Expertise and Complexity in the Social and Engineering Sciences: An Extended Sen's Theorem

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Abstract: Both the social and engineering sciences search for appropriate aggregated outcomes—a search that can be accompanied with complexities and inefficiencies. As shown here, certain difficulties are direct consequences of adopted approaches; e.g., management style, choice of a division of labor, use of expertise and experts, or even the modeling of the physics. Particularly troubling is that even with reasons to believe that "success" has been achieved, this need not be true. These results are motivated by Sen's Theorem from decision theory. More general results motivated by Arrow's Theorem will be given elsewhere.

1 Introduction

Details differ, but a significant amount of what is done in society and in engineering are conceptually similar; both need to create rules, practices, approaches that involve allocations and aggregations and both need to make decisions. To unite themes from these areas, I treat any aggregation or allocation rule that combines "parts" to attain an outcome as a "generalized decision rule." Such rules, then, include statistics, allocation procedures as varied as the economic "supply and demand" story, the aggregation of physical processes to create a macro effect, organizational "divisions of labor," and much of what is done in the social sciences.

Generalized decisions include standard voting rules where the parts are the voters' preferences and the aggregation determines the election outcome. They include location problems, assessments, as well as engineering and corporate decisions where voters are replaced by criteria. Central to organizational design is a "division of labor" assignment of tasks to different groups, which become the parts, to achieve a desired outcome. To address the complexity of engineering, projects normally are divided into parts; the contributions made by different groups to the final product determines a generalized decision outcome. Key to multiscale design is a relationship whereby what happens at nano and micro levels determine valued aspects of macro behavior; this connection makes these techniques "generalized decisions." More generally, multiscale considerations are central to policy decisions such as health; e.g., whenever the policy is determined by ways in which individuals, the parts, interact, it becomes a generalized decision.

To address the mysteries associated with generalized decisions, guidance comes from decision theory. In particular, my conclusions are motivated by re-examining Sen's [9] profound but negative theorem, which asserts that it is impossible to create a decision rule that does what seems to be obviously possible to do. As developed here, a similar assertion holds for generalized decision rules. It proves that certain *current approaches* make it impossible to avoid particular conflicts and

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inefficiencies that arise in societal and industrial settings. Illustrations include the recognized inefficiencies that can occur among sales, design, and manufacturing within an industry, or the difficulty of achieving equilibria in decentralized economic settings. These assertions explain, for instance, why conflicts can occur even when generalized decisions are determined in optimal ways, or by relying on expertise. The relaxed conditions of this theorem ensure that its conclusions encompass a wide selection of social science, engineering, industrial organization, and other considerations.

For simplicity of exposition, the generalized decision rules described here require the space of inputs to be similar to the space of outcomes. The modeled situation could be, for instance, an industry's aggregation of plans combining the design, manufacturing, and sales components of a project into a master plan, it might be a standard decision of where to locate a plant based on various criteria, it could be involve issues about economic reallocation, or supply-and-demand issues. In particular, these results, which permit settings with a continuum of choices, show how notions coming from Sen's Theorem affect a surprisingly broad array of issues coming from several different areas. More extensive results motivated by the more general Arrow's Theorem [1] will be given elsewhere.

2 Sen's result

To develop a sense of what kind of results hold for generalized decision rules, I review what can happen with Sen's Theorem. (For a different interpretation of Sen's Theorem, see Saari [4, 5].) Sen starts with a compatibility constraint defining the admissible inputs for the decision rule: individual rankings over a finite number of alternatives must be "orderly" in that they are complete (each voter ranks each pair of alternatives) and transitive; i.e., a voter who prefers A to B (denoted by $A \succ B$) and $B \succ C$ must prefer $A \succ C$. In decision contexts, replace "voters" with "criteria."

Assumption 1 *(Compatibility) Each voter ranks the $n \geq 3$ alternatives in a complete, transitive manner. There are no restrictions on the choice of a ranking selected by a voter.*

As for the decision rule, the Pareto assumption requires it to respect unanimity.

