive potential for scale-up in size of cooperative groups and community) with (2) the contending abilities to form both emergent centralized social structures and (3) cohesive resistance to invasion or centralized authority. HB dynamical processes that can be phrased in terms of symbolic and social interactions of types (1)-(3) are discussed in Section IV. Central to these issues, John Turner's (2002) *Face to Face: Toward a Sociological Theory of Interpersonal Behavior* [15] presents evidence for the deeply rooted ethological two-sidedness of humans as a species pitting *cohesion* against *resistance*. A reviewer's summary is worth quoting:

Turner forcefully argues that we are not the solidarity-seeking emotional animals that theorists like Durkheim, Goffman, and Mead would have us to be. Nor are we normally the tortured beings of the Freudian perspective. Reflecting our origins among the great apes, we are a deeply ambivalent species of two minds, craving strong emotional attachments and at the same time, bridling against the constraints in closed social circles of even strong interpersonal ties. Turner argues that this two-sidedness is rooted deeply in our biology, and is not simply the product of historically specific ideologies and social structures. Clearly this viewpoint has enormous implications for the study of face-to-face interactions, as well as many other aspects of sociology. However, in his deep respect for the traditional perspectives in this field, these implications are often obscured and hidden in Turner's exegesis of the general problems and principles in this area of study. None of the other theorists analyzed here have created a better model of ambivalence. Capturing the two-sided nature of social linkages was not a key part of theorists such as Mead, Goffman, and Schutz. Freud made ambivalence central to his model, but locked it into a narrow sexual model. As Neil Smelser has argued, the future of sociological theory will depend in large part on its ability to deal with ambivalence, and Turner's model goes a long way in this regard. [16]

Issues of two-sidedness, through a number of steps in logic and measurement, are not unrelated to those of scalability in structural cohesion. To clarify the first three steps in this logic, we can refer to the number of elements in a maximally-sized  $k$ -cohesive group as its  $k$ -cohsize (or *extension*) and so state, for clarity, that  $k$ -cohesion and  $k$ -cohsize= $n$  ( $>k$  by definition) of such a group can vary independently for a given level of  $k$ -cohesion that defines the boundaries of a particular subgroup in a network. The steps are:

Step 1. *Intensive versus extensive aspects of structural  $k$ -cohesion are independent.* Evidence of the causal effect of  $k$ -cohesion is found in empirical studies and is unrelated to  $k$ -cohsize. There are three major tests of this to date, one where the major variance in student attachment to high school [19] (as measured by a half-dozen validated questions) was consistently predicted, in multiple tests (ten American high schools randomly selected from the 100-school sample of U.S. Adolescent Health network surveys [126]), by level of  $k$ -cohesion in which each student was embedded in the school's network of friendships. With complete data on students and networks in each school, replication of this result was achieved in logistic regressions where all other attribute and pertinent network measures competed in accounting for variance. The influence coefficients for  $k$ -cohesion replicated in each of the 10 independent populations [19]; and the  $k$ -cohsize of the friendship groups for individual students did not account for school attachment. Since these groups varied in size for each level of  $k$ -cohesion, this is evidence that the causal effect of  $k$ -cohesion is not diluted by size, that is, it is an intensive predictive property independent of its scalability in size.

In a second major study (Powell et al. 2006 [17]), Attraction to  $k$ -cohesion along with recruitment of diversity were the major predictors in a 12-year time-series analysis of variables accounting for tie-formation probabilities proportional to  $k$  in the biotech industry. Because of the recruitment of new entrants, with fewer ties the overall industry levels of cohesion varied relatively little and oscillated in alternation with 3 to 4-year waves of variation in attracting new recruits. While

$k$  did vary slightly for the maximally cohesive core of the industry, it neither uniformly grew nor uniformly decreased over time. There is a consistency here with the finding that *greater cohesion* was the attractor in tie formation and not greater network centrality as hypothesized in the Barabási scale-free network model [18]. The tie-preference attractor was a *sufficient level of  $k$ -cohesion that is scalable by addition of members to the structurally cohesive group*, as is shown to occur over time in the biotech industry study.

A third study, of cohesive decay (White and Harary 2001 [12]), tested predictions of how a single 4-cohesive group disintegrated into two competing and eventually disconnected groups. With leaders in opposing groups, order of dissolution of ties followed the pattern predicted, as individuals dissolved their ties successively on the side of the leader with whom they had less  $k$ -cohesion, and if cohesion was equal, dissolved these ties to the opposing side that had the longer path lengths. While the larger 4-cohesive group dissolved, ties redistributed to the two smaller 4-cohesive groups that formed around the disputant leaders.

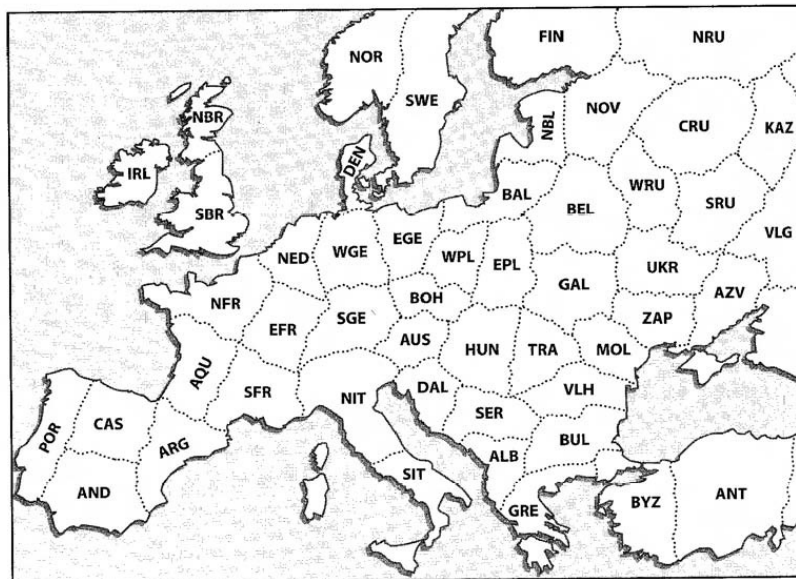
Step 2. *Further evidence for scalability* is that  $k$ -cohesion and  $k$ -cohesion measures find cohesive groups on much larger scales than do density-based measures called *community detection* [135][136] that split networks into mutually exclusive groups such that higher densities are within rather than between them. Calling these density groups “communities” ignores the fact that such groups overlap and form  $k$ -cohesive groups on much larger scales. Community detection lacks the scalability of structural  $k$ -cohesion. White et al. [134] demonstrate how the boundaries of these much larger cohesive groups can be approximated in extremely large networks, which is needed because  $k$ -cohesive blocks computation is  $\dagger$ NP-complete. Further, they show how density-based and row-column correlation-based algorithms fail in 30 out of 31 methodological studies in a meta-analysis of a classical small-network dataset. They also show analytically, as do Harary and White [12], how  $k$ -cohesive components of networks stack hierarchically for successive values of  $k$ , providing core-group centralization in addition to horizontal cohesion within a group. The analytical properties of multiconnectivity (*aka* structural cohesion and also  $k$ -cohesion) as a precisely measurable and scalable concept ( $\dagger$ connectivity- $k$  in graph theory) for hierarchical and overlapping group-cohesion boundaries make it an ideal construct for studying the relation between micro (small group and local network properties) and macro properties of social networks, those of political and other social units, and the social construction of roles and institutions [24]. A large number of studies show cohesive scalability in the ways in which symbols and attachments are deployed in human groups and networks, as will be discussed in Section IV, although some effects are preserved only up to certain scale-up thresholds in group size.

Step 3. *Evidence discussed in Section IV supports the hypothesis that  $k$ -cohesive components of human social networks amplify transmission quality and the utility of information that can be cross-checked from multiple independent paths*. For example, distinguishing carefully between dominance (force or force threat) and prestige (freely conferred deference), generalized prestige rankings are scalable along with the transmission quality of multiple channels in  $k$ -cohesive groups, while dyadic dominance hierarchies are not. Henrich and Gil-White [20] tested and found support from data across the social sciences for the predictions of a prestige model of social learning as opposed to dominance imprinting. This supports their argument that “natural selection favored social learners who could evaluate potential models and copy the most successful among them,” and that prestige rankings were an emergent product of psychological adaptations that evolved to improve the quality of information acquired via cultural transmission. Finally, studies of networks where utility is gained from long-range interactions [21][22][23] show a variety of network topologies that may combine the benefits of centralized hubs (which are often thought in network economics to maximize efficiency by minimizing redundancy) with those of redundancy in  $k$ -cohesive components.

The approach to cohesion taken here—also contrasting to methods for the partitioning of roles [25]—is not that of trying to specify analytical boundaries using matrix-based methods (Newman [26][27]), which are insufficient as tools to capture the precise boundaries and overlaps in the concept of  $k$ -connectivity. The analogy between physical forces and social cohesion or repulsion breaks down because the latter do not involve the kinds of hard-body (Hamiltonian) equations used to describe simple systems such as a bouncing ball, billiard balls, a pendulum, or an oscillating spring. The algorithmic complexity identifying  $k$ -cohesive units given an arbitrary graph is  $\dagger$ NP-complete and not

susceptible to matrix-analytic detection, although humans are often better at perceiving accessible and simple but algorithmically complex patterns than are computers.

