Dynamic Games among Traders in the Online Commerce Market

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Abstract

This paper studies dynamic games among traders through the reputation system in the online commerce market. The first part of the paper investigates how institutions and the reputation mechanism help to sustain trust in the online market. Using eBay as an example of online commerce market, we develop game theoretical models of asymmetric information on product quality and show that the reputation system can support trust if all the reputation reports are observable. By changing reporting costs, the reputation system has different effect on deterring fraud. As mentioned in the literature on online trust, the lack of incentives to report is one of the main reasons for observing partial reputation reports. In the second part of the paper, we designs a mechanism to solve this problem. We suggest that eBay creates an option for sellers to compensate the buyers’ cost on providing reports. We show that there exists a pooling equilibrium where both good type sellers and bad type sellers will choose this rebate option under certain conditions. The sellers’ types are revealed through those reports. This mechanism also helps to induce bad type sellers to put effort in the model of both moral hazard and adverse selection.

1 Introduction

Coase (1988) points out that “fraud increases the profit of the defrauding firm but reduces custom, thus reduces its future business; while in a highly mobile society, it is obvious that there is likely to be less honesty.” The problem of developing trade among remote traders has been discussed for as old as remote trade itself. Possible solutions include law enforcement, judicial institutions and community enforcement. In many cases,
the value of the transactions is too low to be worth settling in court, and in some cases institutions are only responsible for information. In the absence of law or institution enforcement, reputation plays an crucial role in supporting trust in remote trade. Greif (1993) shows that a reputation mechanism among economically self-interested individuals enabled eleventh century Mediterranean merchants to deal with the trust problem in remote trade in southern Europe. The law merchant studied by Milgrom et al. (1990) was a medieval institution of merchant judges and private dispute resolution, which allowed reputations of traders to be developed and spread to other potential traders in the future.

In today’s world, the Internet has changed the long-term relationships in the brick-and-mortar world transactions. The online commerce markets, especially the online auction market, shares essential features of remote trade. Buyers and sellers are remote and anonymous from one another. They know no more about each other than what they see online, the transactions tend to be geographically diffused, and it is very easy to exit and enter these markets by changing an online ID. So, what makes traders believe they can trust the trading partners to provide the service or payment as they promised per agreement made in Cyber-Space? How does the online commerce markets survive in the face of these trust issues?

As the IBM 2002 Global Service Executive Technology Report pointed out, “the value of e-business is fundamentally tied to achieving the trust that allows us to rely on electronic information transmitted over the Internet...specific services must be put in place to establish and help ensure trust before the full potential for e-commerce, collaboration, electronic markets and dynamic partnering can be realized.”¹ Brown and Morgan (2006) point out that “Unlike conventional markets which exhibit network effect, electronic marketplaces face an enormous ‘trust problem’ which may limit their growth.” There is an enormous amount of literature on trust in the online markets, for example, Jøsang et al. (2005), Bolton et al. (2005), Bolton et al. (2003), Wang and Emurian (2005), Dellarocas (2004), Ba et al. (2002), Kinateder and Rothermel (2004), Shankar and Sulta (2002). Briefly stated, all of them discuss various issues regarding trust in the online market.

According to a Federal Trade Commission(FTC) study (Anderson (2005)), online auction fraud complaints made up 41,796 out of 180,000 total complaints filed to the FTC from January 2005 to June 2005, and it consistently ranked near the top of the list for all fraud complaints filed to the FTC from 2000 to June 2005 in a row. The FTC report shows that the dollar value amount of online auction fraud increased from 2.42 million

in 2002 to 3.27 million in 2003. In 2004, the dollar value of the complaints on online auction fraud was 3.88 million which comprised 61 percent of the total dollar value of fraud complaints filed to the FTC. Based on a content analysis of a random sample of internet auction complaints filed in Consumer Sentinel section of the FTC report during calendar year 2004, “Item not received” was the top reason for filing a complain, which comprised 74% of those online auction complains. This was followed by “quality of the item” and “Payment not received” which made up 16% and 6% of the filed complains, respectively. According to the complaints filed in 2004, the items which cost between $251 to $500 have the highest reporting rate, 17.6%, of all the online auction complaints, followed by the items costing $1001 to $2500 which came in at 15.2%. The items which cost $51-$100 and $501-$1000 each accounted for 12.8% of the complaints, followed by 12% for the items ranging from $26 to $50. The items priced between $11 and $25 and over $2500 each made up 6.4% of the filed complaints.

In spite of increasing online auction fraud, the online auction market is still thriving. According to a ACNielsen study on Global Consumer Attitudes Towards Online Shopping in October 2005, more than 627 million people have shopped online, including over 325 million within September 2005 alone. The latest report about online auction by Forrester Research forecasts that online consumer auction sales will reach $65 billion by 2010, accounting for nearly one-fifth of all online retail sales. eBay, the largest online auction site in the world, has 180.6 million registered users, and 71.8 million of them are active users. In 2005, eBay generated consolidated net revenues of $4.552 billion, a 39% increase over the $3.271 billion in 2004. There were 1.9 billion items listed in 2005 alone, and the Gross Merchandise Volume, the total value of all successfully closed listings on eBay’s trading platforms, reached $44.3 billion. eBay’s major competitors Amazon and Yahoo, also launched auction sites in the late 1990s. These three are ranked in the top 10 web sites by the Nielsen/NetRatings.

One critical reason for the success of these online auction sites is the use of online feedback forum as a reputation system to help sustain trust in online market (Resnick and

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3Cumulative total of all users who have completed the registration process on one of eBay’s trading platforms.

4All users, excluding users of Half.com, Internet Auction, Rent.com, Shopping.com, and eBay classified websites, who bid on, bought, or listed an item within the previous 12-month period. Includes users of eBay EachNet in China and eBay India since the migration to the eBay platform in September 2004 and April 2005, respectively.


Zeckhauser (2002), Shankar and Sulta (2002), Ba et al. (2002), Wang and Emurian (2005), Dellarocas (2004), Dellarocas (2005), Jøsang et al. (2005), Brown and Morgan (2006)). Resnick et al. (Forth Coming) provides several advantages for online market to establish reputation: First, any information that is gleaned can be tallied at very little cost on a continuing basis, and written assessments can readily be assembled. Secondly, that information can be transmitted at little cost to millions of potential customers (comparing with word-of-mouth). Third, the Internet could be used for sophisticated processing of information for the consumers. However, online auction fraud still accounts a significant proportion of the complaints filed to the FTC. It is natural for online auction business to ask “how to reduce online auction fraud?” Since the reputation system is an important factor in solving the trust problem in online markets, these markets could reduce the fraud by improving the current reputation system. Thus, we are interested in investigating how well the current reputation system is, what the weaknesses are in the system, and what the possible solutions are.

The layout of this paper is as follows: In section 2, we review the literature on asymmetric information and reputation models. In section 3, we exam the current reputation system in eBay as an example of online market reputation system\(^7\), and then identify potential problems with the system. In section 4, We use simple game theory to model trust problem in trade, and examine the effect of institution alone on inducing cooperation among traders. We provide the benchmark model of reputation system, and propose an incentive mechanism to overcome the lack of incentives to report problem in section 5. We provide the conclusion and possible extensions in section 6.

2 Literature Review

Asymmetric information on quality of product or sellers has a tremendous impact on the market exchange. Akerlof (1970) discusses that in the used car market low-quality products and sellers will drive out high-quality products and sellers if there is a large amount of information asymmetry. Klein and Leffler (1981) develop an analytical model that shows that cheating behavior still could exist when the profit from cheating is greater than the profit from lost future sales due to reputation effects. Shapiro (1982) discusses how and when product quality is reduced if buyers cannot be fully and accurately be evaluated before the purchase in a monopoly market. Shapiro (1983) extended Klein and Leffler (1981)’s model and relaxed their assumption of perfect communication between customers which has an impact on future contracts. Kauffman and Wood (2000) provide

\(^7\)Li (2006) provides an analysis on different rating system in different online auction market, and shows that eBay’s rating system is more reliable than the others.

Since most online transactions require buyers to pay first, then sellers send the product, thus we will only consider the case where sellers have incentive to commit fraud. As we have noted previously, most people who complain to the FTC about Internet auction fraud report problems with sellers who: “fail to send the merchandise,” “send something of lesser value than advertised,” “fail to deliver in a timely manner,” “fail to disclose all relevant information about a product or terms of the sale.”

