A new axiomatic version of set theory is presented that is based on an extension of Aristotle's notion of "potential infinity." Like in several other set theories in the literature, sets are distinguished from proper classes. The set portion of the new theory is very much like the set portions of other theories. However, the proper class portion is very different. Like in other set theories, sets are classes and classes correspond to the extension of properties. However, in the new theory, proper classes and sets are distinguished by their behaviors as the potentiation process unfolds: After some point in the potentiation process, some properties always produce the same extension, while other properties only produce extensions that vary with the unfolding of the potentiation process. Those that after a point in the potentiation process have constant extension are the "sets," and those that produce extensions that vary with the unfolding process are the "proper classes." In the context of the new theory, various ideas and controversies about the nature of classes and sets are discussed.