Because a great many fundamental issues in HB dynamics can be framed in the context of the pairing of cohesion and resistance—similar to but much more complex than the concept of attractive and repulsive force—this pairing is used to organize many of the research questions and findings presented here, not the least of which is related to the problem of the units of analysis needed to for tests of HB dynamics, and how these units interact or embed in one another.



**Figure 1:** Turchin’s [29] 50 cultural regions used as geographical units in the statistical analysis of the relationship between metaethnic frontiers and polity size (courtesy of the author)

How, for example, do scalability and resistance play out in terms of HB dynamics on the larger historical scale? Peter Turchin’s [29] examination of 50 cases (Figure 1) in the historical military expansion of agrarian states in European history over the last two millennia is illustrative as a test of historical DHB theories that engage concepts of social cohesion. What happens when agrarian states or empires invade a sizeable group that differs in major metaethnic markers (multiple cultural differentiations in *religion, language, and ethnicity*) that are internally cohesive for the group invaded? The framing of this problem is given initially by comparison of dynamical equation-based models for ordinary differential equations of zero-order (unbounded growth or decline), first-order (bounded growth or decline), and second-order (oscillatory growth and decline) [28][29]. Empires show growth and collapse that fail to conform to the first two types of dynamical equations, but could be governed by a second-order dynamics in which there are time lags and negative feedback. The next steps in this study engage the ethological issues that will also be examined here. For example: *What accounts for the resistive capabilities of human social groups, e.g., against outside invasion?* (It is useful to recall that this research was finished before 9/11, 2001). The study draws parallels with the dynamical theory of Ibn Khaldun, who used the term *asabiya* for collective solidarity:

Ibn Khaldun was clearly aware of the nested nature of ethnic groups, and that each level has its own *asabiya* associated with it.... [T]he leading or ruling element within a group must be vested in a family or lineage that has the strongest and most natural claim to the control of the available *asabiya* (Ibn Khaldun 1958:I:lxix). Only the leader who controls an *asabiya* of sufficient strength may succeed in founding a dynasty. [20:38-39]

Ibn Khaldun is widely credited as a thoroughly modern sociological scientist of culture, knowledge, politics, and urban life and in his theory of oscillations of Arab and Berber polities. His theory and historical analysis is framed in terms of second-order dynamics:

It [the theory] is held together by his central concept of “*asabiyyah*,” or “social cohesion.” It is this cohesion, which arises spontaneously in tribes and other small kinship groups, but which can be intensified and enlarged by a religious ideology, that provides the motive force that carries ruling groups to power. Its inevitable weakening, due to a complex combination of psychological, sociological, economic, and political factors, which Ibn Khaldun analyzes with consummate skill, heralds the decline of a dynasty or empire and prepares the way for a new one, based on a group bound by a stronger cohesive force. [30]

The thesis of the 50-case study of European military expansion is that “areas where imperial and metaethnic frontiers coincide act as *asabiya* incubators” ([29]:56), areas where new ethnies (i.e., ethnicities, nationalities) are born in the growth of collective resistance. These solidary groups with high *asabiya* have the attributes of *k*-connectivity: “An important element of the theory is the ability of ethnic groups to scale up without splintering into subgroups” ([29]:57). Examples of integrative mechanisms in this particular context of differing ethnies are *religion, society-wide mechanisms of male socialization, and rulership with primogeniture*.

External conflict has long been seen to stimulate cohesion on both sides of the conflict boundaries [31][32], as exemplified in the faultline frontiers in history [33][34] and in the marcher state [35] conflicts along these frontiers. A remarkable display of the dynamics of history for the 50-case study is provided by the maps constructed to show, for the regions included in Figure 1 and for each of the last twenty centuries, the invasions by European empires across metaethnic frontiers and the resultant appearance of new nationalities as resistive movements and states [36].

This mathematical model for empire expansion lacks “a well-developed theory that would connect micro-level individual actions”—like those deriving from structural cohesion—“to macro-level dynamics of *asabiya*” [29] although the altruism of *asabiya* is seen to follow a conditional altruist model (like that of kin-selection [56]) that depends on cohesion with other altruists—discussed in Section IV below. A provisional model (later improved) is given, in its simplest form ([29]:64-66), for a polity with a spatial scale  $h$  of power projection (imperial “reach”) over an area  $A > 0$  and the resistant cohesion  $0 < S < 1$  of *asabiya* with an everpresent/constant minimum geopolitical pressure  $a$  from the hinterland across a metaethnic frontier of size  $b$ . This is given as two dynamical equations with negative feedback that give an unstable equilibrium with a single boom/bust cycle ( $c_0$  and  $r_0$  in these equations are constants):

$$\begin{aligned}\dot{A} &= c_0 A S \left(1 - \frac{A}{h}\right) - a \\ \dot{S} &= r_0 \left(1 - \frac{A}{2b}\right) S(1 - S)\end{aligned}$$

Here, change in area is a function of the polity area and of cohesion, limited by overextension, while the function for change in cohesion has an inbuilt oscillatory dynamic affected by the size of the metaethnic frontier. These dynamics, although intended only to characterize the problem, are informative as to how its parameters play into dynamical complexity. If “reach”  $h$  is not much greater than frontier width  $b$  the empire can reach a stable equilibrium, while if  $b < h/4$  the boom/bust cycle, but only one can occur, not more. Only when the model incorporates the discounting of expansionary power with distance, in a spatial simulation, is a second-order type of effect is obtained, that of oscillatory growth and decline.

Rather than having this model serve to study attacks and resistance, and the influence of *relative* cohesiveness in outcomes of politicomilitary contests (which is difficult to measure), Turchin’s frontier theory is tested instead with the time-lagged prediction for each of two millennia that *when the metaethnic frontier is intense in the first half of the millennium, for one of the cultural areas in Figure 1, then large territorial polities (empires) will originate in the second* [29]. The evaluation is whether the expansive tendency *originated* in a contest of respectively cohesive entities rather than trying to predict the outcome of the battle, the more relevant outcome being that—having developed its cohesion through external conflict—the unit that is initially attacked may eventually enlarge to become an empire. This holds for 11 out of 15 cultural regions that were on the metaethnic frontier, while out of 34 regions that were not on the frontier, only 1 developed an empire in the first millennium AD; and it holds for 22 of 28 Figure 1 frontiers in 500AD-1500 and empires in 1000AD-2000. The 4 exceptions in the first case and the 6 in the second were regions incorporated into an empire centered in a neighboring region.

Table 1: Cross tabulations for polities that start on frontiers and end as empires a millennium later

| 0-1000CE       | Starts as Frontier | No Frontier | 50 regions | 1000-1900CE    | Starts as Frontier | No Frontier |            |
|----------------|--------------------|-------------|------------|----------------|--------------------|-------------|------------|
| Becomes Empire | 11                 | 1           | p<.0000004 | Becomes Empire | 22                 | 3           | p<.0000004 |
| No Empire      | 4                  | 34          |            | No Empire      | 6                  | 19          |            |

How many empires *were observed that lacked the temporal precondition of a metaethnic frontier* (with subsequent growth of resistant cohesion)? The exceptions are 1 and 3 for the two periods, respectively. The first, and two of the latter cases, occur where the existence of the frontier was of short duration. One polity (Savoy-Sardinia, founding Italy) remained as a true exception, in a population formed by Celts and Romans, but with no clear causal path from metaethnic frontiers to polity expansion. But the major result is that the empires of the later periods *did* (and not just may) result in almost all cases from cohesive resistance to the attacks of the previous period, at long time scales.

Thus, these results are fully consistent with the scalability of structural cohesion as a basis of sociopolitical support for military expansion of polities (potentially into empires) but more importantly are supportive of the theory that *k*-cohesive structural resistance, which grows slowly on the metaethnic frontiers of expanding empires in such a way as to facilitate the growth of resistive “nationalistic” ethnic solidarity and eventually of consolidation of resistive metaethnic frontier groups themselves into expanding polities and empires, with long time-lags in their development.

Lim, Metzler and Bar-Yam [127] analyze local conflicts between distinct ethnic or cultural groups within multiethnic states (India and former Yugoslavia), matching actual conflicts to spatial population structure in a simulation model of type separation, where cohesion emerges through movement to more homogeneous regions and through avoidance of conflict. Conflicts are predicted due to the structure of boundaries rather than between the groups themselves, consistent with Turchin’s [29] findings. The local ethnic patch serves as an “order parameter” to which aspects of behavior are coupled in the dynamics of a universality class of collective behavior. Similarly, the multilevel evolutionary model of Garcia and van den Bergh [123] shows how parochialism, as altruistic behavior specifically targeted towards in-group members, can result from group selection operating on direct conflict between groups.