In general there are two types of asymmetric information models: adverse selection and moral hazard. Using bilateral trade as an example: in adverse selection models, nature begins the game by choosing the sellers’ type (e.g., some sellers are more capable or honest than others), unobserved by buyers. A seller and a buyer then agree to a contract, and the seller behaves according to his type (e.g. honest type sellers consistently provide the product as promised, whereas dishonest type sellers often provide inferior product or fail to deliver the product). With the moral hazard model, a buyer and a seller begin with symmetric information and agree to a contract, but then the seller takes an action unobserved by buyers (e.g., the seller has incentive to undercut quality of a product to maximize his profit). Reputation mechanism plays different roles in these two settings. Dellarocas (2003a) points out that in the adverse selection setting, the role of the reputation mechanism is to help the community learn the (initially unknown) attributes of community members (such as their ability, honesty, etc.); while in moral hazard setting, the objective of reputation mechanisms is to promote cooperative and honest behavior among self-interested economic agents by the threat of future punishment (e.g., in the form of lower bids following the posting of a negative rating on a trader’s reputation profile) to induce cooperation. Cabral (2005) summarizes two typical reputation mechanisms that lead to economic notions of trust and reputation. The typical reputation

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8There are several problems with buyers, such as bid siphoning, second chance offers, bid shielding, and shill bidding. For more information, please see: http://www.ftc.gov/bcp/conline/pubs/online/auctions.htm
mechanism models are to elicit “reputation” (it is the situation when agents believe a particular agent to be something) are based on Bayesian updating of beliefs and possibly signaling. This so called “Bayesian Mechanism” models feature adverse selection, for example, Klein and Leffler (1981) and Shapiro (1983). The other essential reputation models to elicit “trust” (it is the situation when agents expect a particular agent to do something) are based on repeated interaction and the possibility of “punishing” off-the-equilibrium actions. These so called “Bootstrap Mechanism” models feature moral hazard, for instance, Kreps and Wilson (1982), Milgrom and Roberts (1986), Diamond (1989).

Reputation and trust have been studied in numerous literature. Cabral (2005) provides an overview on research related to economics of reputation and trust; MacLeod (2006) conducts a survey on enforcement of incomplete contracts, and focuses on comparing legal enforcement to enforcement via relationships and reputation; Jøsang et al. (2005) provides a survey of Trust and Reputation Systems for Online Service Provision. Reputation is also studied as a attribute of a firm in several papers. Tadelis (1999) develops a model which considers reputation as a tradeable asset, and no such equilibrium exists in which only good types buy good names under adverse selection. Mailath and Samuelson (2001) examines reputation as a commitment device for firm to solve moral hazard problem, and it shows that relatively capable firms tend to buy medium reputations, leaving high reputations to be acquired and utilized by less capable firms. Brown and Morgan (2006) conduct a case study on eBay to investigate the market of reputation in the online auction market and find that there are sellers to “buy” reputations online.

3 Current Online Market Reputation System

Since eBay is the dominant online auction site, and Li (2006) finds that eBay’s binary rating system is more reliable than Amazon’s 1-5 points rating system, based on this finding, we use eBay as an example to study the online auction market reputation system.

3.1 How does eBay’s current reputation system work

eBay might be best thought of as an e-commerce web site which provides a “virtual” flea market of new and used merchandise from the world of buyers and sellers to trade via auctions. eBay provides solely a platform for sellers listing and bidders bidding, eBay launched Set-price site, eBay Express (http://www.express.ebay.com/), in April 2006 to supplement bid-and-wait online auctions. From Wall Street Journal April 24, 2006.
and it plays no role in the actual exchange of items at the end of the auction. The winning bidders and sellers complete the transaction by themselves. In order to increase trust and help facilitate transactions among strangers, eBay uses a feedback system. The founder of eBay, Pierre Omidyar, announced the initiating of a feedback system on eBay’s AuctionWeb www.auctionweb.com on Feb 26, 1996. After each transaction is completed, the seller and the winning bidder can send feedback about the other party to eBay. The rating can be +1 (positive), 0 (neutral), and -1 (negative), along with brief textual comments. Each trader has a profile which contains this reputation information.

As in Figure 0, there are several summary statistics in the current member’s profile. “Feedback Score”, which always appear next to the trader’s ID, represents the number of eBay members who have completed a transaction(s) with this particular member. The score is usually the difference between the number of members who left a positive rating and the number of members who left a negative rating. If a member has had several transactions with the seller and leaves more than one positive rating, eBay will still only count it one time. In the example shown above, the feedback score is 3531 - 3 = 3528. “Positive Feedback” represents positive ratings left by members as a percentage. “Members who left a positive” and “Members who left a negative” represents the number of unique members who have given the seller a positive rating or a negative rating respectively, and they do not double count repeated ratings from the same member who has given the same rating. “All positive feedback received” represents the total number of positive feedback received for all transactions, including repeat trade partners. “Recent ratings” table shows all of the ratings left for this member during the past month, 6 months, and 12 months. In addition, eBay also provide the entire feedback record with the information on both the seller and the buyer, time of the comment, transaction ID and textual comments.12

When eBay’s feedback system launched in 1996, it did not contain so much information about a trader’s reputation. Here are the changes eBay made after the introduction of feedback system:

1. In 1999, eBay moved away from non-transaction based feedback by preventing mem-

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10 It is a dead link now, but you can see an early version of the AuctionWeb home page at http://forums.ebay.com/db2/thread.jspa?threadID=410122850. eBay was launched on the Labor day of 1995.

11 The transaction data can only be stored on eBay for 90 days, so it is impossible to track the transaction and see the product and the value of the product for the duration of the transaction.

12 The information was accessed from eBay web site at http://pages.ebay.com/help/feedback/evaluating-feedback.html
bers from leaving negative non-transactional feedback. By March 2000, all feedback became transaction based.

2. From June 16th, 2001, eBay introduced buyer/seller labels on the Member Profile page, which helped people distinguish the context in which a member had received feedback.

3. In January 2003, eBay introduced the Seller Information Box on the item page. This snapshot view of a seller contains information about the seller’s feedback score and positive feedback percentage.

5. Also in 2003, eBay introduced an additional page that members had to read prior to leaving a neutral or negative feedback comment in order to make sure members understood the impact of leaving negative or neutral feedback.

6. On March 1st, 2003, eBay began reporting “Percentage of Positive” which is the ratio of positives received by the seller in her entire ebay history, “Seller’s Age” which is the date when the trader registered on eBay.

7. On Feb 9th, 2004, ebay modified the Feedback Removal policy to provide members the ability to mutually withdraw feedback.

8. From September 20th, 2005, eBay removed the feedback left by the users who are indefinitely suspended within 90 days of registration.

9. Late 2005, eBay added two more changes. One was neutralizing feedback left by members who don’t participate in the issue resolution processes, and another requiring new members to complete a tutorial before leaving neutral or negative feedback.

3.1.1 How Well is the Online Reputation System Working?

There are many empirical papers that try to quantify the market value of reputation and quantify distortion from asymmetric information in the online auction market. Resnick et al. (Forth Coming) and Bajari and Hortacsu (2004) provide a very good survey of empirical analysis of the reputation mechanism used by eBay. Enriching the literature review table in Resnick et al. (Forth Coming) and Bajari and Hortacsu (2004), we provide a summary of literature on the feedback effect on price in the attachment.
All of the papers try to empirically examine the effect of a seller’s feedback profile on prices. Besides addressing this issue, some look for the component of eBay’s feedback profile that can better explain buyer behavior (Ba and Pavlou (2002), Lee and Malmendier (2005), Dewan and Hsu (2004)); some of them also examine the effect of a seller’s feedback profile on the probability of sale (Eaton (2002), ?). In addition, many empirical studies of eBay’s reputation mechanism tend to focus on the buyer’s response to the published feedback aggregate, some papers also investigate the effect of a seller’s feedback profile on the probability of sale (Bajari and Hortacsu (2003), Eaton (2002), Livingston (2005), Resnick and Zeckhauser (2002)). Among the literature, only two papers Cabral and Hortacsu (2006), and Jin and Kato (2005)) focus on the seller’s equilibrium behavior by incentives created through eBay’s feedback system.

Most of the results show that positive feedback increases the price of the item, and negative results decrease price or probability of the sale (Melnikt and Alm (2002), Dewan and Hsu (2004), House and Wooders (Forth Coming), Kalyanam and McIntyre (2001) Cabral and Hortacsu (2006)), only a few papers show that no significant feedback on price (Lucking-Reiley et al. (2000), Kauffman and Wood (2000)). Resnick et al. (Forth Coming) point out the omitted variable problem for other researches, and use controlled experiments to hold constant quality of goods, skill at listing, responsiveness to inquiries, and all potential confounds in previous observational studies. However, the experiment did not control for the potential confounds of private reputation information and volume of seller listings. They found that buyers are willing to pay 8.1% more for pairs of lots-batches of vintage postcards - from an established seller rather than new venders. A subsidiary experiment followed the same format, but compared sales by relatively new sellers with and without negative feedback. Surprisingly, one or two negative feedbacks for new sellers did not affect buyers’ willingness-to-pay. Although Resnick et al. (Forth Coming) find that a seller with a strong reputation received a price premium, it is not clear whether the price premiums reflect a reputation equilibrium. As summarized in Dellarocas (2003a), the principal conclusions derived from a collective reading of these works are:

- Feedback profiles seem to affect both prices and the probability of a sale. However, the precise effects are ambiguous; different studies focus on different components of eBay’s complex feedback profile and often reach different conclusions.
- The impact of feedback profiles on prices and probability of sale is relatively higher for riskier transactions and more expensive products.
- Among all the different pieces of feedback information that eBay pub-
lishes for a member, the components that seem to be most influential in affecting buyer behavior are the overall number of positive and negative ratings, followed by the number of recently (last seven days, last month) posted negative comments.