Assumption 2 *(Pareto) If for some pair of alternatives, all voters have the same ranking, then that is the pair's societal ranking.*

My interpretation of Sen's third assumption differs from the traditional intent, which is to explore whether there is a fundamental conflict between the rights of an individual and those of society. "Rights" are important concerns (that I explore with Petron in [6] and Li in [3]), but my current goal is to examine the broader issues that are associated with generalized decisions. So, in this article, treat Sen's "decisive agent" as anything—a physical or economic force, an expert—that reflects expertise. What I have in mind is captured by an amusing television ad where a frightened man, with a sharp knife pressed against his midsection and a phone to his ear carefully listening to a doctor's instructions how to perform an operation on himself, nervously questions the doctor, "But, shouldn't you be doing this?" Namely, expertise matters; for certain issues, expertise should determine that part of the societal or industrial outcome.

Assumption 3 *(Minimal Liberalism) Each of at least two individuals, called a decisive agent, is assigned at least one specific pair of alternatives. Each decisive agent's ranking of an assigned pair determines the pair's societal ranking.*

The last assumption imposes structure on the outcome; it merely requires that the societal ranking of the alternatives does not admit cycles.

Assumption 4 *Societal outcomes do not have cycles.*

While these seemingly innocuous conditions appear to be the kind that should be satisfied by all group decision rules, Sen's Theorem states that no rule can always satisfy them. As a side comment, all verifications of this theorem that I have seen involve creating examples; the first general proof (based on the geometry of a cube) of this seminal theorem is in Li and Saari [3].

Theorem 1 *(Sen [9]) With $n \geq 3$ alternatives and at least two decisive agents, no rule exists that satisfies Assumptions 1-4.*

To interpret this result in the context of decision theory, consider a situation where the ranking of a particular pair is determined by the Department of Health criterion, while the ranking over another pair is determined by the EPA criterion. This theorem ensures that problems can occur. Namely, Sen's Theorem becomes applicable whenever a rule allows the rankings for even two pairs to be determined by certain criteria, or experts, or .... By considering "expertise," it becomes reasonable to extend the result from the use of individual experts to groups providing the expertise. Such extensions can be made [3]; the conclusion holds even where separate groups, even the full group, can decide with certain rules, such as a majority vote. (As shown in Saari and Sieberg [7], similar problems arise when "pairs" are replaced with "parts.") Let me recommend Batra and Pattanaik [2] who nicely developed extensions for group rights. Also, see Salles [8] comments.

The impact of this result comes from the reality that we do rely on expertise; we do use experts. As such, Theorem 1 guarantees that situations will arise whereby one of the four assumptions is violated. We control the first three assumptions, so we must expect cyclic outcomes. A troubling, pragmatic consequence of cycles is how they can cause us to inadvertently make "bad decisions." This is particularly so with the "path dependency" phenomenon where, rather than discovering excellence as determined by the data, the outcome of a decision rule may more accurately reflects the path—the order—in which alternatives are compared.

To illustrate this behavior with seven alternatives, suppose in a decision problem that the rankings over three criteria are

$$\begin{array}{c} A \succ B \succ C \succ D \succ E \succ F \succ G, \\ B \succ C \succ D \succ E \succ F \succ G \succ A, \\ C \succ D \succ E \succ F \succ G \succ A \succ B \end{array} \tag{1}$$

As each criterion ranks $C \succ D \succ E \succ F$ over G, the data clearly identifies G as the inferior choice. As this more general information typically is not known, the alternatives may be compared pairwise to save on costs. Herein lies the problem; rather than ensuring optimality, *each* alternative—even G—can be selected as the "optimal" choice just by using different orders of comparisons.