#### IV. COOPERATION, CONNECTIVITY-K AND “CRITICAL MASS” IN COLLECTIVE ACTION

If structural cohesion is scalable, what are the factors, aside from external conflict, that would prevent or facilitate the scale-up of cohesion? Or of group size generally, assuming some modicum of cooperation? [37] The major problem in explaining why cooperation should occur at all in human groups, in the absence of external conflict, is that of the benefits of selfishness to free-riders when others bear the cost of altruism. One component of “The Tragedy of the Commons” [38] is that collective goods [39] are nonexcludable: Once achieved (like peace, clean water or air, public transport, or wage contracts) they are available to everyone. Many if not most such goods have jointness of supply (available to all), i.e., their cost does not increase proportionally to group size. The initial problem is that if it takes only some initial investment and costs by those who bring such goods into existence, why should anyone else bear these costs when they can have them for free? This creates “the dilemma of cooperation” [39] and of collective action. And the larger the group the easier it is to ride free. Evolutionary game theory [40], with a replicator dynamic that favors those with lower cost for the same benefits, predicts that without some compensation for altruism, even starting from a small number of free-riders in a population, selfishness becomes the norm. The secondary problem of collective goods is who will bear the costs to maintain them?

Reputation may attach positively to altruism and negatively to free-riding. In this respect, two recent experimental papers are strongly supportive (although unaware) of connectivity-*k* in helping to explain cooperation in human groups [41]. In one study the judged veracity of gossip is shown to increase considerably if it came from more different sources [42], not if one source kept repeating the same gossip, while another relates gossip to reputation and cooperation in general [43]. James West in *Plainsville* (1945) [44] was the first to connect gossip and the maintenance of the unity of groups.

According to Gluckman ([45]:308), however, West misinterpreted the extent to which “gossip does not have isolated roles in community life, but is part of the very blood tissue of that life.” Gluckman refers to Colson’s *Makah Indians* [46] ethnography to illustrate the importance of gossip to the unity of groups.

While diffusion of reputation along the node-disjoint paths of  $k$ -components can provide benefits to altruism, its influence diminishes as groups grow larger and average network distance grows large, reducing the scalability of  $k$ -components with high levels of cooperation. Further, if a group has *too much*  $k$ -connectivity (as in completely connected cliques), the benefits of reputation diminish because of the “echoing” effects of conformity and diminution of independent sources of information [48][49][50]. In cliques or overly-connected groups, single dominant individuals have the potential to influence everyone and thus to distort the robust veracity of information. Further, studies of human friendships and other long-term relationships show that the success of reciprocal strategies (such as tit-for-tat) relies on a combination of medium-term accounting, forgiveness, and propensity to defect with strangers if they already have an adequate number of partners [50].

Benefits of punishment within a group have a similar profile of optimality to reputation. Like gossip and reputation, punishment can be effectively delivered through  $k$ -cohesive independent paths and thus diffuse coherently respecting the boundaries of  $k$ -cohesion [41] (although for a given  $k$ -cohesive group, the paths used to diffuse reputation need not be the same as those used to deliver punishments). This works best for groups with moderate average distances in the network and with (or defined by) moderate connectivity- $k$ . Similar to reputation, “cohesion extends punishment even beyond the community network and protects insiders against trouble-making outsiders, [especially] when community members come to defend fellow community members against norm-violating outsiders” [41], while incidental defectors at the margins of the cohesive group may have little impact on behavior within the group. Henrich et al’s [51][52] ethnographic-psychological study of 15 societies from five continents, representing the breadth of human production systems, found that willingness to use punishment in the †dictator game covaries with altruism across populations in a manner consistent with coevolutionary theories. But while appropriate punishments diminish the relative rewards of free-riding, they also incur costs to the enforcers.

Punishment as third-party intervention tends to rely more on dominance and perceptions of the use of force, entailing higher risks in some cases, than on reputation in the modeling of advantages of cooperative behavior. After observing a group of macaques in captivity in which a small number of individuals fulfilled policing tasks, making interventions into dyadic conflicts, temporary experimental “network knockout” removals of the policing monkeys showed it was their presence that prevented the group from falling apart into small clusters [53][54]. Here, Jessica Flack and coworkers note [55], “the degree to which one individual perceives another as capable of using force is communicated using a special dominance signal. Group consensus about an individual’s capacity to use force arises from the network of signaling interactions.” Consistent with studies of  $k$ -cohesion, this research found that “coarse-grained information stored at the group level—behavioral macrostates—“was more useful than detailed information at the individual level”. Because “successful intervention relies on consensus among combatants about the intervener’s capacity to use force,” use of “a formalism to quantify consensus in the network,” and with consensus as a measure of power, showed that “the power distribution is fat tailed and power [here: consensus] is a strong predictor of social variables including request for support, intervention cost, and intensity.” This modeling of power distributions shows how dominance *signaling* strategies “promote robust power distributions despite individual signaling errors” [54].

Third-party interventions in conflicts resemble recognition of community membership in that such interventions rarely occur with respect to outsiders. Recognized community boundaries (as distinct from  $k$ -cohesion, which may extend beyond these boundaries) provide the most probable context for the dyadic construction of cooperativity through reciprocity [56], dominance in third-party intervention, and dyadic game theoretic strategies that achieve cooperativity (such as tit-for-tat or lose-shift in Prisoner’s dilemma) [57]. These, together with generalized reciprocity [58], i.e., altruism in the expectation of indirect return, are also among the most potent constructors of community-building strong ties in social networks [60], especially if they are navigable [137], as in many elite groups and nonwestern [59] societies. Bowles and Gintis [91] summarize the game-theoretic work on

cooperation showing that the critical condition for cooperative outcomes, which otherwise deteriorate with increases in group size, is the presence of *strong reciprocators*, who cooperate with one another and punish defectors, even if they sustain net costs, provided that they are more likely to interact with one another than at random. Thus, network structure and preferences (positive assortment) prove to be central to an evolutionary path to large-scale cooperativity. Pepper and Smuts [125] show how positive assortment through environmental feedback can play the same role. There are, then, evolutionary paths to the scale-up of  $k$ -cohesion for indefinitely large groups.

Putting together these principles of primate (reciprocity, policing) and human social networking, we can also see compatibility of  $k$ -connectivity with the theory of “critical mass” in collective action [61][62][63]. Group size *does* increase the probability of a critical mass of people who develop common goods through collective or cooperative action. This relates directly to the scalability of  $k$ -connectivity, wherein as the size  $n$  of such a structurally cohesive group expands, it is still only  $k$  links per person that are needed for  $k$ -connectivity. But there is always an expected excess of ties, upwards of  $k$ , for some members of such a group, and an increase in  $n$  increases the probability of formation for a group with a critical mass of connectivity  $k+1$  or higher ( $k+l>1$ ). This relates to the “paradox of group size” for collective action groups: “When groups are heterogeneous and a good has high jointness of supply [i.e., with cost that does not increase proportionally to size], a larger interest group [size  $n$ ] can have a [relatively] smaller critical mass,” which could also be a critical mass with connectivity  $k+l$ . The problem of mobilizing collective action is whether there is a mechanism that connects enough people with appropriate interests and resources so that they can act to construct a collective good [62]. Structural cohesion provides just such a mechanism [17][19][92].

An extended feature of this model of critical mass, which has been investigated through simulation [62], is an *accelerative function* for what has been called network externality [64], where every new participant in creating a collective good makes it more attractive for the next participant to join. Different forms of collective action have some mix of this source of nonindependent decisions and/or a *decelerative function* wherein free-riding is more likely on the belief that others will do the job. Since success in collective action is partly a problem of coordination, there is some advantage to members of a critical mass in collective action having greater centrality. But again, if the collective action group at large has connectivity  $k$  and the leadership critical mass has connectivity  $k+l$ , the latter is achieved by the hierarchical embedding of higher orders of connectivity and not necessarily by greater centrality of a single leader.

So there are two aspects to consider for the dynamics of growth and decline in size of cooperative human groups: (1) reinforcement mechanisms of community, which tend to be self-limiting with respect of structural cohesion, and (2) critical mass in collective action or positive assortative reciprocators [91], which both tend to be self-enabling. While collective action to produce a collective good also requires a model of group process that cannot be deduced from simple models of individual behavior [62], the problem for understanding how large-scale societies and polities can achieve sustainability may be solved by assortative strong reciprocators [91]. The former problem—of sustainability of cooperation in a community—is different. All of the mechanisms there—reciprocity, third-party intervention, reputation, and punishment—depend on relative stability of community membership. Prior to the electronic age (which poses somewhat different problems of stability in virtual communities but makes for more independence of  $k$ -connectivity from local density and geographic distance), stable communities carried designations such as “settlement” or “nomadic group” as nominal indicators of relative stability in proximal spatial interactions among their members.