3.1.2 The Weaknesses of the Online Current Reputation System

There are several potential problems regarding the current reputation mechanism systems in the online markets. Resnick and Zeckhauser (2002) report some interesting properties about the feedback score by using the data from eBay during Feb 1, 1999 to June 30, 1999. 1. Most trading relationships are one-time deals: 89% of all buyer-seller pairs conducted just one transaction during the five-month period covered by the data set. 2. Buyers left feedback on sellers 52.1% of the time; sellers on buyers 60.6% of the time. 3. Feedback is overwhelmingly positive; of feedback provided by buyers, 99.1% of comments were positive, 0.6% was negative, and 0.3% was neutral. First, from these statistics, we can tell that the participation rate of leaving feedback is not very high, once the transaction being completed, the transaction partners usually have no direct incentive for leaving feedback about the other party, and there might exist free-riding problems in the community, such as that some traders read reports but not leave reports (Dellarocas et al. (2003)). Another explanation is that there are costs associated with reporting, and it might be the opportunity cost of typing comments, or it might be the possibility of being retaliated against by the other party if one were to leave a negative message. Second, the feedbacks are bias towards the positive. This might be because of an exchange of courtesies or avoiding retaliation (Resnick and Zeckhauser (2002)). Third, because of the change of identity issue, bad reputation traders can easily change their online ID and start over as a new member. There are also many other problems, for example, there exists a market of reputation in which traders can manipulate their reputation score by participating in the market on eBay (Brown and Morgan (2006)), abuse of the reputation system such as unfair rating, ballot stuffing (a seller colludes with other buyers to undertake fake transactions in order to enhance her reputation), bad mouthing (a seller is targeted by a group of buyers who deliberately lower her reputation). In this paper, we focus on solving the first three problems, especially focusing on solving the lack of incentive problem.
4 Model

4.1 Basic Model

As we know, online Consumer-To-Consumer (C2C) transactions are anonymous, easy to enter and exit, and geographically diffused. We use a very simple model to capture the trust problem in C2C transactions. Since buyers usually first pay, and then sellers send out the product, sellers have more incentives to deviate. In this paper, we only consider the problem of sellers who commit fraud.

In order to capture the essential feature of the trust problem, we look at a situation of a fixed trading pair, a seller and a buyer, and the price is fixed by the seller. The stage game is shown in the following Table 1, where the set of player: \( I = \{ \text{buyer (b)}, \text{seller(s)} \} \); the set of strategies for the buyer is \( S_b = \{ \text{buy (B), not buy (NB)} \} \); the set of strategies for the seller is \( S_s = \{ \text{send(S) if the buyer buys, send(S) if the buyer does not buy, not send (NS) if the buyer buys, not send (NS) if the buyer does not buy} \} \). We use the most extreme case of deviation for sellers in this basic model, and we will relax it in our benchmark model later.

[ Insert Table 1 Here ]

The extensive form of the game is as in figure 1.

[Insert Figure 1 Here]

Where \( V_b \) is how much the buyer values the product, \( P \) is the settled price, \( V_s \) is how much the seller values the product, and \( V_b \geq P \geq V_s \). In each period, the buyer (b) and the seller (s) choose the strategies to optimize their utilities. We assume players are risk neutral, so the utility is proportional to the payoff. Here, we can think \( V_b - P, P - V_s, -P \), and \( P \) represent utility levels.

The buyer does not buy and the seller does not send, \( (\text{NB, NS}) \), is the weak Nash Equilibrium in this basic stage game. According to game theory, we would not expect any trading between buyers and sellers at the Nash equilibrium. Cooperation (The seller chooses S) could be reached if the game repeats infinitely or indefinitely (players do not know when the game ends, but know the probability of ending the game). Suppose the seller’s discount factor is \( \delta \in (0, 1) \), and the buyer uses a trigger strategy where he won’t deviate (not buy) until the seller deviates and thereby continues to deviate. Here is a
standard result from folk theorem.

**Proposition 4.1** If the basic game is played infinitely or indefinitely, the buyer uses the trigger strategy and the seller’s discount factor is \( \delta > \frac{1}{P} = \delta^* \), there exists an equilibrium where the buyers will buy and the sellers will send the product.

Proof: see appendix A.

A lot of experimental results show that when players play repeated non-cooperative games in fixed pairs (play with the same opponents for every period), the cooperation is easier to sustain; while in randomly matched pairs, cooperation is more likely to collapse. (Schmidt et al. (2001), Duffy and Jack Ochs). One crucial reason is that the information about the opponents’ past history is fully revealed in the case of random matched pairs. There are two ways to induce players to cooperate when they are randomly matched, one is through institutions (Milgrom et al. (1990), Hill (2004)), and another one is through the reputation mechanism (Kadori (1992), Schmidt et al. (2001), Duffy and Jack Ochs, and Bolton et al. (2003)). Bolton et al. (2003) tested the cooperation level under the first order information (what players did last time) and the second order information (what the player’s rival did before he met this rival, and what players did the last time), the results suggest that the more information about reputation, the more cooperation. To distinguish effect from institutions and the reputation system, we examine them separately.

### 4.2 Basic Model with Institutions

We put institutions into the basic game, and the institutions in this model do not play any role of spreading the sellers’ reputation. The institutions can be the legal system or the online market site, or both. We also assume the fixed price and fixed pair of a seller and a buyer. After the transaction, the buyer can report to the institution about the transaction, if he does not receive the product, then he will get a form of compensation from the institution, and the seller will be punished by the institution. The set of strategies for the buyer is buy and report (BR), buy and not report (BNR), not buy and report (NBR), not buy and not report (NBNR), the set of strategies for the seller is \( S_s = \{ \text{send(S) if the buyer buys, send(S) if the buyer does not buy, not send (NS) if the buyer buys, not send (NS) if the buyer does not buy} \} \)

The graph 2 shows the basic model with institutions.
Where $C$ - Reporting cost/reward for the buyer. If $C > 0$, reporting is costly; if $C = 0$, it is costless to report; if $C < 0$, the buyer gets reward if he reports. $\gamma \in [0, 1]$ represents the level of compensation to the buyer if he is defrauded. There are two cases: one is that the compensation is provided by eBay, so it’s a type of insurance provided by eBay; another one is that the if cheating seller is caught and he will refund the buyer. $\phi \in [0, 1]$ - represents the level of punishment for the cheater. It can be imposed by either eBay or the legal system. So far, eBay does not have this kind of punishment for the cheaters. We include this element into the model to see how it can induce people to be more cooperative. New path to think about $\phi$ is that it can be interpreted as the probability of being caught and paying a fine or going to prison. In order to simplify the model, we assume that the buyer reports in accordance with the truth. It means that if the seller cheats, the buyer will report the cheating, and will not report anything negative if the seller is honest. To have some insights on how the reporting costs affect cooperation, we consider three situations where the reporting costs are 0, positive and negative, respectively.

### 4.2.1 Case 1: There is no reporting cost (i.e. $C = 0$)

The game tree is simplified as shown in figure 3.

[Insert Figure 3 Here]

The set of strategies for the buyer is buy and report (BR), buy and not report (BNR), not buy and report (NBR), not buy and not report (NBNR); the set of strategies for the seller is send (S), not send (NS).

If $P - \phi P > P - V_s$, there is a SPE (short for "Sub-game Perfect Equilibrium") in this stage game: (NBR, NS). If $P - \phi P < P - V_s$, then the SPE of the stage game is (BR, S). We are interested in the case where the stage game SPE is (NBR, NS). One way to have cooperation is to repeat the game. Using the similar argument in the baseline model, we can assume that the game is played infinitely/indefinitely, and the reporting is costless in this case, the buyers would choose to report if they are cheated by the seller. Reporting becomes a threat to the seller, and the SPE for the repeated game is $(\sigma_{BR}, \sigma_s) = (BRBR..., SS..)$.

**Proposition 4.2**: If the basic game with institutions is played infinitely/indefinitely, and the buyers use the trigger strategy, and the discount factor of the seller is $\delta_s > \frac{V_s}{P(1-\phi)} - \frac{\phi}{1-\phi} = \delta^{**}$, then there exists a unique equilibrium where the buyer will buy and report, and the seller will send.
Comparing the basic model and the basic model with institutions, it is clear to see that adding the institutional instruments reduces the requirement of cooperation for sellers, so it reduces fraud in the market if there are many sellers with different discount factors.

**Corollary 1**: Adding institutions into the basic model will deter more sellers from cheating than in the basic model alone. Also, if the punishment for cheating (larger $\phi$) is higher, there is a larger range of discount factors at which cooperation is sustainable.