To illustrate, the following path selects G where all outcomes are either unanimous, which satisfies Pareto, or the winning choice has two-thirds of the vote: compare the winner of $\{E, F\}$, which is E, with D, compare that winner (D) with C, compare that winner (C) with B, compare that winner (B) with A, compare that winner (A) with G to determine the final outcome—G.

It is easy to convert this example into one representing Sen's result: instead of a vote, assign appropriate decisive agents for the $\{A, B\}, \{B, C\}$ and$\{A, F\}$ pairs. What results is that the order in which the experts make their decisions can lead to the selection of G. Indeed, the reader should now be able to construct any number of examples illustrating how a decisive agent—an expert—can create inefficiencies. (References [4, 5] show how to do so; in [3] likelihoods of this behavior are compared with that of majority votes.)

3 A generalized inconsistency theorem

To provide a flavor of the kinds of generalized theorems that I have developed, in this paper I concentrate on settings where the space of "parts" is divided into three or more independent components $\{\mathcal{C}^j\}$. (Examples of component spaces follows.) While each component must have at least two elements, the elements in a component could constitute a discrete set of alternatives, a continuum of possibilities or degrees of comparison, various design proposals, the set of possible parts that could be used to construct a particular portion of a project, or the elements of whatever is being modeled. Different components involve different kinds of elements; in engineering, for instance, the design component $\mathcal{D}$ may consist of all possible ways to design a product, the manufacturing component $\mathcal{M}$ may consist of all possible ways to manufacture it, and the sales component $\mathcal{S}$ may consist of all possible ways to market it. In a design project where the effort is divided among three or more units, all options available for each unit $\mathcal{D}^j$ constitute the elements of a component. In a multiscale design problem, each component consists of one type of the possible micro effects that are to be combined.

Although the component parts are independent, they must be related through compatibility conditions. After all, it is easy to imagine a $(d, m, s) \in \mathcal{D} \times \mathcal{M} \times \mathcal{S}$ reflecting an incompatible combination of design, manufacturing, and sales proposals; it is easy to imagine an incompatible $(d_1, d_2, d_3) \in \mathcal{D}^1 \times \mathcal{D}^2 \times \mathcal{D}^3$ consisting of the component parts proposed by three different units for a design project.

The compatibility constraints are intended to avoid what we do not want to occur; e.g., to avoid inefficiencies, impossible arrangements, etc. As such, the precise definition of "compatibility" depends on what is being examined. But whatever conditions are adopted, for efficiency of analysis, a first requirement is to avoid trivial settings; e.g., include only components $\mathcal{C}^k$ that are meaningful for the modeled process. By this I mean that the compatibility condition is not satisfied unless an element from each $\mathcal{C}^k$ is included.

Only the relevant parts of a component $\mathcal{C}^k$, as determined by the compatibility condition, are to be included. To be suggestive, the compatibility conditions may be given by, say, physical laws, engineering principles, or economic constraints, so use these conditions to drop all parts from each $\mathcal{C}^k$ that never would be used. With $\mathcal{D} \times \mathcal{M} \times \mathcal{S}$, for instance, eliminate all designs d^* that always lead to a product that is impossible to manufacture and/or sell. Stated more abstractly, should it be true for all $m \in \mathcal{M}, s \in \mathcal{S}$ that (d^*, m, s) does not satisfy the compatibility condition, then drop d^* from $\mathcal{D}$. Conversely, to avoid "obvious" settings where something always is true so it can be incorporated into the definition of the generalized decision rule, require each $c_k \in \mathcal{C}^k$ to be in some unacceptable combinations. Illustrating again with $\mathcal{D} \times \mathcal{M} \times \mathcal{S}$, this condition means that each design $d \in \mathcal{D}$ is compatible with certain manufacturing and sale plans, but it is not compatible with certain other manufacturing and sales plans.