There is a herd cohesion solution to the stability problem (follow the *surest* neighbor!) [93][94], but also an advanced social cohesion solution (the formation of  $k$ -connectivity groups with a stable core) [41]. Empirical studies of neighborhoods [65][66] show that a certain threshold of residential stability is a crucial factor for the efficacy of mechanisms for community-level enforcement of the cooperative norms of third-party intervention, reputation, and punishment (all but strictly dyadic reciprocity unless it is strong and assortative). “For cooperation to be maintained at the community level, the network as a whole must be relatively more stable than patterns of individual actions” [41]. Combining community mechanisms and critical mass in collective action, we have the foundations for an evolutionary theory of cooperativity and cohesion in human groups. Many of these features

(but not structural cohesion) have been brought under the Darwinian umbrella in a way that shows how the co-evolution of culture and genes jointly influence cultural transmission (dual inheritance theory) through the vehicles of human behavior and psychology [67][68]. This framework allows the integration of work on kinship, friendship, reciprocity, reputation, social norms, and ethnicity into a generally applicable mathematical characterization that may contribute to solving the problem of cooperation and extending on to the evolution of evolution and of economic systems [51][69].

Beyond adding  $k$ -connectivity into dual inheritance theory, there are also newer models of achieving minimum punishment and maximum crime reduction through policing concentration on an arbitrary push-down set of offenders [70]. The theory here, validated in simulation and case study, is that a fair and effective law enforcement strategy can only succeed if it approximates one with a stable target set of offenders at whom punishment is directed until recidivism ceases, individual by individual, replacing each nonrecidivist on the pushdown list by another known offender chosen with a probability *proportional* to rate of current offenses but otherwise *arbitrarily*, i.e., *fairly*. Policing an arbitrarily stabilized set of offenders mirrors the requirement for stability in cooperative neighborhoods. *Stability seems to be a key ingredient for cooperativity.*

What are the implications of these findings for considerations of the scalability of human communities and of human polities? Although  $k$ -connected groups are scalable, the properties of third-party intervention, reputation, and punishment to maintain cooperativity are not scalable, nor is dyadic reciprocity except under very special conditions [59][60]. Scalability through conflict—resistance to threat—and through collective action to produce collective goods, organized by a “critical mass” or through assortative strong reciprocity [91] is, however, scalable.

So why do human groups not simply grow larger at all scales [37], as challenged by competing groups, or by possibilities such as establishing collective goods capable of sustaining growth? Pre-state societies only rarely sustain continued growth in size, but rather split, and then remix through intermarriage and mating (fission and remixing), with transition to a higher-order political form occurring extremely sporadically. It might be thought that if politically independent groups fission but still remain linked, through intermarriage or  $k$ -connectivity, then fusion into larger political groups would be easy. Many anthropological theories assume a stage-wise progression such as band to tribe or tribe to chiefdom [95]. Comparative ethnographic, historical, and archaeological studies, such as those of Wright [71][72], however, make it clear that passages from band to tribe (concepts with serious conceptual problems) to chiefdom to state are extremely difficult and unlikely transitions. And as we have seen [29], growth in state societies and empires is followed by collapse and the rise of other polities instead. Models of political fission might provide necessary conditions for transitions in successions of forms of leadership as polities develop with different sets of roles. New role set configurations might also create founder effects in the emergence of economic or political forms.

## V. TRANSITION MODELS WITH THRESHOLDS

Transitions such as chiefdom to state can be modeled in an evolutionary dynamics of human behavior framework that includes the interaction of ethological characteristics—general human behavioral tendencies—and forms of sociopolitical organization. Social anthropologist Christopher Boehm [73][74], whose field studies range from Montenegro [75] to wild chimpanzees at Gombe, called attention to the *human tendency to resist domination* (consistent with Turchin’s findings [29] in section III), which is not shared with other great apes (consistent with Henrich and Gil-White [20]). In a substantial cross-cultural survey of societies in a wide variety of social and ecological settings, Boehm selected those with egalitarian behavior, and found that their behavior was not shaped by these settings but rather was deliberately shaped by their members, guided by a nearly universal ethos in these societies “that disapproves of hierarchical behavior in general and of bossiness in leaders in particular.” His survey reveals the wide variety of means by which “the political rank and file” evict leaders who evince excessive authoritarian tendencies. This “creates a reverse dominance hierarchy, a social arrangement that has important implications for cross-phylogenetic comparisons and for the theory of state formation” [73] that might be called a “law of human ethological resistance”, consistent with [29]. One of these mechanisms of resistance is fission, the break-away from a group that is growing large or with too many settlements under a single leader.

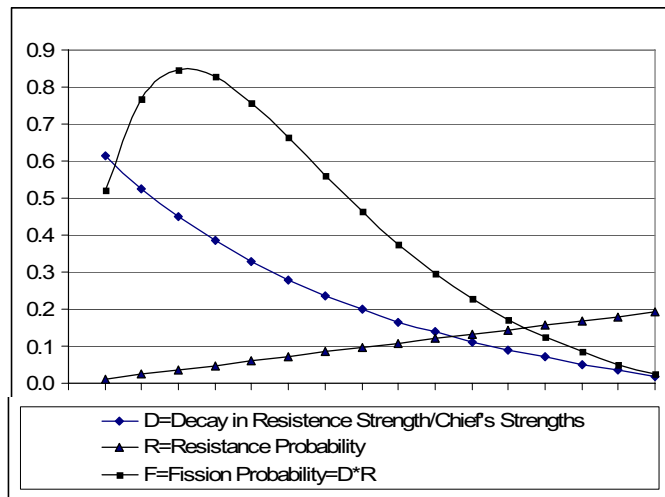
Surveys of archaeological, historical, and ethnographic cases not only show transitions from chiefdoms to states to be very rare but also show that states are based on a radically different principle of a hierarchy of roles to which decision-makers are recruited. Henry Wright [71][72] shows that primary and secondary states have three or more levels of mobilization of resources upwards and passing of information both upwards and downwards through a hierarchy of divided offices and a division of political labor (Spencer 2004 [130]). Chiefdoms, unlike states, are characterized by paramount leaders who delegate as little authority as possible, in contrast to states with their delegated division of labor for authority [72]. Paramount chiefs may govern subdivided territories with village chiefs and ritual specialists but there are nearly always no more than *two* levels of chiefly resource mobilization conducting directly to the chief and *all political decisions are integrated into the chiefly persona*.

To assume a simple quantitative increase in network size and complexity as chiefdoms develop into states is therefore inappropriate. Chiefdoms are also characterized by a reverse ranking hierarchy [73][74], not an actual reversal of dominance but one of prestige ranking [20] in which leaders are expected to exhibit altruism to followers through redistribution of goods or forms of reciprocity and bestowal of favors or gifts to counterbalance the processes whereby resources were concentrated through interpersonal network ties, although the reciprocity is rarely balanced in any material sense [128][129][138]. In their dynamics of growth, chiefdoms—with their structural cohesion and cohesive hierarchies based on intermarriage, exchange, and cross-cutting ties—tend to increase in size through internal growth or annexation of settlements, then to give way to fission at times of crisis following growth, especially if these crises coincide with issues of political succession. There is no tendency in these dynamics for gradual cumulative evolution in complexity toward state organization. The mosaic of sub-chief territories mapped into the chiefly ranking are segments that recurrently separate and then re-form in successive periods of political change

Griffin and Stanish (2007) ([76]:2,24) provide evidence and a model for a tipping-point synchronicity threshold in the transition from chiefdoms to emergent pristine states in the Lake Titicaca case of Tiwanaku, c. 500 AD (outgrowing the territorially larger political formation at Pucara). The transition occurred archeologically and in a detailed simulation model after a long period of cycling in which multiple chiefdoms climb the population size gradient only to be fragmented by fission. There is strong empirical evidence for cycling in growth and fission. During the period of cycling, primary centers, population concentrations, and increase in both the overall productivity and population of the region occur sporadically without synchronization. Then, in one rapid burst, archeologically and in repeated probabilistic simulations, these previously unsynchronized features emerge synchronously, pushing past a probability threshold for fission. Figure 2, reflecting results from the simulation model, shows the variables affecting the fission of chiefdoms plotted against time for growth; then, as cycling occurs, setting the cycling time back to that of an earlier equal scale in size. The simulation data were also reaggregated for  $X$  as number of settlements under the chief, and could be estimated analytically for other variables such as  $X$  for communication time from center to furthest outlier. The first variable,  $Y_1$ , is one of exponential decay in the ratio of resistance strength to the leader's strength, which can be expressed as an exponential probability density function supported on the interval  $[0, \infty)$ , where  $\lambda > 0$  is the rate parameter of the distribution. Since this is a discrete exponential distribution with  $X \geq 1$ :

$$p(X; \lambda) \sim \lambda e^{-\lambda X}$$

The second variable,  $Y_2$ , increases with  $X$  on the assumption that the likelihood of resistance to the chief increases with time, or variables that cycle with time, such as number of settlements. The third variable,  $Y_3 = Y_1 \cdot Y_2$ , is the probability of fission due to resistance, which is humped because it is a product of distributions that have higher resistance probabilities at opposite magnitudes of  $X$ . The shape of the  $Y_3$  distribution emerges as an average over many simulation runs, and opens further questions for investigation. What emerged in the Lake Titicaca region, consistent with the simulation, were two dominant polities, separated in time, one an incipient state, the other smaller in population but larger in territory, along with extensive trade networks including the smaller centers. This is a typical multisite trading configuration of early states [77].