Proof: See Appendix C

### 4.2.2 Case 2: There is a cost of reporting, i.e. $C > 0$

The cost of reporting can be the time spent on writing a report or the opportunity cost of doing such reports. It is natural to think that there is a cost of reporting. If there is a reporting cost for the buyer, the game will look like in figure 4.

![Insert Fig 4 Here]

1. If $\phi P < V_s$ and $C < \gamma P \leq P$, the SPE is (NBR, NS).
2. If $\phi P > V_s$ and $C > 0$, the SPE is (BNR, S).
3. If $\gamma P < C$, the SPE is (NBNR, NS).

Comparing (1) and (2), if $\phi P$ is large, the seller will send the product, and the buyer will buy. It makes sense because $\phi P$ is the utility loss due to the punishment for cheating. If the punishment is greater than the value of the product to the seller, the seller will not choose to cheat. Comparing (1) and (3), if the $C$ (reporting cost) is low, the buyer will not report.

### 4.2.3 Case 3: There is a Reporting Reward (Negative Cost), i.e. $C < 0$

If eBay provides some incentives to encourage buyers to report, then the reporting offers rewards to the buyers. In this case, the game tree looks the same as in figure 4 and only the value of $C$ changes from positive to negative. Under this setting, let us look at the potential SPEs.

1. If $\phi P < V_s$ and $0 < \gamma P \leq P + C < P$, the SPE is (NBR, NS).
2. If $\phi P < V_s$ and $C + P < \gamma P \leq P$, the SPE is (BR, NS).
If $\phi P > V_s$ and $V_b - C > P$, the SPE is (BR, S).

If $\phi P > V_s$ and $V_b - C < 0$, the SPE is (NBR, S).

We compare cases (1) and (2): no matter how great the punishment, the only thing that affects the buyer’s decision is the compensation of loss. The larger the compensation is (i.e. the larger value of $\gamma P$ is), the more likely they are to buy. That may be why eBay needs to provide insurance to attract consumers to buy. Comparing (1) and (2) with (3) and (4), we see that the only thing which makes sellers cooperate is the value of $\phi P$ (i.e. the punishment for cheating). It doesn’t matter how much compensation is offered to the buyer or reporting reward.

Comparing the results from the three cases (1) No Reporting Cost (2) Reporting Cost (Positive Reporting Cost), and (3) Reporting Reward (negative reporting cost), we find that only substantial punishment to the seller that can induce cooperation. For the buyers, the promise of a compensation for the loss greatly affects the buyers’ decisions. All the SPEs are listed in table 2.

[Insert table 2 here]

The interesting thing is that in the case (C) Reporting Reward setting, the buyer buys and report and the seller does not send the product (BR, NS) is an equilibrium where the buyer pays for the product, but doesn’t receive the product. This equilibrium is the one we do not want to appear at all. Many theory papers suggest that the more complete the information about players’ history, the more cooperation can be achieved. In order to have more complete information about the seller, we need more buyers to report. One affective way to induce buyers to report is to give them rewards. However, if the reward is too high, there may appear the undesirable equilibrium.

In reality, traders are more likely to meet randomly in the online market. Reputation plays a critical role on transferring information of the past history among traders through the reputation system. The next section will focus on investigating the effect of reputation system.
4.3 Benchmark Models with the Reputation System

4.3.1 A Brief Review on the Reputation Mechanism Design

Much of the literature, institutions play a role in distributing reputations about traders, e.g. Milgrom et al. (1990), Hill (2004). Here, we exclude this aspect, and only focus on the reputation effect through the online reputation system, and see how the online market can be self-sustained by traders without institutional intervention.

As we see in the online auction market, the reputation system helps to sustain trust and cooperation among traders. However, current online reputation systems are not perfect: From the FTC reports regarding online auction complaints, online auction fraud complaints consistently rank near the top of the list of all fraud complaints filed to the FTC from January 2000 to June 2005 consecutively. There are several major problems about the online reputation system, such as: low incentive for providing a rating; there exists bias toward positive ratings; abuse of reputation system, and ease to change identities. Given these problems of the online reputation system, the natural objective would be to think of solutions for these issues. This paper focuses on solving the problem of the lack of incentives to provide reports.

Since the online reputation system control the form of information they publish, aggregation information format, and what information is available for public, it is important to design an incentive compatible mechanism to elicit truthful feedback. Dellarocas (2003a) provides the two most concrete evaluation criteria of a feedback mechanisms performance: (1) the expected payoffs of the outcomes induced by the mechanism for the various classes of stakeholders over the entire time horizon that matters for each of them, and (2) the robustness of those outcomes against different assumptions about the participants behavior.

There is much literature pertaining to the topic of mechanism design. Resnick et al. (2000) and Dellarocas (2006) provide an overview on the reputation mechanism. Dellarocas (2001) analyzes economic efficiency of eBay-like online reputation mechanisms. Bakos and Dellarocas (2002) compare conventional litigation with the online reputation system as quality assurance mechanisms. Ba et al. (2002) provides an incentive mechanism to build trust in the online auction market. Lin et al. (2003) study reputation, reputation System and reputation distribution in Online Consumer-to-Consumer auctions. Josang et al. (2003) simulate the effect of the reputation system on the e-market, and the simulation confirms the hypothesis that the presence of the reputation system improves the quality of the market. Bhattacharjee and Goel (2005) present a study on the robustness of binary feedback reputation systems to ballot stuffing and
bad mouthing, and they find that an inflation resistant reputation premium ensures that there is no incentive for sellers to fake transactions to enhance their reputations, and transaction costs ensure that a family of reputation premiums are inflation resistant. Dellarocas (2004) introduces a number of novel “immunization mechanisms” for countering the undesirable effects of such fraudulent behavior. Dellarocas (2003b) proposes charging a listing fee contingent on a seller's announced expected quality and rewarding the seller contingent on both his announced quality and the rating posted for that seller by that period's winning bidder. Miller et al. (2005) propose the peer prediction method to elicit honest feedback. Dellarocas (2005) studies eBay-like binary reputation mechanisms with noisy monitoring of quality and pure moral hazard. He suggests considering the missing reporting as good report.

4.3.2 Model Setup - Adverse Selection Model

We set up a benchmark model to capture the essence of the online reputation system. The assumptions are as follows: Suppose there are \( M \) sellers and \( N \) (\( N \) is a very large number) buyers in the entire market. We focus on one auction listing where a seller \( s \) lists a good (\( g \)), and the seller lists the same good (\( g \)) at each period. There are two realizations of the transaction, high quality level (\( Q_H \)) and low quality level (\( Q_L \)). The high quality transaction includes that the good is received by the buyer, the quality of the good is the same as the seller promised, and that the good is shipped on time. The low quality transaction is one which fails in any of just discussed conditions. There are two types of sellers, good type (\( G \)) and bad type (\( B \)). The prior of good type sellers is \( \mu_0 \), and the prior of bad type sellers is \( 1 - \mu_0 \). The probability of a high quality transaction is provided by a good type seller is \( \alpha \), and it is \( \beta \) for a bad type seller (\( 0 \leq \beta < \alpha \leq 1 \)). One example is that the good type sellers are very careful and honest, so they pack the good carefully and ship it on time; while the bad type sellers are lazy and not careful.

In this model, the nature chooses the transaction outcomes for the sellers, so the sellers do not control the transaction outcomes. We will consider the cases where the sellers can choose to put in efforts that affect the transaction outcomes in the session of both adverse selection and moral hazard. For each period, there are \( K \) buyers randomly draw from the \( N \) buyers, the valuation of the good by the buyers \( V_b \)'s are uniformly distributed from 0 to \( V_b \), \( 0 = V_b(1) < V_b(2) < \cdots < V_b(K) \), the buyer \( K \) wins the bidding, and the price is settled at \( P \). We use eBay as an example of the online auction market, and eBay uses the Vickrey auction method, i.e. the winning bidder pays the second highest bid, so \( P = P(V_b(K - 1)) < P(V_b(K)) \). We assume that there are many bidders for each auction\(^{13}\), and a good is worth 1 to the winning bidder \( B \) if it is a high quality

\(^{13}\) We can also use the auction model in Cabral and Hortaçoşu (2006) to show that the winning bid is an increasing function of buyer’s willingness to pay.
transaction \( (Q_H) \), and the good is worth 0 if it is a low quality transaction \( (Q_L) \). After the auction, there is only one seller \( (s) \) and one buyer \( (b) \) in the transaction.