It remains to introduce flexibility into the compatibility conditions. For intuition about how to do so, consider standard behavior associated with equilibria settings. Namely, while changing one force may destroy the equilibrium, it may be possible to compensate and return to equilibrium by appropriately changing another force. Similarly, budget constraints require the amount spent on a commodity bundle to agree with the available income. While buying more of one commodity might violate the budget constraint, buying less of another can restore it. As a third illustration, it is easy to envision compatible combinations (d, m, s) from $\mathcal{D} \times \mathcal{M} \times \mathcal{S}$ where, for various reasons, instead of using s, it is preferred to use the sales program $s' \in \mathcal{S}$. But, it may be that replacing the original s with the preferred s' makes the combination (d, m, s') incompatible. The flexibility part of the compatibility conditions permit making compensating adjustments in some other component; e.g., it may be possible to use, say, the different manufacturing approach $m' \in \mathcal{M}$ to compensate for s'

in that (d, m', s') is compatible.

The above discussion is formalized by the three parts of the following definition. These conditions reflect standard practice, so it is easy to construct any number of illustrating examples.

Definition 1 *A compatibility condition imposed on combinations $(c_1, \dots, c_n) \in \mathcal{C}^1 \times \dots \times \mathcal{C}^n$ is "acceptable" if it satisfies the following three conditions:*

1. *(Completeness) For a combination to satisfy the condition, it must have a term from each $\mathcal{C}^k$.*
2. *(Meaningful) For any combination satisfying the compatibility condition, there is some component where a particular change in that entry creates an incompatible combination. For each component $\mathcal{C}^k$, each $c_k \in \mathcal{C}^k$ is in some combinations that satisfy the compatibility conditions and in some that do not. Namely, for each $c_k \in \mathcal{C}^k$, there exist a combination $(c_1^*, \dots, c_{k-1}^*, c_k, c_{k+1}^*, \dots c_n^*)$ that satisfies the compatibility condition. However, there is a compatible combination $(c_1', \dots, c_{k-1}', c_k', c_{k+1}', \dots c_n')$ where, by changing c_k' to c_k, the combination $(c_1', \dots, c_{k-1}', c_k, c_{k+1}', \dots c_n')$ fails to satisfy the compatibility condition, $c_j^*, c_j' \in \mathcal{C}^j$.*
3. *(Compensative) For any two components, j and k, there exists a combination $(c_1, \dots, c_n)$ satisfying the compatibility condition where a change can be made in the j^{th} component to create an incompatible combination, but a compensating change can be made in the k^{th} component to make the new combination acceptable.*

An acceptable combination is called a "plan." It is denoted by $\mathbf{p} = (c_1, \dots, c_n) \in \mathcal{A}$.

A generalized decision rule determines an outcome based on information, or plans, coming from different "participants." In Sen's setting, the participants are the voters, in a standard decision rule, participants could be the different criteria, in a design project, it might be the different participating units, in $\mathcal{D} \times \mathcal{M} \times \mathcal{S}$, the three participants could be the design, manufacturing, and sales units, in a physical behavior or multiscale design, it could be competing physical forces that contribute to the final effect.

Assumption 5 *(Compatibility; inputs) Assume there are $m \geq 2$ participants. Each is required to select a plan $\mathbf{p} \in \mathcal{A}$; there are no restrictions on the choice of each participant's plan. The list of plans for all of the participants is called a profile; it is represented by $\mathbf{P} = (\mathbf{p}_1, \dots, \mathbf{p}_m)$.*

While this assumption implicitly requires the same compatibility conditions to be imposed on each participant's plan, this is not necessary. Instead, with only slight modifications, everything extends to where each participant is assigned a different compatibility constraint as long as the conditions satisfy Definition 1.

A similar comment applies to the next assumption about the compatibility of outcomes; while the discussion given here addresses those natural settings where the outcomes have the same form as the inputs, this is not necessary; e.g., in multiscale design the nature of the outcomes could differ significantly from that of the inputs.

