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Functional Equations Arising in a Theory Of Non-Commutative Joint Receipt
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The functional equations arise from a model of utility. Assume a preference weak order $\succsim$ over binary gambles and over their joint receipt (or possession), $\oplus$, coupled by a simple interlock called segregation. Earlier work assumed a familiar rank-dependent utility representation (RDU) *U* and the commutativity and monotonicity of $\oplus$. This led to a specific form for *U* over $\oplus$ and from that associativity of $\oplus$ followed. Recent psychophysical data (unpublished) concerning an alternative interpretation of the mathematics failed to support commutativity, which led us to investigate what happens when it is dropped. The present paper replaces commutativity with a quite general form for $\oplus$ and also replaces RDU with a more general form. From these, the following functional equations are arrived at:

$$\xi(s,v)\phi(w) = \phi\left(\frac{w\,\xi(s,v)}{\xi(sw,v)}\right) \qquad (s \in [0,k[, v \in]0,\infty[, w \in [0,1[)$$

and

$$G[vF(z)] = A(v)G(vz) + G(v) \qquad (v \in [0,\infty[, z \in [0,\infty[).$$

The former is solved under monotonicity assumptions and leads to the latter equation, which is solved with added differentiability assumptions. An open problem is to solve it without the differentiability assumptions. The resulting non-commutative case leads to $\oplus$ being bisymmetric and so to a utility representation that is weighted additive form.