For each good listing, we consider it as one period. There is one seller \( (s) \), and \( K \) buyers random drawn with replacement from \( N \) buyers in the market. The seller lives infinitely, and the buyers only live for one period, and the \( K \) buyers will be replaced by the other buyers in the pool of the buyers. Each period \( t \) consists of a sequence of moves in the following order:
1. Nature chooses sellers type \( \theta \in \{\theta_G, \theta_B\} \). The seller’s type is chosen in the first period, and it persists for the seller for the rest of the game.
2. The buyers choose the bid, and the winning bid equals the highest willingness to pay, \( P \geq 0 \). \(^{14}\)
3. The seller chooses to accept or reject \( p \) based on his reservation price. If he rejects, the game ends. If he accepts, then the game continues to the next step. For simplicity, we assume the good type sellers’ reservation price is \( V^G_S \geq 0 \) and the bad type seller’s reservation price is 0.
4. Nature chooses the quality of the transaction that buyers get from different types of sellers, \( Q_H \) or \( Q_L \). The quality of transaction is a new draw in every period.
   \[ q(\theta) = \text{probability of providing } Q_H \]
   \[ q(\theta_G) = \alpha \]
   \[ q(\theta_B) = \beta \]
   \[ 0 \leq \beta < \alpha \leq 1 \]
5. The Buyer chooses \( \{NR, GR, BR\} \). Buyers can choose to give a good report \( (GR) \), a bad report \( (BR) \), or no report \( (NR) \); the reporting cost is \( C \) for all buyers. Assume buyers report honestly if they decide to report, i.e. \( GR \) for \( Q_H \) and \( BR \) for \( Q_L \).
6. Payoff received for period \( t \).
   \[ U_s(\text{Accept}) = P - 0 = P; \]
   \[ U_s(\text{Reject}) = 0; \]
   \[ U_b(P, NR, Q_H) = 1 - P; \]

\(^{14}\)eBay uses the Vickrey auction, so the winning bidder pays the second highest bid. We assume there are many bidders for each auction, winning bid is only \( \varepsilon \) higher than the second highest bid, and the bid equals the willingness to pay, so we use the highest willingness to pay as an approximate of the second highest bid, and it is the price that the winning bidder will pay to the seller. We assume the highest valuation of the high quality transactions is 1, and 0 for low quality transactions.
4.3.3 Adverse Selection Model Without the Reputation System

If there is no reputation system, buyer can not distinguish the bad type sellers and good type sellers. Since good type sellers provide good transactions with probability $\alpha$, and bad type sellers provide good transactions with probability $\beta$. The prior of meeting a good type seller is $\mu$. The buyers’ willingness to pay is $P = P_1 = \mu_0 \alpha + (1 - \mu_0) \beta$ for every period. If the population of the sellers are fixed at $M$, and eBay’s revenue is an increasing function of sale value, then eBay’s revenue $R_{eBay}$ is proportional to $\mu_0 M \alpha + (1 - \mu_0) M \beta$. If the good type seller’s reservation price is higher than $P_1$, then the good type sellers will not sell on this market, and only bad type sellers stay in the market, so the buyers’ willingness to pay will be $\beta$. If the population of the sellers are fixed at $M$, and $\mu_0$ of them are good type sellers, and eBay’s revenue is an increasing function of sale value, then eBay’s revenue $R_{eBay}$ is proportional to $(1 - \mu_0) M \beta$.

In order to keep the good type sellers ($V_{G}^{S} > P_1$) in the market, we need to provide a means to increase the buyers willingness to pay to those sellers, thus we need the reputation system to help to identify those good type sellers.

4.3.4 Adverse Selection Model With the Reputation System

By allowing the reputation system, in each period, the new buyers (who are new to the seller, but may not be new in the market) observe the reputation history of the seller. First, let us examine the case where there is no reporting cost, i.e. $C = 0$. The buyer’s willingness to pay is $P_{t+1} = \mu_t \alpha + (1 - \mu_t) \beta$, where $\mu_t$ is the prior of meeting a G type seller at period $t$. For instance, at period $t = 1$, the prior is $\mu_0$, the buyer’s willingness to pay is $P_1 = \mu_0 \alpha + (1 - \mu_0) \beta$. If the buyer receives a $Q_{H}$ product and reports $GR$ (we assume buyers not only report but also report honestly if there is no reporting cost) in period $t = 1$, the buyer in period $t = 2$ observes the reports and updates his beliefs on the seller’s type by Bayes’ rule, so the prior in period $t = 2$ is

$$\mu_1 = Pr(\theta_G|GR) = \frac{Pr(GR|\theta_G)Pr(\theta_G)}{Pr(GR|\theta_G)Pr(\theta_G) + Pr(GR|\theta_B)Pr(\theta_B)} = \frac{\mu_0 \alpha}{\mu_0 \alpha + (1 - \mu) \beta}$$

If the buyer receives a $Q_{L}$ product and reports $BR$ in period $t = 2$, the buyer in period $t = 3$ observes the previous reports and updates his beliefs according to Bayes’ rule, so
the prior in the period $t = 3$ is
\[
\mu_2 = Pr(\theta_G | GR, BR) = \frac{Pr(BR | \theta_G, GR) Pr(\theta_G, GR)}{Pr(BR | \theta_G, GR) Pr(\theta_G, GR) + Pr(BR | \theta_B, GR) Pr(\theta_B)} = \frac{Pr(BR | \theta_G) Pr(\theta_G)}{Pr(BR | \theta_G) Pr(\theta_G) + Pr(BR | \theta_B) Pr(\theta_B)} = \frac{Pr(BR | \theta_G) Pr(GR | \theta_G) Pr(\theta_G) + Pr(BR | \theta_B) Pr(\theta_B)}{\mu_0 \alpha (1 - \alpha) + (1 - \mu_0) \beta (1 - \beta)}
\]

Where $Pr(\theta_G) = \mu_0$, $Pr(\theta_B) = 1 - \mu_0$, $Pr(GR | \theta_G) = \alpha$, $Pr(BR | \theta_G) = 1 - \alpha$, $Pr(GR | \theta_B) = \beta$, $Pr(BR | \theta_B) = 1 - \beta$, and the reports are independent with each other, so $Pr(GR, BR | \theta) = Pr(GR | \theta) Pr(BR | \theta)$.

In general, if there are $t_{GR}$ good reports and $t_{BR}$ bad reports about the seller before the period $t$, so $t = t_{GR} + t_{BR} + 1$, the buyer’s prior of meeting a G type seller at period $t$ is
\[
\mu_{t-1} = \mu_{t_{GR} + t_{BR}} = \frac{\mu_0 \alpha^{t_{GR}} (1 - \alpha)^{t_{BR}}}{\mu_0 \alpha^{t_{GR}} (1 - \alpha)^{t_{BR}} + (1 - \mu_0) \beta^{t_{GR}} (1 - \beta)^{t_{BR}}} \tag{1}
\]

Where $t_{GR}$ is the number of periods that the seller gets good reports, and $t_{BR}$ is the number of periods that the seller gets bad reports.

Proof: See appendix C.

Each period, the seller’s payoff is $U_s = P = \mu_t \alpha + (1 - \mu_t) \beta$, the buyer’s payoff is $U_b(P, NR/GR/BR) = 1 - P = 1 - (\mu_t \alpha + (1 - \mu_t) \beta)$. As the game repeats, $\mu$ converges to 1 for good type sellers, and converges to 0 for bad type sellers; buyer’s willingness to pay converges to $\alpha$ for G type sellers and converges to $\beta$ for B type sellers. eBay’s expected revenue is proportional to $\mu_0 M \alpha + (1 - \mu_0) M \beta$ given that there are $M$ sellers in the market.

Secondly, let us examine the case where there is reporting costs on both $GR$ and $BR$. The reporting costs may be time or energy spent on writing reports, or the opportunity cost during that time, or the retaliation by the seller if the buyer leaves a bad report. For simplicity, we transfer all the reporting costs in terms of dollars, and the highest reporting costs for the buyers is $C$.

Apparently, not reporting ($NR$) dominates reporting ($GR$ and $BR$) for buyers, so there is no report about the seller’s previous history and reputation. A buyer can not update her beliefs, so the buyer’s willingness to pay is $P = P_1 = \mu_0 \alpha + (1 - \mu_0) \beta$ for every period $t$. In equilibrium, the buyers’ strategy is $(P_1 = P_1, NR)$, and the seller’s
strategy is (accept if \( P_i \geq 0 \)). If we assume that both type sellers’ reservation price is 0, sellers will always accept the bid. However, the G type sellers get less than what she could get in the case of no reporting cost, and the B type sellers get more than what she could get in the case of no reporting cost. There is a wealth transfer from G type sellers to B type sellers. If G type sellers’ reservation price is higher than \( P_1 \), then only B type seller accepts the bid, thus G type sellers will be driven out of the market, and buyers’ willingness to pay will drop to \( P_i = \beta \), and eBay’s expected revenue will also drop. This is similar to Akerlof (1970)’s lemon car market, good quality cars are driven out by lemons because of asymmetric information of product quality. Here, good sellers are driven out by bad sellers due to the asymmetric information about the seller’s types which determine the quality of the products. Another way to think about it is that if there are two online markets, one has 0 reporting cost, and other one has positive reporting cost, then G type sellers want to get the higher price so that they would move to the markets where there is no reporting cost. As a consequence, there is another separation of buyers. Those who want cheap things and do not care about low quality transactions will stay in the market, and those who care about the quality of transactions will go to another market where there are many good type sellers.