Assumption 6 *(Compatibility; outputs) The components for the space of outputs are the same as those for the imputs. The generalized group outcome $\mathbf{O}$ based on profile $\mathbf{P}$ satisfies the compatibility conditions of Definition 1. This acceptability property is denoted by $\mathbf{O} \in \mathcal{A}$.*

The generalized decision rule combines plans in the profile to create a particular outcome $\mathbf{O}$. How this is done is determined by the physics, economics, organizational, or engineering principles. What follows are two assumptions that appear to occur with many of these rules. The first assumption for this rule, or an organizational structure, or a description of a practice used within a corporation, is that unanimity rules.

Assumption 7 *(Pareto) If for a profile* $\mathbf{P}$*, there is some* k *where the* k^{th} *entry in all* $\mathbf{p}_j$ *plans is the same* c_k*, then this agreed upon term is the* k^{th} *entry in* $\mathbf{O}$.

As an illustration, if the plans put forth by the design, manufacturing, and sales parts of an industry are, respectively, $((d_d, m^*, s_d), (d_m, m^*, s_m)$, and $(d_s, m^*, s_s))$, then the universal agreement among three plans with respect to the manufacturing component of m^* requires m^* to be the $\mathcal{M}$ component for the outcome $\mathbf{O}$.

The next decentralization, or "expertise" condition is where, as long as selective participants strictly adhere to the compatibility constraints, each is free to make the decision over a designated component. In our standard $\mathcal{D} \times \mathcal{M} \times \mathcal{S}$ example, for instance, as long as the manufacturing unit strictly adheres to compatible plans, it is reasonable to permit this unit to make the decision about the outcome's $\mathcal{M}$ component. More generally, the participant may be an "expert" who can make an informed choice for a particular component's outcome. The participant may be a particular physical force; e.g., the component may describe the angular momentum of each part while the participant is the aggregation determining the system's angular momentum.

Definition 2 *(Decentralization) A decisive participant is one that is assigned a particular component for the outcome; this participant determines the component's entry for* $\mathbf{O}$.

As with Sen's result, the above minimal assumptions seem to be innocuous, but they lead to a contradiction.

Theorem 2 *With more than one decisive participant, no rule exists where the plans and outcomes satisfy the respective compatibility assumption 5 and 6, and the rule satisfies the Pareto assumption 7.*

A simple example capturing the idea of the proof with $\mathcal{D} \times \mathcal{M} \times \mathcal{S}$ is where (d_1, m_1, s_1) is an unacceptable outcome. Suppose there are two decisive participants where the first participant is decisive over $\mathcal{D}$ and the second over $\mathcal{S}$. The following table specifies each participant's plan while the dash indicates irrelevant information because another participant is decisive.

Participant	**Plan**	$\mathcal{D}$	$\mathcal{M}$	$\mathcal{S}$
1	(d_1, m_1, s_2)	d_1	m_1	$-$
2	(d_2, m_1, s_1)	$-$	m_1	s_1
	Outcome	d_1	m_1	s_1

(2)

Both plans are compatible, but the (d_1, m_1, s_1) outcome is not. To compute this outcome, the $\mathcal{D}$ and $\mathcal{S}$ components are determined by the decisive participants, and the $\mathcal{M}$ component is determined by Pareto. The general proof is much the same.

Proof. Suppose there are two decisive participants; the first determines $\mathcal{C}^j$ and the second determines the $\mathcal{C}^k$ component of the outcome. Select any plan $\mathbf{p} = (c_1, \ldots, c_n)$ satisfying the compensative condition of Definition 1 with respect to the specified j and k. Plan $\mathbf{p}$ is selected by the first decisive participant, who determines the $\mathcal{C}^k$ component, and by all other participants except for the second decisive participant. The second decisive participant, who selects the $\mathcal{C}^j$ component, selects plan $\mathbf{p}_j$ where c_j is replaced with c'_j to create an incompatible combination, so c_k is replaced with the compensating c'_k to create a compatible plan.