**Figure 2: The transition threshold from Chiefdom to State**

$Y_1 \sim$  Exponential Decay in the ratio of Resistance strength/Leader's strength

$Y_2 \sim$  Increasing probability of Resistance as number of settlements increases

$$Y_3 = Y_1 \cdot Y_2 = \text{the probability of fission}$$

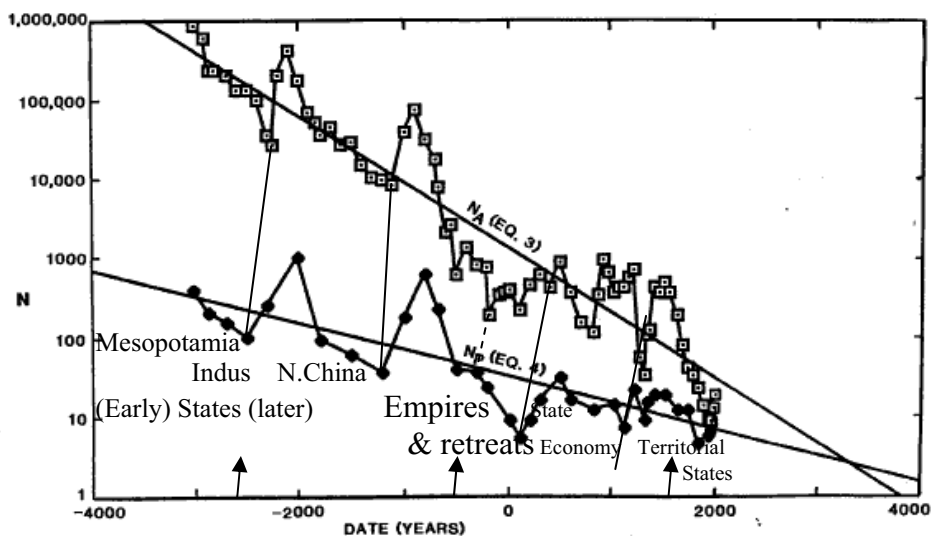
One can speculate as to whether resistive transition thresholds might occur also from band to tribe or big man (having occasional fission), to chiefdom and from chiefdoms (having occasional fission) to minimal states. Fissioning is by no means universal, and one study of the Titicaca region itself shows that village but not chiefly fissioning had ceased long before state formation with emergence of a regional religious tradition [79]. Are there resistive transition thresholds from minimal states (having occasional fission) to urbanized states, from urbanized states (having occasional fission through colonization) to dynastic states, or from dynastic states (with occasional fission with the death of a ruler and partition of domains under obligatory personal inheritance) to territorial agrarian states? Or do nonterritorial state expansions collapse, replaced by others? The territorial state, given institutional sovereignty over territory, is less likely to fission at a size threshold and its growth dynamics are shown in section VI to involve shrinkage following times of scarcity in population/resource ratios. This creates amplifications of inequality and internal conflict [78]. Modern mega-corporation growth is often arrested by national and international legal regulations mandating breakup of monopolies but there are no early barriers against corporate growth in size, although some corporations do fission for reasons other than size constraints.

The temporal scaling of long-term transitions in populations and sizes of the largest polities does show clear transitions, over 5,000 years of world history, as shown on Figure 3 [80]. The lower line in the figure is an exponential fitting of the effective number of polities (Laakso-Taagepera †concentration index  $1/\sum_i p_i$ , where  $p_i$  is the effective proportion-weighting for each unit) weighted by their populations, and the upper line by their geographical areas. More even proportions for  $p_i$ , such as  $\{.4 .3 .2 .1\}$ , compared to higher concentrations like  $\{.7 .1 .1 .1\}$ , will have a higher effective numbers, 3.33 versus 1.92, while extremely concentrated proportions, e.g.,  $\{.97 .01 .01 .01\}$  with effective number 1.06, approach unity. The declining slopes in Figure 3 show a *decrease* in effective polity numbers  $1/\sum_i p_i$  with greater concentration of population than of area (slopes differ by 2, the fitted exponential population roughly the square root of area). Over these five millennia the fitted effective number of political entities weighted by area decreased from circa one million to circa 64, and from circa one thousand to circa 8 weighted by population. For Figure 3:

Three sudden increases in polity sizes occur: [fewer large polity concentrations] around 3000 BC [urban revolution in Mesopotamia], 600 BC, and AD 1600 [the seafaring trade revolution]. This study tests the exponential model against area and population data [for polities] over five millennia. It also gives tables and graphs of area versus time for all major polities since AD 600. The median duration of large polities at more than half the peak size has been 130 years, and it has not changed over 5,000 years (Taagepera 1997:475) [80].

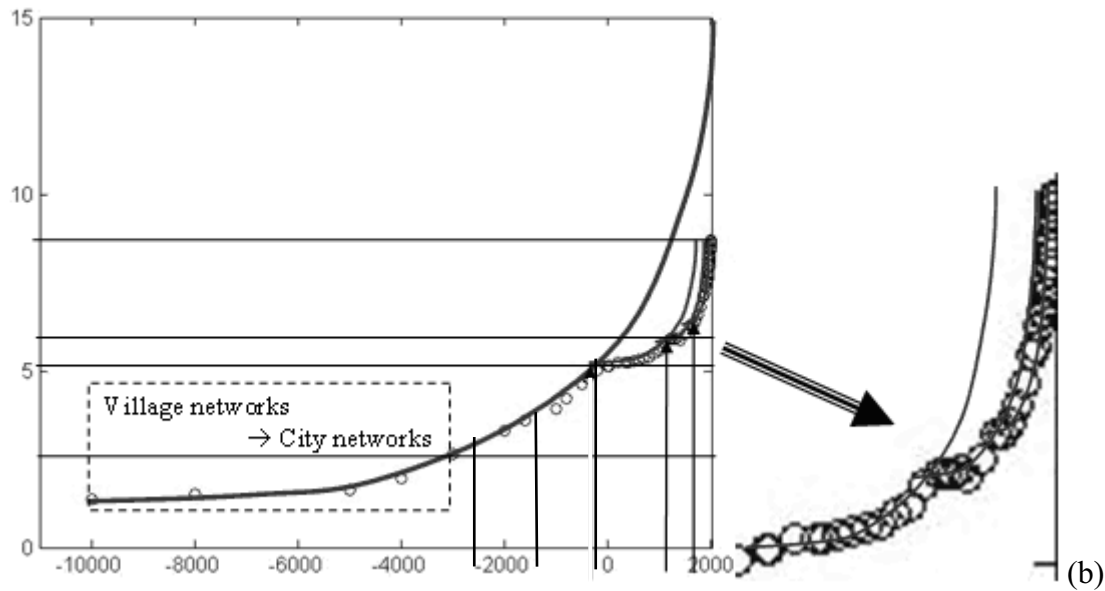
Two of the three solid lines superimposed on the original figure show how two of the three elbows of change in the lower of the empirical data lines (circa 2600 BC, 1200 BC, and 200 AD), from polity population concentrations to dispersals, are followed with short time-lags by two similar elbows of

change in the upper line from polity area concentrations to dispersals over the next hundred years. Whether these transitions represent eras of crises in urban empires is unclear, as are most extrapolations from so few data points. The first case might reflect the short-lived breakup of early Bronze-age polities (Mesopotamian and Indus) and the second the breakup of North China states. At 50 AD the third elbow of transition from population concentration to dispersion (e.g., for classical empires such as those of the Romans, Svataphana and Han, which actively discouraged market developments) occurs in the dispersion phase commensurate with that of polity area. Population and area reconcentration in the next phase (ca. 500-850 AD) are also roughly commensurate. A downward spike of population concentration recurs circa 1050 AD when again it has a lagged effect on polity area concentration. It might be surmised that changes in population-area interactions in the era of power-law city growth are increasingly subject to market-driven trade routes (e.g., Silk Roads in Eurasia from 100 BC-1300 AD). This is a context, from 900 AD forward, of new national market economies that diffuse from Sung China to the west, where the Abbasid, Carolingian and related polities encouraged widely articulated market systems. Such changes are studied and modeled by Modelski and Thompson (1996) [81]. Variability in the ways that markets change these political oscillations is particularly evident from 1800 and the industrial revolution, as nationalism and markets consolidate the effective number of polities geometrically weighted by size, up until 1990 with the breakups of the Soviet Union. There has been little dynamical modeling of the multiple causality in these coupled/decoupled oscillations. Taagepera [80:488] notes that while population concentration can continue to increase in the present era area concentration must stabilize because jumps to higher concentrations in earlier eras occurred with acquisition of control over large and sparsely populated areas (desert, steppe, deserts, tundra, respectively for Sargon, Mongol, British, Russian Empires), and such areas are much less available now.