Third, let us see what will happen if there exists asymmetric reporting costs. Suppose there are three sellers in the market, and the following table shows their rating profiles on eBay.

[Insert Table 3 Here]

Seller Ann has 18 positive, 18 neutral and 27 negative ratings, so the overall rating for her is \(-9\); seller Bob has 10 positive, 30 neutral and 9 negative ratings, the overall rating is 1 (According to eBay, the overall rating is the sum of all the positive, neutral and positive ratings); seller Cindy has 21 positive, 1 neutral and 13 positive ratings, so the overall rating for her is 8. If we can observe all the information about their ratings, then we can see seller Cindy has the highest overall rating, and followed by Bob and Ann (i.e. \( Cincy \succ Bob \succ Ann \)). If there exists reporting costs on good reports, \( GR \), then we will not observe \( GRs \), and the overall ranking will reflect that Bob is a better seller than seller Cindy and Ann, respectively (i.e. \( Bob \succ Cindy \succ Ann \)). If there exist reporting costs on bad reports, \( BR \), then the overall ranking will be \( Cindy \succ Ann \succ Bob \). In fact, when we can observe all the information, the overall rating is consistent with the ranking by using Borda Count voting rule; when we can not observe \( GRs \), the ranking is the same as using the anti-plurality voting rule; when we can not observe \( BRs \), the ranking is the same as using the plurality voting rule. Many researches have shown that the Borda Count has the least amount of problems among those voting rules (Saari
and Sieberg (2001), Saari (1999), Saari (2001), Li (2006)). The best case is when we can observe all the reports and sum them up. If buyers know that there are reporting costs, no matter the cost is on GR or BR, they can not update their beliefs by using the information from rating profiles, thus their willingness to pay is still $P_1$. The next section will discuss how to induce buyers to report.

4.4 Benchmark Model with the Reputation System and Incentive Mechanism

If there exist reporting costs, theoretically speaking no buyer wants to report. While in reality, there are still many buyers to report. It might be because altruism, social norms, emotional expression, or that different buyers have different reporting costs (Bolton et al. (2003), Xiao and Houser (2005), Resnick et al. (Forth Coming)). However, as long as there exists some reporting costs to some of the buyers, the information about the sellers reputation profile is not complete. If the buyers are not rational enough and use the incomplete information to make bidding decisions, they might bid higher than what they should bid, so they might then suffer the "winner’s curse". It is important to provide incentives for the buyer to report and make all the information available to everyone. How might this be achieved? One way is to eliminate the reporting costs for the buyers. Will eBay want to compensate buyers for their reporting costs? It seems to be impossible, even providing 1 cent on each transaction, it would be a huge cost to eBay due to the millions of transactions on eBay per day. What about sellers providing incentives to buyers? Will sellers want to compensate the buyers’ reporting costs? What type of sellers would be willing to compensate this costs to the buyers? G type sellers are more likely to sell the same product at a higher price if there are full reports, however B type sellers are more likely to get lower prices if there are full reports available to the consumers. Therefore, it appears that G type sellers would be more inclined to provide a rebate, but not the B type sellers. If so, it seems that there exists a separating equilibrium which can help us to identify the seller’s type. But before we draw this conclusion, we need to do more analysis on equilibria.

To see whether sellers want to provide incentive for buyers to report, we assume that all sellers can choose a rebate, $r$, which is greater or equal to $C$. The game played in every period is described as the following:

1. Nature chooses sellers type $\theta \in \{\theta_G, \theta_B\}$

2. The seller chooses to rebate $r$ or not, where $r > C$, the seller’s actions set is

---

\[^{15}\text{Bajari and Hortaçsu (2004) write "The winners curse occurs when bidders do not condition on the fact that they will only win the auction when they have the highest estimate."} \]
\{Rebate(R_S), NoRebate(NR_S)\},

3. Buyers choose bid, and the winning bid equals the highest willingness to pay, \( P \geq 0 \).

4. The seller chooses to accept or reject \( P \) based on his reservation price. If he rejects, the game ends. If he accepts, then the game moves onto the next step.\(^{16}\)

5. Nature chooses whether \( Q_H \) or \( Q_L \).
   \( q(\theta) = \) probability of providing \( Q_H \).
   \( q(\theta_G) = \alpha \)
   \( q(\theta_B) = \beta \)
   \( 0 \leq \beta < \alpha \leq 1 \)

6. The buyer chooses \( (NR, GR, BR) \). The buyer can choose to give a good report \((GR)\), bad report \((BR)\), or no report \((NR)\); the reporting cost is \( C \) for all buyers. Assume all buyers report honestly if they decide to report, i.e. \( GR \) for \( Q_H \) and \( BG \) for \( Q_L \).

7. Payoff received for period \( t \).
   \( U_s(\text{Accept, } R_S) = P - 0 - r = P - r; \)
   \( U_s(\text{Accept, } NR_S) = P - 0 = P; \)
   \( U_s(\text{Reject, } R_S) = 0; \)
   \( U_s(\text{Reject, } NR_S) = 0; \)
   \( U_b(P, NR, \text{ if } Q_H) = 1 - P; \)
   \( U_b(P, NR, \text{ if } Q_L) = -P; \)
   \( U_b(P, GR, \text{ if } Q_H) = 1 - P - C + r; \)
   \( U_b(P, BR, \text{ if } Q_L) = -P - C + r; \)

To find possible equilibria, we use the guess and verify method to look for PBE(Perfect Bayesian Equilibrium).

First, let’s examine the separating equilibrium where \( G \) type sellers choose \( R_S \), and \( B \) type sellers choose not to report, \( NR_S \). If it is an equilibrium, then buyers can identify the seller’s type by observing whether the seller chooses the rebate option or not. If the seller chooses it, then she is a \( G \) type seller, the buyer’s willingness to pay is \( \alpha + r - C \), and the good seller’s payoff \( \alpha - C \). If the seller does not choose the rebate option, then she is a \( B \) type seller, the buyer’s willingness to pay is \( \beta \), and bad seller’s payoff is \( \beta \). If \( \alpha - C < \beta \), then both good and bad type sellers choose not to rebate,\(^{16}\) For simplicity, we assume the seller’s reservation price is 0.
\( NR_S \). If \( \alpha - C \geq \beta \), we need to check whether any type sellers want to deviate from the separating equilibrium. Apparently, a \( B \) type seller would get the higher payoff \( \alpha - C \) instead of \( \beta \) if she pretends to be a \( G \) type seller by choosing the rebate option. Thus, the separating equilibrium does not exist.

Another separating equilibrium is that \( G \) type sellers choose \( NR_S \) and \( B \) type sellers choose \( RS \) does not exist either. The payoff to the good type seller is \( \alpha \), and the payoff to the bad type seller is \( \beta - C \), and \( \alpha > \beta \), so that the bad type seller has incentive to deviate from this separating equilibrium. An easier way to check the existence of separating equilibrium is through checking the single-crossing property. Since there is no single-cross property, i.e. the rebate costs the same for the two type sellers, there exist no separating equilibrium.

Second, we examine the pooling equilibrium where both type sellers choose to report, \( RS \). In this case, buyers can not update their beliefs by observing the sellers’ choice on providing rebate. Since both types of sellers provide rebates, all buyers will provide reports. The future buyer can, by using the information about seller’s previous history, update her beliefs on the seller’s type. If the buyer does not report, her willingness to pay at period \( t + 1 \) is \( P_{t+1} = \mu_t \alpha + (1 - \mu_t) \beta \); while if she choose to report, her willingness to pay is \( P_{t+1} = \mu_t \alpha + (1 - \mu_t) \beta - C + r \). Since buyers bid for the product, and rebate on the report is more than the cost of the report, \( r > C \), the winning bidder will report and the bidding price will automatically take the rebate and reporting cost into consideration, otherwise the buyer can not win the bid. The bidding price in period \( t + 1 \) is:

\[
P_{t+1} = \mu_t \alpha + (1 - \mu_t) \beta + r - C
\]

The payoff for the seller is:

\[
U_s(\text{Accept}, \ RS) = P - r = \mu_t \alpha + (1 - \mu_t) \beta - C
\]

It is less than the case of the benchmark model without the reporting cost case. Thus, the reporting cost is transferred to the sellers.

The buyer’s payoffs are

\[
U_b(P, \ GR, \ if \ Q_H) = 1 - P - C + r = 1 - (\mu_t \alpha + (1 - \mu_t) \beta + r - C) - C + r = 1 - (\mu_t \alpha + (1 - \mu_t) \beta)
\]

(2)

\[
U_b(P, \ BR, \ if \ Q_L) = -P - C + r = -(\mu_t \alpha + (1 - \mu_t) \beta + r - C) - C + r = -(\mu_t \alpha + (1 - \mu_t) \beta)
\]

(3)
They are the same as in the case of benchmark model without the reporting cost. Thus, all the reporting costs become the sellers’ burden if the incentive mechanism is adopted.

