We present a scheme that uses quantum catch data from reflected lights to recover simultaneously surface reflectance functions and illuminant spectral power distributions. This linear method works in situations beyond those captured by the two-stage recovery procedures analyzed by D'Zmura and Iverson. We determine conditions under which bilinear models can be used to recover color properties uniquely with the new procedure, and formulate an algorithm for checking whether a particular bilinear model provides perfect color constancy. Results of checking color constancy problems are presented, so completing the classification of linear methods for color constancy begun earlier. Polychromatic visual systems can use this general linear recovery scheme to determine spectral descriptions of arbitrarily high dimension.