**Figure 3:** Transition thresholds for States and Empires (effective number of polities, based on area and on population (Taagepera 1997 Fig. 5, courtesy of the author, who notes that individual polities that expand slower tend to last slightly longer [80:475]; arrows mark his dates for large polity concentrations; others are marked by lighter lines)

To express some of the consequences of the transition to networks of cities connected by trade routes, and eventually to market-driven trade, Figure 4 sketches the suggestion that world population begins to grow not exponentially but in power-law growth spurts, correlated initially with the transitions noted in Figure 3 [84].



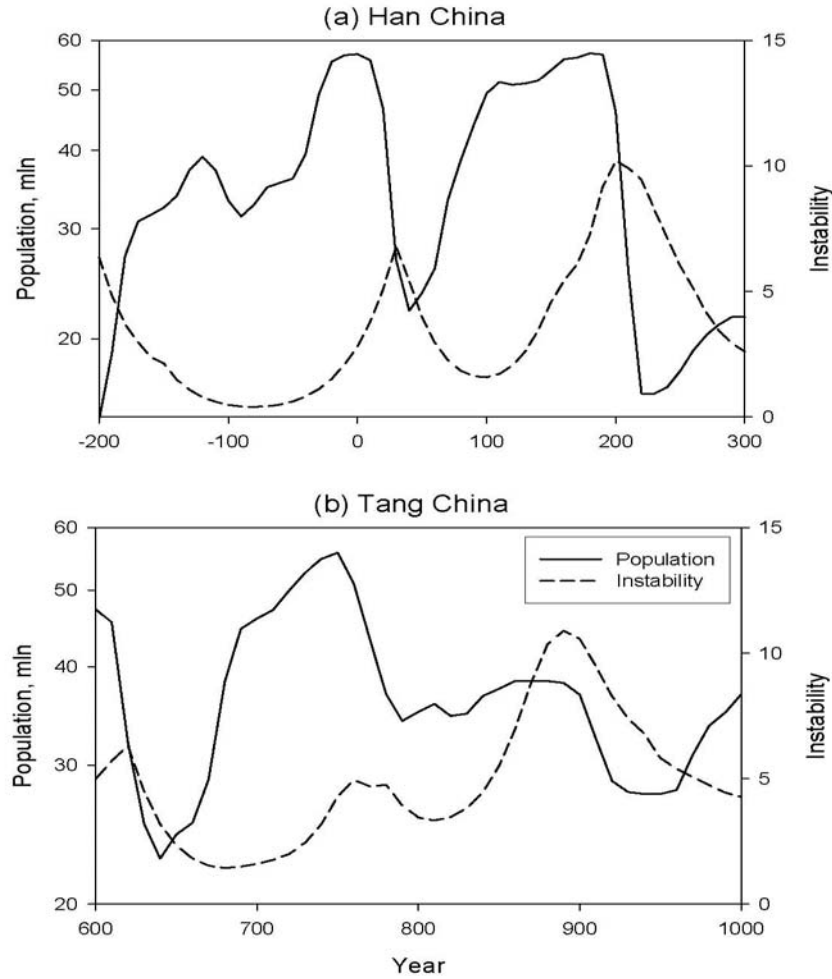
**Figure 4:** World Population Power-Law Growth Spurts and Flattening as shown in a semilog plot of Kremer's (1993) [83] data with successive power-law fits

Cities act as attractors for skilled, unskilled and intellectual labor as well as entrepreneurs and merchants, with a concomitant drain on settlements of smaller size. This enables power-law growth, at growth rates proportional to city size, i.e., cities as “attractors” as in the †scale-free network model of Barabási 2000 [18] (but see [139]), while rural areas and smaller settlements do not diminish their population but with elevated birth rates can replenish their losses from outmigrants. This pattern allows world population to grow in power-law spurts, but power-law growth is self-limited by population crashes as it would otherwise grow to infinity in a finite time [82]. The places where the polity transition crises occur in Figure 3, e.g., 2600-2400 BC, 1200-1100 BC, and 200-100 BC, and 1300 AD correspond to those crises in the larger states and cities where power-law growth in their (and world) population hits some sort of limit, growth flattens, and resets the starting parameters for a new upswing of †power-law growth (a pattern first noticed but not explained in Korotayev et al. 2006 [96]). The largest world empires, of the Golden Horde Mongols and the British, appear as the result of two of these more recent upswings (a topic currently under investigation by Christopher Chase-Dunn in one of the National Science Foundation's Human Social Dynamics research awards).

## VI. AGGREGATE (“SUFFICIENT UNIT”) EQUATION-BASED MODELING

This approach aggregates to the unit size and boundaries at which to define causal variables and interactions and to attempt to explain behavioral dynamics of these units by appropriate equations. This requires the “sufficient unit” condition that the aggregate units of study have the kinds of cohesive mass or entitivity for causal interactions to act on their aggregate characteristics. Time-series will have periods in which this condition is satisfied because of relative “endogeneity” of interactions where there are few external disturbances or exogenous shocks to the unit.

Using this approach, Peter Turchin [85][86] extended a realistic and empirical approach to historical processes—not caricatures of imperial collapse—for basic Malthusian models of population pressure on resources and time-lagged negative feedback effects with internal conflict (see also Turchin and Korotayev [88]). He simply uses “standard quantitative methods of natural sciences, such as time-series analysis, regression, and cross-validation. The statistical analysis reveals strong and repeatable patterns in the data on population numbers and the intensity of internal war. And history of science suggests that strong empirical regularities are usually associated with the action of fundamental laws” [87], some yet to be discovered for complex systems science. Examples from the Han and Tang China data [85][86][29:Chapter 8] are shown in Figure 5 for population and sociopolitical instability (internecine wars), which are related by time-lagged feedback effects.



**Figure 5:** Population and Sociopolitical Instability for Han and Tang China (Turchin 2005a [85], courtesy of the author.)

For the stationary  $X$  (population) and  $Y$  (internecine wars) variables in these figures, standard time-lagged regression is used to estimate regression constants  $a_i = \{ a_0, a_1, a_2 \}$  where  $\tau$  is the time lag of 30 years (approximating a human generation),  $t$  is time, and  $\varepsilon_t$  is an error term assumed to be normally distributed [89]:

$$X(t) = a_0 + a_1 X(t - \tau) + a_2 Y(t - \tau) + \varepsilon_t \quad \text{Model (1)}$$

(and an analogous model for  $Y(t)$ , reversing the definitions of  $X$  and  $Y$ ). Further:

One possible objection to the procedure outlined above is that there is some positive autocorrelation between  $X(t)$  and  $X(t - \tau)$  due to the time-series nature of the data, and it is conceivable that the excellent correlations between the observed  $X(t)$  and predicted  $X(t)^*$  are entirely due to this “inertial” effect. To eliminate this possibility [the analyses were redone] with a different dependent variable,  $\Delta X(t) = X(t) - X(t - \tau)$ .  $\Delta X(t)$  is a measure of the rate of change, and by using it we break the autocorrelation arising from the time-series nature of the data. In fact,  $\Delta X(t)$  is none other than the realized per capita rate of population change, which is the standard dependent variable in the analyses of population data.... There can still be some predictive relationship between  $\Delta X(t)$  and  $X(t)$ , so we need to compare two alternative models:

$$\Delta X(t) = a_0 + a_1 X(t - \tau) + \varepsilon_t \quad \text{Model (2)}$$

... the *inertial* model (with an analogous Model (2) for  $Y(t)$ ), and

$$\Delta X(t) = a_0 + a_1 X(t - \tau) + a_2 Y(t - \tau) + \varepsilon_t \quad \text{Model (3)}$$

... the *interactive* model (with an analogous Model (3) for  $Y(t)$ ). The interactive model has an extra parameter, but in a cross-validation setting this does not matter (if the extra independent variable does not have a systematic influence on the dependent variable, then adding it to the model actually *decreases* to the ability of the model to predict out-of-sample data). [85].

The comparisons of the inertial and interactive predictions in Table 2 show consistent effects of dynamical time-lagged interactions between population and sociopolitical instability (civil conflict) that cannot be attributed simply to the inertial dynamics of each of these variables separately. The interactive effects, documented in detailed case studies (Turchin and Nefedov (2008) [78]), are those of oscillations: rising population creating resource scarcity, which amplifies inequality, making the value of property rise while that of labor falls, which, if lasting longer than a generation, causes civil unrest and conflict, causing population in turn to decline, with a lag until the cycle recurs as civil conflict ceases, allowing population to rise again (see Goldstone 1991[140]). Replications of similar findings are obtained by Turchin for the English Tudor Cycle (1485–1730) [85], the Medieval English Plantagenet Cycle (1150–1485), French Capetian Cycle (1150–1450) and Valois (1450–1660) cycles, Roman Republican (350–30 BCE) and Principate (30 BCE–285 CE) cycles, Russian Muscovite and Romanov cycle (Turchin and Nefedov 2008) [78], and the Pueblo cycle, where Kohler et al. [90] examine the Turchin model with data from Southwest Colorado between A.D. 600 and 1300. They find that “it fits well during those periods when this area is a more or less closed system. It fits poorly during the time from about A.D. 1000-1200 when this area is heavily influenced first by the spread of the Chacoan system, and then, by its collapse and the local political reorganization that follows. The model is helpful in isolating periods in which the relationship between violence and population size is not as expected.”