In the model of this section, we do not allow sellers to change ID. As the period $T$ goes to infinite, $\mu$ for the good seller goes to 1, and the buyer’s willingness to pay converges to $\alpha$; while $\mu$ for the bad seller goes to 0, and the buyer’s willingness to pay converges to $\beta$. If $\alpha - C \geq \mu_0\alpha + (1 - \mu_0)\beta$ and $\mu_0\alpha + (1 - \mu_0)\beta - C > \beta$, then the good type seller wants to choose the rebate, $R_S$, and the bad type sellers also will give the rebate, $R_S$, until their payoff $\mu_T\alpha + (1 - \mu_T)\beta - C$ is less than $\beta$, and choose no rebate afterwards.

If $\alpha - C < \mu_0\alpha + (1 - \mu_0)\beta$, both types of sellers want to choose no rebate, $NR_S$. If we allow sellers to change ID and start over as new sellers, that would be another story. The following section will discuss the changing ID issue.

Another pooling equilibrium is that both types of sellers choose not to rebate, $NR_S$, and the off-equilibrium path belief is that anyone who chooses $R_S$ must be B type sellers. In this case, the buyers willingness to pay is the same for all the periods, $P_t = P_1 = \mu_0\alpha + (1 - \mu_0)\beta$. Seller’s payoff is $P_t = \mu_0\alpha + (1 - \mu_0)\beta$ for every period. If $\alpha - C \geq \mu_0\alpha + (1 - \mu_0)\beta$, this equilibrium does not exist if we use the intuition criteria. Since the G type sellers want to separate from the B type sellers, G type sellers have incentives to give rebates, thus making the buyers report. So the off-equilibrium belief is not feasible. If $\alpha - C < \mu_0\alpha + (1 - \mu_0)\beta$, then the pooling equilibrium that both types of sellers choose to not rebate, $NR_S$, exists.

In summary, if the reporting cost $C$ is smaller than $(1 - \mu_0)(\alpha + \beta)$ and $\mu_0(\alpha - \beta)$, there would be a pooling equilibrium where both type sellers choose to rebate on reporting, and buyers could learn the sellers’ types by observing the reports. If the reporting cost $C$ is greater than $\alpha - (\mu_0\alpha + (1 - \mu_0)\beta)$, then both types of sellers would choose not to rebate, $NR_S$.

4.4.1 Changing ID

In the case that $C < \alpha - (\mu_0\alpha + (1 - \mu_0)\beta)$, good type sellers always choose to rebate, and bad type sellers choose to rebate until their payoff is less than $\beta$. If we allow sellers to change ID and start over as new sellers, then bad type sellers will change their ID if their future expected price minus reporting cost is less than $\beta$. Suppose in period $T$, the bad type seller will get $P_T = \mu_{T-1}\alpha + (1 - \mu_{T-1})\beta$, and his payoff is $U_s = \mu_{T-1}\alpha + (1 - \mu_{T-1})\beta - C > \beta$. With probability $\beta$, a bad type seller provides a $Q_H$ in period 1, and he gets a good report, $GR$. The buyer’s belief on his type in the
second period is
\[ \mu_1 = \frac{\mu_0 \alpha}{\mu_0 \alpha + (1 - \mu) \beta} \]
With probability \(1 - \beta\), the B type seller provides a \(Q_L\) in period 1, and he gets a bad report, \(BR\). The buyer’s belief on his type in the second period is
\[ \mu_1 = \frac{\mu_0 (1 - \alpha)}{\mu_0 (1 - \alpha) + (1 - \mu)(1 - \beta)} \]
Since buyer’s willingness to pay is \(P_{t+1} = \mu_t \alpha + (1 - \mu_t) \beta\), then B type seller’s expected payoff is
\[
E(P_2) = \alpha \left[ \frac{\beta \mu_0 \alpha}{\mu_0 \alpha + (1 - \mu_0) \beta} + \frac{(1 - \beta) \mu_0 (1 - \alpha)}{\mu_0 (1 - \alpha) + (1 - \mu_0)(1 - \beta)} \right] + \\
\beta \left[ 1 - \frac{\beta \mu_0 \alpha}{\mu_0 \alpha + (1 - \mu_0) \beta} + \frac{(1 - \beta) \mu_0 (1 - \alpha)}{\mu_0 (1 - \alpha) + (1 - \mu_0)(1 - \beta)} \right] - C
\]
If she is free to change ID, the B type seller can start as a new seller, and she will get \(P_1 = \mu_0 \alpha + (1 - \mu_0) \beta - C\) in the second period. We can see that \(E(P_2) < P_1\).

proof:
\[
E(P_2) - P_1 = (\alpha - \beta) \left[ \frac{\beta \mu_0 \alpha}{\mu_0 \alpha + (1 - \mu_0) \beta} + \frac{(1 - \beta) \mu_0 (1 - \alpha)}{\mu_0 (1 - \alpha) + (1 - \mu_0)(1 - \beta)} - \mu_0 \right] - \beta \left[ \frac{(\alpha - \beta)}{\mu_0 \alpha + (1 - \mu_0) \beta} \right] - C
\]
Since \(\alpha > \beta\) and \(0 < \mu < 1\), it is clear to see that \(E(P_2) < P_1\) for B type sellers. The bad type seller can change IDs once their next period’s payoff is less than \(P_1\).

In order to discourage B type sellers from changing ID, the online market can impose a cost for changing ID, \(k\). If the B seller does not change ID, his expected payoff in period 2 is \(E(P_2)\), and his total expected payoff in the two period is \(P_1 + E(P_2)\). If he changes ID, he needs to pay \(k\), and the B type seller’s total payoff over the two period is \(2P_1 - k\). If \(2P_1 - k \leq P_1 + E(P_2)\), then the B type seller will not change ID. In other words, in order to prevent B sellers to change ID, we need to set the cost of change ID is significant high enough, \(k \geq \mu_0^2 (1 - \mu_0)(\alpha - \beta)^3\), to discourage the sellers from changing IDs.
Let us take a look at the G type sellers.
The G type sellers will get $P_1 = \mu_0\alpha + (1 - \mu_0)\beta$ in period 1. With probability $\alpha$, a G type seller provides a $Q_H$ in period 1, and he gets a good report, $GR$. The buyer’s belief of his type in the second period is

$$\mu_1 = \frac{\mu_0\alpha}{\mu_0\alpha + (1 - \mu)\beta}$$

With probability $1 - \alpha$, G type seller provides a $Q_L$ in period 1, and he gets a bad report, $BR$. The buyer’s belief on his type in the second period is

$$\mu_1 = \frac{\mu_0(1 - \alpha)}{\mu_0(1 - \alpha) + (1 - \mu)(1 - \beta)}$$

Since the buyer’s willingness to pay is $P_{t+1} = \mu_t\alpha + (1 - \mu_t)\beta$, the G type seller’s expected payoff is

$$E(P_2) = \alpha\left[\frac{\alpha\mu_0\alpha}{\mu_0\alpha + (1 - \mu_0)\beta} + \frac{(1 - \alpha)\mu_0(1 - \alpha)}{\mu_0(1 - \alpha) + (1 - \mu_0)(1 - \beta)}\right] + \beta[1 - \left(\frac{\alpha\mu_0\alpha}{\mu_0\alpha + (1 - \mu_0)\beta} + \frac{(1 - \alpha)\mu_0(1 - \alpha)}{\mu_0(1 - \alpha) + (1 - \mu_0)(1 - \beta)}\right)]$$

If she is free to change ID, the G type seller can start as a new seller, then she will get $P_1 = \mu_0\alpha + (1 - \mu_0)\beta$ in the second period. To see whether a G seller wants to change her ID, we need to compare $E(P_2)$ with $P_1$.

$$E(P_2) - P_1 = \alpha\left[\frac{\alpha\mu_0\alpha}{\mu_0\alpha + (1 - \mu_0)\beta} + \frac{(1 - \alpha)\mu_0(1 - \alpha)}{\mu_0(1 - \alpha) + (1 - \mu_0)(1 - \beta)}\right] + \beta[1 - \left(\frac{\alpha\mu_0\alpha}{\mu_0\alpha + (1 - \mu_0)\beta} + \frac{(1 - \alpha)\mu_0(1 - \alpha)}{\mu_0(1 - \alpha) + (1 - \mu_0)(1 - \beta)}\right)]$$

$$-\left[\mu_0\alpha + (1 - \mu_0)\beta\right]$$

$$= (\alpha - \beta)\mu_0\left\{\frac{\alpha^2}{\mu_0\alpha + (1 - \mu_0)\beta} + \frac{(1 - \alpha)^2}{\mu_0(1 - \alpha) + (1 - \mu_0)(1 - \beta)}\right\}$$

$$= (\alpha - \beta)\mu_0(1 - \mu_0)((\alpha - \beta)^2(1 - \mu_0) + \mu_0(\beta - 1))$$

Since $\alpha > \beta$ and $0 < \mu < 1$, it is clear to see that $E(P_2) < P_1$ for G type sellers if $(\alpha - \beta)^2(1 - \mu_0) > \mu_0(1 - \beta)$, and the G type seller wants to change ID. If $(\alpha - \beta)^2(1 - \mu_0) < \mu_0(1 - \beta)$, the G type seller does not have incentive to change her ID.

