Let's not promulgate either Fechner's erroneous algorithm or his unidimensional approach

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Murray several times cites Luce and Edwards's (1958) of Fechner's derivation of Fechner's Law from Wel (sects. 1.1, 1.2, and Note 2). Unfortunately, he see have absorbed our message and he makes the same logical error where it matters some. Fechner corrects Equation 2, \( \Delta S(I)/\Delta I = C/I \), as the functional equation of his hypothesis of equal subjective jnds and the notion of Weber’s Law. To solve for \( S(I) \), he proposed a solution calling it a 'mathematical auxiliary principle' replacing \( \Delta S(I)/\Delta I = C/I \) by the differential \( dS(I)/dI \). Thus, \( E \) is transformed into a (well-understood) differential whose solution is Equation 4. Our point was that the algorithm happens on a correct solution when either Law or the generalization implicit in Equation 5 holds arrive at the correct solution for any other Weber scale (1985) provides a good discussion of algorithm.

This may seem a bit of esoterica, but the issue reap more significant way in section 1.3.3 where Murray and Fechner later considered a reformulation (see Equation 11) of the initial problem. To “solve” Equation 10 using the \( S(I) \), rewrite it as \( \Delta S(I)/\Delta I = C/I \). Replace the differences by a differential to get \( dS(I)/dI = C/I \). Equivalent to \( d[\log S(I)]/dI = Cd[\log I]/dI \). Integrating yields Equation 11. Murray accepts this not only as historically accurate, but also mathematically correct. About the latter he is wrong. One d

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If one accepts that measurement is a one-dimensional matter, the committee was right. 1 For a careful contemporary treatment of the one-dimensional approach and its ambiguities, see Narens (submitted). Falmagne (personal communication) points out that if one is willing to deal with choice probabilities rather than orderings, it is possible in principle to construct a binary operation over the probability space. The difficulty is that because most choice probabilities are 0 or 1, one is forced to piece together the global scale from locally available data.

What the Ferguson committee failed to acknowledge, and many psychophysical scalers seem to continue to ignore, is that something very like one-dimensional measurement becomes feasible when two or more independent variables affect the same attribute. One can use the resulting trade-off between the independent variables as a source of measurement of the attribute and how the factors combine. Indeed, trade-offs typically induce mathematical concatenation operations on the components. This was completely familiar in physical measurement of such quantities as momentum, energy, density, and so on, but it had not been axiomatized in a fashion analogous to the turn-of-the-century axiomatizations of extensive measurement. The lacuna was corrected in the early 1960s (see Chapter 6 of Krantz et al. 1971 and Chapter 19 of Luce et al. 1990 for historical details).

Thoroughgoing behavioral examples of the trade-off approach are Luce’s (1977) axiomatization of power and logarithmic functions when the data are orderings of stimulus pairs, such as loudness to binaural tones; the functional measurement procedures advocated and applied to psychophysical as well as other psychological problems by Anderson (1981; 1982); and the entire complex literature on axiomatizations of preferences and/or judgments of uncertain alternatives. In each case, one uses the trade-off between variables to establish simultaneously the measures involved and the law relating them.

If axiomatics are not to one’s taste, another approach is to increase the dependent variables from just choice and/or judgments about psychophysical stimuli to include the time taken to respond. Here the models are much more process oriented and have little in common with the measurement-theoretic models. The measures of sensation are parameters of the model and are estimated from behavioral data. Again, trade-offs are the name of the game, in these cases often between two or more sensory variables such as signal duration and signal intensity but also between a sensory variable and some sort of motivational criterion. The models simultaneously develop measurement and theory. Summaries of various of these models can be found in Luce (1986) and Townsend and Ashby (1983).

My only point is that one is probably misguided to continue to fuss at the one-dimensional measurement case; unless a relevant concatenation operation or some other rich internal structure can be found, the situation is simply too underdetermined to be of much theoretical interest.

**NOTE**

1. At the time, the only cases that were understood involved operations with additive representations. Later work (see Chs. 19 and 20 of Luce et al., 1990, for a summary) yields a whole family of inherently nonadditive operations. Homogeneity, described in Luce and Narens (1987), underlies this result; this paper, which is purely expository, gives references to the original contributions.