Table 2: Comparing Out-of-Sample Predictions of the Inertial and Interactive Models

| Source of data | Dependent variable | Correlation between predicted and observed |             |                      |             |
|----------------|--------------------|--|-------------|----------------------|-------------|
|                |                    | 1st half => 2nd half                       |             | 2nd half => 1st half |             |
|                |                    | inertial                                   | interactive | inertial             | interactive |
| England        | population         | -0.57                                      | 0.94        | -0.07                | 0.44        |
| England        | instability        | -0.13                                      | 0.80        | -0.53                | 0.89        |
| Han China      | population         | 0.45                                       | 0.57        | 0.73                 | 0.48        |
| Han China      | instability        | 0.39                                       | 0.87        | 0.37                 | 0.68        |
| Tang China     | population         | 0.56                                       | 0.80        | 0.61                 | 0.90        |
| Tang China     | instability        | 0.57                                       | 0.78        | 0.66                 | 0.92        |

(Turchin 2005a [85], courtesy of the author)

## VII. INSTITUTIONS, NETWORK ECONOMICS AND EXPERIMENTS: TESTING CAUSALITY

Studies of historical HB dynamics often lead to different conclusions. In many cases these differences result from the aspects of social process that are focused upon. Contending views may have more general points of consensus when we look at these processes more abstractly. The concepts of structural ( $k$ -)cohesion and resistance may help to provide more points of consensus.

There are many views of the formative processes of a market economy based on impersonal exchange and its prior institutional bases. Conceptualized as a network, a market economy requires  $k$ -cohesiveness simply to attain  $k > 3$  alternatives for buyers and sellers, the minimum “many” players in the market without which the advantages of competitive pricing cannot be obtained. Competition itself, however, is simultaneously a *resistive* as well as a cohesive process, a differentiation of the interests and identities of the competitors. The goods exchanged, for competitive markets, must be alienable, which entails a change of hands in property rights. Players at one time and place may be groups or corporations as property owners party to exchange. At other times they are individuals; or, parties to exchange may be a heterogeneous mix of individuals and groups. For parties to exchange

they must have rights: rights to hold property and to alienate property, rights that can be agreed upon by contract, rights and *institutions* that can enforce the contract. Effective “coercion-constraining” institutions that prevent the abuse of others’ property rights “influence whether individuals will bring their goods to the market in the first place” (Greif 2007b:727 [97]). These give rise to agency, as the capacity for human beings to make choices within a social world and to enforce the rights that those choices impose on the world, whether agency is for the selfsame agent or on behalf of another. The social world is complexly layered at the level of rights, obligations, agents, agency—and institutions as cohesive and resistant social constructions exist for the enforcement of norms. Competing views and agendas are entailed.

These kinds of interlocking components of social worlds do not fall into place quickly, but are built up incrementally over time, just as social networks are built up incrementally and their structural configurations may change slowly even while specific individuals come and go. Market institutions, for example, “co-evolve through a dynamic inter-play between contract-enforcement and coercion constraining institutions” ([98]:727) along with resistive social movements, movements to create collective goods against the resistance of free-riders, and more episodic events.

The institution-building perspective is one that has received very detailed effort in modeling actual social networks and institutional change in their historical context, abstracting the ways that social players and agents have come to effectively optimize their interactions from their multiple interests and perspectives. One of the most formidable projects of this sort over the last decade, building on the earlier work of North (2005) [102], has been to trace social foundations and historical development of institutions in pre-modern Eurasia that facilitate impersonal exchange and lead to paths toward competitive markets, while other developmental paths lead in a variety of other directions [99]. In the words of one reviewer, this work, of Avner Greif:

strips economic transactions down to their elements [and] focuses on the core question: who (or what) were the watchdogs that allowed the merchants to trust one another and to bear with the princes who could confiscate the fruits of all their efforts? And who (or what) were the watchdogs' watchdogs? [The work] repeatedly and carefully relates these questions to economic theory [and] illustrates them with real transactions of medieval merchants. He takes the right approach to economic development, and thereby achieves an original and important new perspective on its causes (Akerlof 2007 [100]).

In each of Greif’s case studies, dynamical game theory is used to test the fit between the observed data and the known historical development of institutions as well as the cultures and behaviors of the players and actors. One of the shortcomings of Greif’s work is that the early modern merchants did not face the same problems as those developing markets *de novo* in early Mesopotamia, India, China, and Mesoamerica. But further evaluation of the replicability of Greif’s model is carried on by network economists using experimental real-world simulations that engage participants in the knowledge, payoffs, and choices of the context that is modeled, testing the experimental models against the observed or recorded historical processes and outcomes (Kimbrough, Smith, Wilson 2006 [101]). To quote:

North (2005) argues that belief systems and the stock of local knowledge, the internal representations of the human experience, are intimately intertwined with the external institutions that humans build. We investigate this relationship by varying the degree to which property rights are enforced in yesterday’s institutions before the opportunities for long-distance trade present themselves with perfectly enforced property rights. Specifically, in the new experiment we report here, three-fourths of the subjects in an economy are drawn from two different treatment histories in *Build8* sessions, one in which property rights in personal goods are perfectly enforced for all of the participants, *though they must rely on trust and repeat interactions to enforce exchange agreements*, and another in which no property rights of any kind are enforced. Hence, in both sets of history-inducing sessions, there is no external enforcement of exchange contracts and, as found [in an earlier experiment], no need for such. (Kimbrough, Smith, Wilson 2006) [101].

The findings of the experimental study are “that a history of un-enforced property rights hinders our subjects’ ability to develop the requisite *personal* social arrangements necessary to support specialization and effectively exploit *impersonal* long-distance trade.” Thus we might understand through network economic experiments some replicable elements of the origin of impersonal market system. These, like cooperativity, require but go beyond structural cohesion to the social constructions of institutions that secure trust and the benefits of interpersonal trade, i.e., network elements that reinforce the scalability and benefits of structural *k*-cohesion as discussed above.

In Greif’s analysis, while the institutional supports for impersonal long-distance trade only developed slowly in Medieval Europe (and elsewhere) the full protections of “coercion-constraining”

institutions “that prevent the abuse of others’ property rights” and “influence whether individuals will bring their goods to the market in the first place” were still not in place even in England after the “Glorious Revolution of 1688,” which did not secure such rights beyond “the landed, commercial, and financial elite” (Greif [97]:786). Rarely is linear progression of rights entailed in the ups and downs of the precursor elements of fully competitive markets that vary from one country to another. In England, after 1688, for example, although

parliament gained supremacy, it was not in the business of protecting property rights per se. Its policy reflected the interests of those who controlled it.... The subsequent history is thus marked by gross abuses of property rights.... Yet, a state controlled by its landed, commercial, and financially elite and later empowered by the Industrial Revolution was a boon for the extensions of markets. The evolution of the modern markets reached its Zenith.... Europeans shared a common heritage of individualism, self-governance, a broad distribution of coercive powers, and man-made laws. Reversing their institutional developments and enabling market extensions was relatively easy ([97]:775-776).

Eurocentrism is not intended in the use of this example, as this project entailed equally detailed historical and modeling analyses of China and the Muslim world.

Greif’s analysis of land-based institutions and exchange example provides a contrastive comparison against Erikson and Bearman’s 2006 [103] network study of English maritime trade between 1600 and 1831. Here an entirely different account is given of the emergence of the competitive market system. The shared elements are the *k*-cohesive extensions of trade routes, extensive by sea as by land, and the institutional development of English rights in property, commercial exchange, protection, and agency. Here, however, the resistive element is paramount, and the “new economy” arises through malfeasance of the sea captains of the English East India Company. Their work is carried on preemptively, out of self-interest, exploiting the opportunity of delay. Instead of bringing English goods to the orient and returning with oriental goods in one single return cycle, in order to stay beyond the time when the ships could return by the monsoon winds, they traded from port to port on their own behalf with their own goods and retained the profits. Over time, the density of this network became so great that the sheer volume of overlapping circular routes, crisscrossing the net of visited ports, pushed the *k*-connectivity of the market exchanges beyond 0, 1, or 2 for different subregions often up to 7-8, a veritable revolution in creating new market opportunities and competitive market pricing through sheer volume of malfeasance behavior: malfeasance because this was all conducted against the policy of the home company, which was powerless to prevent it.