For each $i \neq j, k$, the Pareto condition determines the $\mathcal{C}^i$ component for the outcome to be the common c_i. For the k^{th} component, the first decisive participant determines the outcome, which is c_k. But for the j^{th} component, the second decisive participant selects c'_j creating an incompatible outcome. The same construction applies to settings with any number of decisive participants. □

4 Path dependency and other consequences

A reasonable expectation is that incompatible outcomes are caused by profiles that exhibit a strong disagreement among the plans. This expectation is what makes the proof of Theorem 2 disturbing; incompatibility can arise even in surprisingly orderly settings where the plans deviate only slightly from unanimity! Indeed, in the proof, all but one participant agreed on all aspects of the plan, and the deviating participant agreed on all but two of the elements.

Theorem 2, then, states that even when participants, or experts, carefully use information that satisfies the specified compatibility conditions, it is possible to encounter disarray, which may be manifested in terms of inefficiencies or incorrect interpretations. Notice that the result does *not* assert that incompatibility *must always* occurs; it occurs only with certain settings of inputs. This means it is very possible for particular practices that satisfy the conditions of the theorem to have satisfactory outcomes. This "success," however, could lead to the mistaken opinion that a physical law, a organizational principle, or whatever is being modeled, has been established, whereby success is guaranteed. But this need not be the case. The problem is that should an incompatible outcome occur, which is guaranteed by the theorem, focus may be misplaced. Rather than examining whether the fault is due to the nature of the generalized decision rule, it may be misplaced by worrying whether, for instance, the data or inputs are at fault.

A natural way to try to correct this situation is to impose more stringent constraints on how to select the outcome for each component. One approach is to require a greater level of interaction, communication, or exchange of information among the participants. For instance, when deciding the outcome for a specific component, maybe information relevant to that component coming from the plans of *all* participants should play a role.

Even here, problems can arise. Consider, for instance, a three component situation, say $\mathcal{D}, \mathcal{M}, \mathcal{S}$, where each component has two entries. The eight possible combinations are

$$\begin{array}{lll} (d_1, m_2, s_1), & (d_1, m_1, s_2), & (d_1, m_2, s_2), \\ (d_2, m_1, s_1), & (d_2, m_2, s_1), & (d_2, m_1, s_2) \\ (d_1, m_1, s_1), & (d_2, m_2, s_2). & \end{array}$$

It is easy to show that the smallest set of admissible plans, $\mathcal{A}^s$, consists of three plans having the $\{(d_2, m_1, s_1), (d_1, m_2, s_1), (d_1, m_1, s_2)\}$ format; the five remaining combinations are incompatible. At the other extreme, the largest possible choice, $\mathcal{A}^l$, consists of six plans; with an appropriate labeling of the indices, the only two incompatible plans are (d_1, m_1, s_1) and (d_2, m_2, s_2). As it can be shown, with a relabeling of subscripts if necessary, any set of compatible conditions $\mathcal{A}$ contains $\mathcal{A}^s$ and is contained in $\mathcal{A}^l$. Even more, any $\mathcal{A}$ satisfying $\mathcal{A}^s \subset \mathcal{A} \subset \mathcal{A}^l$ is a set of admissible plans.

Suppose the plans proposed by design, manufacturing, and sales are as in Eq. 3: to ensure that each participant's plan influences each component's outcome, the choice is determined by a majority vote. The incompatible outcome is as described in the last row:

Component	$\mathcal{D}$	$\mathcal{M}$	$\mathcal{S}$
Design	d_1	m_1	s_2
Manufacturing	d_2	m_1	s_1
Sales	d_1	m_2	s_1
Outcome	d_1	m_1	s_1

(3)

Because these plans come from $\mathcal{A}^s$ and because $\mathcal{A}^s \subset \mathcal{A}$, it follows that this incompatibility feature must be admitted by all choices of $\mathcal{A}$. Moreover, this conclusion extends to where each component has any number of elements, even a continuum of possibilities. As shown elsewhere, this incompatibility, which is a cause of inefficiency, holds for a wide selection of natural aggregation rules.