If there exit a cost of changing ID, $k'$, the G seller will not change ID if $P_1 - k' \leq E(P_2)$, in other words, $k' \geq -(\alpha - \beta)\mu_0(1 - \mu_0)((\alpha - \beta)^2(1 - \mu_0) + \mu_0(\beta - 1))$.

Then , to show $k > k'$ or $k' > k$. If $\mu_0[1 - \beta - (1 - \mu_0)^2(\alpha - \beta)^3]$, then $k > k'$. 27
4.5 Adverse Selection and Moral Hazard Model

4.5.1 Benchmark model of Adverse Selection and Moral Hazard

Suppose there exist two types of sellers, good type \((G)\) and bad type \((B)\). If both types of sellers put forth efforts \((e = 1)\), they will provide high quality products \((Q_H)\) at probability 1; if they do not put forth an effort \((e = 0)\), then they will provide a low quality product \((Q_L)\) for sure. Assume G type sellers’ cost of making an effort is 0, \(C_{\theta_G}(e = 1) = e(0) = 0\), and B type sellers’ cost of making an effort is \(C_{\theta_B}(e = 1) = e(1) > 0\). To simplify, we assume G type sellers always make effort, because it costs 0 to them.

The game played in every period is described as the following:
1. Nature chooses sellers type \(\theta \in \{\theta_G, \theta_B\}\), the prior of meeting a \(\theta_G\) seller is \(\mu_0\).

2. The seller chooses \(r \in Rebate(R_S), NoRebate(NR_S)\), where \(r > C\).

3. Buyers choose bid, and the winning bid equals the highest willingness to pay, \(P \geq 0\). \(^{17}\)

4. The Seller chooses to accept or reject \(p\) based on his reservation price. If he rejects, the game ends. If he accepts, then go to next step. \(^{18}\)

5. The Seller chooses to put forth an effort or not, \(e = 1\) or \(e = 0\).

6. The Buyer chooses \((NR, GR, BR)\). Buyers can choose to give good report \((GR)\), bad report \((BR)\), or no report \((NR)\); the reporting cost is \(C\) for all buyers. Assume all the buyers report honestly if they decide to report, i.e. \(GR\) for \(Q_H\) and \(BG\) for \(Q_L\).

7. Payoff received for period \(t\).
\[
\begin{align*}
U_s(\theta_G, e = 1) &= P; \\
U_s(\theta_B, e = 1) &= P - e(1); \\
U_s(\theta_B, e = 0) &= P; \\
U_b(P; NR, if Q_H) &= 1 - P; \\
U_b(P; NR, if Q_L) &= -P; \\
U_b(P; GR, if Q_H) &= 1 - P - C;
\end{align*}
\]

\(^{17}\)We assume there are many bidders for each auction, and the winning bid is only \(\varepsilon\) higher than the second highest bid, so we use \(V_B\) as an approximate for the second highest bid.

\(^{18}\)For simplicity, we assume the seller’s reservation price is 0.
\[ U_b(P; BR, i f Q_L) = -P - C; \]

If there is no reporting cost, \( C = 0 \), all buyers report. When \( B \) type sellers do not make efforts, he will get a bad report, \( BR \). If the game repeats \( T \) period, \( B \) type seller will not make effort at the last period.

Buyers willingness to pay is \( P_t + 1 = \mu_t + (1 - \mu_t)\hat{e}_t \), where \( \hat{e}_t \) is the buyer’s expected effort put by the seller.

\[
\mu_1 = \frac{P(\theta_G|GR)P(\theta_G)}{P(GR|\theta_G)P(\theta_G) + P(GR|\theta_B)P(\theta_B)}
= \frac{\mu_0}{\mu_0 + (1 - \mu_0)P(e_1 = 1|\theta_B)}
= \frac{\mu_0}{\mu_0 + (1 - \mu_0)e(1)}
\]

In period \( t \), the updated prior of meeting a good type seller is:

\[
\mu_{t-1} = \frac{\mu_{t-2}}{\mu_{t-2} + (1 - \mu_{t-2})e_{t-1}}
\]

In the last period, \( T \), buyers’ willingness to pay is \( \mu_t \).

If \( T = 2 \), the seller’s strategy can be \((e(0), e(0))\) or \((e(1), e(0))\), where the first element represents the action in period \( t = 1 \), and the second represents the action in period \( t = 2 \). To examine which strategy for is right the seller, we need to calculate the payoffs.

If the bad type seller chooses \((e(0), e(0))\), his total payoff in the two period is \( U_s = \mu_0 \).

If he chooses \((e(1), e(0))\), his total payoff is

\[
U_s = P_1 - e(1) + \delta P_2
= (\mu_0 + (1 - \mu_0)\hat{e}_1 - e(1) + \delta \frac{\mu_0}{\mu_0 + (1 - \mu_0)e_1}
= 1 - e(1) + \delta \mu_0
\]

If \( 1 - e(1) + \delta \mu_0 > \mu_0 \), i.e. \( e(1) < 1 - (1 - \delta)\mu_0 \), then the bad type seller’s best strategy is making an effort in the first period but not in the second period, \((e(1), e(0))\).

In general, for \( T \) period game. The payoffs to bad type sellers in each period is as the following:

At \( t = 1 \), \( V_1 = P_1 + \delta I(e_1)V_2 - e_1(1) \)

If \( e_1 = 1 \), \( I(e_1) = 1 \), and \( V_1 = 1 + \delta V_2 - e(1) \).
If $e_1 = 0$, $I(e_1) = 0$, and $V_1 = P_1 = \mu_0 + (1 - \mu_0)e_1 = \mu_0$

At $t = 2$, $V_2 = P_2 + \delta I(e_2)V_3 - e_2(1)$
If $e_2 = 1$, $I(e_2) = 1$, and $V_2 = 1 + \delta V_2 - e(1)$.
If $e_2 = 0$, $I(e_2) = 0$, and $V_2 = P_2 = \mu_1 + (1 - \mu_1)e_2 = \mu_1 = \mu_0$

....

At $t = T - 1$, $V_{T-1} = P_{T-1} + \delta I(e_{T-1})V_T - e_{T-1}(1)$
If $e_T = 1$, $I(T - 1) = 1$, and $V_{T-1} = 1 + \delta V_T - e(1)$.
If $T - 1 = 0$, $I(T - 1) = 0$, and $V_{T-1} = P_{T-1} = \mu_0$

At $t = T$, $V_T = P_T = \mu_{T-1} = \mu_0$.
In order to induce the bad type sellers to choose $e_t = 1$ for every period prior to $T$, the condition $e(1) < 1 - (1 - \delta)\mu_0$ has to be satisfied.

Thus, as long as $e(1) < 1 - (1 - \delta)\mu_0$, bad type sellers will continue to make genuine efforts, but will cases to do so in the last period.

If there exist reporting costs, then no buyer will be inclined to report. In this case, the buyers’ willingness to pay is $P_i = \mu_0$. The good type sellers would get worse off, and the bad type sellers would not make an effort in any period.

If we use the incentive mechanism we proposed in the pure adverse selection model, both type sellers would choose to give a rebate, and bad type sellers would put forth efforts as long as their payoffs are more than $\beta$.

Thus, the incentive mechanism can help to induce bad type sellers to cooperate, and it helps to sustain a trustful trading environment. Together with the reputation system, the online market could be self-sustainable by the sellers and buyers, thus reducing the cost to the market designer.

5 Conclusion

This paper develops game theoretical models to understand the current reputation system in the online auction market, and proposes resolutions confronting the problem of
the lack of incentives for the buyers to report. Along with this problem, there is also the issue of buyers being hesitate to report negatively for fear of retaliation by the seller. These two conditions help to create an environment where buyers are reluctant to report which leaves them a lack of adequate information about the sellers, thereby allowing the bad type sellers to commit fraud more freely. We show that giving sellers the option to compensate the buyers’ reporting costs could lead to a pooling equilibrium where both types of sellers choose to compensate the reporting costs, and since the reports reveal information about the sellers past history, thus help the buyers to learn the types of sellers in pure adverse selection setting and induce bad type sellers to make efforts in the setting of both adverse selection and moral hazard. This incentive mechanism also helps buyers to distinguish the good type sellers from the bad type sellers in the pure adverse selection setting, thus it will distribute prices fairly to the different types of sellers. As a result, the good sellers would be compelled in this market instead of leaving, and the bad type sellers either leave the market or perform good and ethical behavior in the market.

References


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