## VIII. FUTURE DIRECTIONS

The topics covered here, of cohesion and resistance as measurable social forces in human behavior, and the multiple ways that these two social forces dynamically interact—and what enhances or limits scale-up and scale-down of both cohesion and conflict or resistance—leaves open many researchable questions. Lim, Metzler and Bar-Yam's (2007) study supports a more advanced even if partial view of the dynamics of cohesion and resistance, group separations, and segregative conflicts along insufficiently demarcated boundaries. Other parts of the human cohesion/resistance dynamics covered in this review show some of the other ways in which cohesion and resistance interact. Human capacities for structural cohesion, for example, support cultural differentiation of groups. Transition thresholds characterize evolutionary bouts of scale-up in group size through central authority, oscillating against resistance from egalitarian preferences for autonomy. With scale-up in size, expansions of political units encounter boundaries of cultural and ethnic differentiation where resistance scales up as oppositional cohesion in positive feedback cycles, creating further expansion of polities that began only as resistive groups. These support growth of population sizes, which lead in turn to scarcity relative to resources within regions. With generational time lags there develop both greater differential inequality and conflictual resistances to inequality. Large polities develop institutional and economic frameworks that can provide benefits to internal differentiation, while the enhanced potential for cohesiveness and economic growth can find ways, as in the biotechnology industry illustration, to utilize the recruitment of diversity to create innovation (Page, 2007 [131]) while stabilizing the costs of cohesive integration.

The problems of modern states and institutions may be seen to devolve on how to minimize the costs of the conflicts that are generated by the oscillations between oppositional cohesion and

integrative cohesion. For HB dynamics more generally, solid causal analysis using the most advanced techniques is only possible with current and future data collected systematically on historically documented entities compared over different time scales, up to millennial time series. These data can be analyzed with processual models, network analytic models, institutional, cultural, and evolutionary game-theoretic and economic analyses. Many of the algorithms needed at this level of complexity have developed in computer science, e.g., by Pearl [3][104] and, by including the crucial element of agency in a new econometrics framework (Halbert White [4][105][106]) economics —the otherwise dismal science— can be investigated by causal modeling algorithms. In the modeling of causality that is relevant here to HB dynamics, the analytical power recently gained in econometric models may be neatly illustrated by a comparison of statistical results and conclusions reached by fractal economics, survivorship analysis of successful mutual fund managers, bootstrap models of the same problem, and market simulations of intelligent agents that place orders to trade at random. The first case involves the discovery of †fractal pricing in cotton markets (Mandelbrot 1963 [107]) and the Dow-Jones (Mandelbrot and Hudson [108]), contradicting the standard assumption in economics hypothesized by Bachelier (1900) [109] that Brownian movement (Gaussian price deviations) is descriptive of market price dynamics. If volatility is predictable in markets, but not price and direction, the implication is that value might not be useful as a concept in economics [107] (consider market collapse when no trader wants to trade in an uncertain market (Jorion 2007 [142:3-4]). Parallel evidence from experimental studies rejected the reference-independent framing of judgments for the “value” of expected utility theory as originally framed by Bernoulli (1738) [110], and questioning the assumption that utilities are stable (Kahneman and Tversky 2003 [111]). Similarly, survivorship analysis of successful mutual fund managers showed no evidence that the top ranked funds were any better than random as they lacked measurable persistence (Carhart et al. [112]). Finely tuned bootstrap estimation models that are oriented toward testing causal models in econometrics, however, showed that while income fund managers did no better than random, growth fund managers showed persistence in their ability to pick stocks (Kosowski et al. [113]). And finally, a baseline market simulation model of for intelligent agents that place orders to trade at random, with only one free parameter, accounted for 96% of the best buying and selling prices (the spread), and 76% of the variance of the price diffusion rate (Farmer et al. [114]), which “demonstrates the existence of simple laws relating prices to order flows, and in a broader context, because it suggests that there are circumstances where the strategic behavior of agents may be dominated by other considerations.” “One of the virtues of this model is that it provides a benchmark to separate properties that are driven by the statistical mechanics of the market institution from those that are driven by the strategic behavior of agents. It suggests that institutions strongly shape our behavior, so that some of the properties of markets may depend more on the structure of institutions than on the rationality of individuals.” These examples are all indicators of complex dynamics.

The challenge of HB dynamics is to assemble better data related to aspects of the problems modeled, including those of competing hypotheses: more complete data, data better grounded in diverse historical circumstances, and more contextual detail. A second challenge is to have better statistical estimators, identification and correction for sources of bias, attention to †nonindependence, careful modeling of richly grounded historical data, attention to causal modeling, and multiple-level models. These efforts are facilitated by sharing of data, collaborative analysis of potential biases in data, sharing of documentation of software and source code, verification of source code, and replication of results. Extensive effort has gone into the archaeological and geocoded data related to the evaluation of the model in Figure 2 of transition probabilities. Ten years of effort went in locating and coding the data in Figure 3, for example. Good data on population numbers at all levels, such as aggregated in Figures 3 and 4, is extremely important for modeling, hard to come by, and demands careful analysis for bias detection, bias correction and data reconstruction. The data on internecine warfare in Figure 5 were patiently transcribed episode by episode in two compendia of scholarly work over millennia. These are but a few of many thousands of databases, many of which have not been made sharable or are not conserved or not well documented. Also needed are data analytic routines able to make accurate estimates using probabilistic bootstrap methods with small samples and for different kinds of data, e.g., continuous or discrete, nominal or ordinal. A great deal of documented open source code is now available.

Causal modeling is the core of dynamical analysis, and future directions will include modeling of the types illustrated here, and many more, but with integrated datasets for different foci and levels of analysis. A host of intersecting and mutually enriching integratable time-coded longitudinal datasets—like Turchin’s data for the 50-region data in Figure 1 [29], or the sufficient size data for Figure 2 [85]—are needed from comparable local contexts and processes up to the global, e.g., google-earth-like sharable data structures, equipped with analytic routines for time-series causal modeling and testing. There are many separate projects on shared issues, but overall integration is needed. Geographic Information Systems (GIS), for example, need to be reintegrated around open source code (e.g., GRASS, written in open source R) that includes temporal and network modeling. Autocorrelation and other techniques and models for dealing with nonindependence of cases will figure heavily in causal modeling.

Among new network analytic methods that are becoming standard in many disciplines are the censuses of different types of cycles (“motifs”) in large networks that make up cohesive  $k$ -components, and accessible software to compute  $k$ -connectivity. Analysis of these sorts of data allow testing of where and how the internal micro and middle-range structures come from in  $k$ -components [115][116][117]. Which structures come from preferences and which from the marginals or limits on how data were collected or spatially distributed? This kind of work is now being done in biology but also in anthropological network studies, where new software packages have been developed that are specifically designed to deal with certain problems, such as kinship networks or the kinds of generative kinship computations that people actually use in their social cognition [11][118].

Entropy maximization “open system” models conditioned on biased random processes will increasingly become integrated with HB dynamics as we come to understand how to connect them to foundational problems in the human sciences, some of which are discussed in the ENTROPY entry in these volumes. Simple entropy models, for example, are currently being used to fill in missing data from what is known from an archaeological site [119]. Tsallis entropy [120], in contrast, would provide a one-parameter modification for least energy maximization channeled through networks, with diffusion gradients that have multiplicative effects. Generative network models for cohesive cycles, as studied to date [121] show consistent distributions of numbers of links that are all in the Tsallis entropy family, so there are promising avenues in this research area.

As seen in examples here, more integration of theory and data is needed, from macrohistorical models where large-unit aggregation relates to sufficient statistics for causality, through cascades of spatio-temporal processes to the micro level of interactions between individuals [141]. Kirman (1999) [122], for example, argues that “The emergence and evolution of the networks that govern the interaction in the economy plays a crucial role and it may well be the case that the standard notion of equilibrium is irrelevant in such as context.” Multilevel network analyses will be aggregated structurally in new ways for which new modeling techniques are needed to analyze the composition of units and of processes [124].

The construction of theory and hypotheses in this article illustrate only a few potential causal links among major topics on evolutionary and historical dynamics, institutional and economic models, game theory and social networks, organized around a few core dimensions of ethological importance (structural cohesion and resistance). The point of these illustrations has been that there are truly major forces in history— $k$ -cohesion and cohesive resistance for example—but these have very different properties than forces in physics, or the dynamics of chemistry and biology (although one LANL biologist, asked to pinpoint the major threats to survival, responded with “human behavior” [143]), and they require very different measurements and theory. But there is no closure on the topics of HB dynamics: rather, there is an abundance of theory and results in social, historical, and simulation modeling that lend themselves to the evaluation of causality. Taking causality and dynamics seriously rather than dismissively leads to very different theoretical and analytical perspectives. Longitudinal data on human social, historical, and network phenomena are sufficient to support high-level theoretical and integrative research that can further benefit from the most advanced of methods in the complexity sciences. But there is a pressing work to be done in analytic methods and in constructing valid and reliable datasets and variables on appropriate and comparable units and processes under analysis.

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