Let me now turn to a particularly troubling result, which is based on the fact that a natural way to create an outcome is to use a step-by-step process. Namely, in discussions with experts in multiscale engineering design, I learned that some use the approach where they determine the optimal choice for as many components as possible and then adjust the choices for remaining components to ensure that the resulting combination is compatible. For instance, using the design of a product and depending on the flexibility of the compatibility conditions, after finding the optional choices for $\mathcal{D}$, it might be necessary to select appropriate manufacturing and sales plans in order to have a compatible combination; e.g., with $\mathcal{A}^s$, selecting d_2 mandates that m_1 and s_1 must be selected. More flexible compatibility conditions, such as given by $\mathcal{A}^l$, permit exploring optimality by selecting any desired choices from $\mathcal{D}$ and $\mathcal{M}$, but it might be necessary to appropriately adjust the sales plan to create a compatible combination.

While "step-by-step" strategies and their variants are common, these methods can generate inefficiencies. One must wonder whether the earlier path-dependency behavior, with its inherent inefficiencies or wrong conclusions, holds for the general rules addressed in this paper. It does.

To demonstrate this path dependency phenomenon, it can be shown that, with a relabeling of the indices if necessary, with seven components an $\mathcal{A}$ includes combinations where all but one subscript is "1," while the other one is a "2;" e.g., the following three plans are in $\mathcal{A}$ where $(a_1, b_1, c_1, d_1, e_1, f_1, g_1)$ is an incompatible combination. (The choices mimic Eq. 1.)

$$\begin{array}{ccccccc} a_1 & b_1 & c_1 & d_1 & e_1 & f_1 & g_2 \\ a_2 & b_1 & c_1 & d_1 & e_1 & f_1 & g_1 \\ a_1 & b_2 & c_1 & d_1 & e_1 & f_1 & g_1 \end{array} \tag{4}$$

To create a compatible outcome, select an order in which choices are made; over each component, make an "optimal" choice. The choices for any remaining components are selected to ensure a compatible outcome. Just as with Eq. 1 where each of the seven alternatives could be judged as "optimal" depending on the order of comparisons, by using different orders in which this "step-by-step" decision process is carried out, *seven* different outcomes emerge; they differ by which alternative has the subscript 2! For instance, if the process starts with selecting an a_j and working down through the alphabet, the selected entries are $a_1, b_1, c_1, d_1, e_1, f_1$ until the g_j choice is to be made. To be compatible, g_2 must be selected leading to the $(a_1, b_1, c_1, d_1, e_1, f_1, g_2)$ outcome. To have an outcome where c_2 must be selected, use the (d, e, f, g, a, b, c) order; the "optimal" outcome is $(a_1, b_1, c_2, d_1, e_1, f_1, g_1)$. It is unlikely that all seven different outcomes are equally "optimal," or that all seven capture the appropriate behavior. In general, expect inefficiencies because path dependency is an unavoidable problem with many generalized decision rules.

Illustrations are easy to find. For instance, it is common for a lecturer on optimal engineering practices to put forth a complicated path diagram demonstrating the order in which decisions are made. The above comments forces one to wonder whether "optimality" truly has been found or whether we are experiencing consequences of "path dependency." Similarly, when questioning experts in multiscale design in engineering, a commonly described course of analysis is where individual forces of physics determine portions of the final outcome, and then adjustments are made to achieve a compatible outcome. This description, of course, closely resembles the above example warning of the dangers associated with path dependency.

5 Final comment

The conditions of the theorem are not demanding; they are the kind we expect in economics, physics, engineering, organizational design, and so forth. Moreover, the ways in which outcomes often are determined satisfy the "expert" or "special physical force" conditions described above. As such, inefficiency and impossibility can be an accompanying feature. Elsewhere more general results that have been developed will be described.

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