Spousal Guarantees*

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Abstract

Lenders often require of small investors that they sign a personal guarantee before forwarding funds, and if the borrower's own funds or assets are insufficient backing for a guarantee, then a third party may be asked to sign. Since strangers do not guarantee each other's debts, it is in the nature of such guarantees that they straddle the private and business worlds within any relationship. Important relationship assets (such as the family home) are often at stake, and courts struggle with the policy tradeoffs inherent in such deeds or contracts between 'freedom of contract' and a concern with the potential for 'coercion' of the one signing. This paper explores the nature of the optimal third party guarantee within the incomplete contracting environment inaugurated by Grossman and Hart (1986). The optimal contract trades off the ex post amount of relationship asset to be foreclosed by a bank against the desirability of ensuring the ex ante release of funds to promote the exploitation of viable investment opportunities. A measure of coercion is developed and it is shown that while the number of projects financed increases with the amount of coercion within a relationship, beyond a certain amount such coercion becomes sub-optimal.

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1 The problem

It is well-known that the debt structure of the majority of small to medium sized businesses is bank debt.[1] Such businesses have limited access to capital markets and are much more affected than larger firms by business cycle-related fluctuations.[2] It is known that in the event of financial distress banks rarely forgive principal.[3] Empirically, the more bank debt a company has, the more likely that asset sales will be forced upon it during bankruptcy.[4] Banks rarely lend to small businesses without the comfort of guarantees or collateral.[5] Even when such businesses operate via a corporate form that legally provides limited liability protection, banks generally insist on personal guarantees.[6]

This paper is concerned with the contractual phenomenon of third-party guarantees. If for some reason (such as the business being a start-up) the person seeking a loan does not possess business assets to act as collateral, a third party may be asked to act as guarantor in their stead. Since strangers do not act as guarantors for each other's debts, it is intrinsic to such guarantees that they are signed within the context of a longer-term, continuing, personal relationship. Concrete examples of people likely to be asked to act as a third-party guarantee include wives guaranteeing the loans of husbands (and vice-versa), parents the loans of children, grandparents the loans of grandchildren. A problem potentially arises when the asset which acts as collateral is an important relationship asset like the family home, which has a value to its occupants greater than the market value a foreclosing bank might receive. Although it might be thought that guarantors worried about the future loss of such an important relationship asset should then not sign a guarantee, such an approach

[1] See Gertler and Gilchrist (1994) and Petersen and Rajan (1994) for details and evidence.

[2] See Gertler and Gilchrist (1994).

[3] See ??.

[4] See David Brown and Mooradian (??).

[5] In a 1983 survey of its members, the National Federation of Independent Business found about 60 percent of firms with commercial bank loans provide collateral as security for the loan agreement. They also found that collateral secured 78 percent of the total volume of small business loans. (See NFIB 83). Similarly, the Interagency Task Force on Small Business Finance (1982) found some form of collateral securing almost 80 percent of the dollar volume of large and small business loans from all sources (See ITFSB 82). These figures cited in Leeth and Scott (1989).

[6] See Chesterman (1982), and also Petersen and Rajan (1994) (summarizing economtric analysis of NFIB (1983)): "The owner's reputation is apparently more important than that of the business" (at page15).

ignores the potential for coercion in close domestic relationships. With perhaps an excessive regard for the ex post regret obviously felt in those instances when loans or loved ones turn sour, such third party guarantees have been dubbed in some legal scholarship a form of 'sexually transmitted debt'.[7]

During the nineties courts in the Anglo-American world grappled with the policy trade-offs involved in permitting the enforceability of third-party guarantees.[8] As an example the leading House of Lords case involved a wife suing to prevent a bank foreclosing on the matrimonial home. She had co-signed a guarantee as backing for business interests in which her husband was involved (and which did not directly involve her). In their decision the law lords were aware that any desire for paternalistic circumvention of the usual legal and economic norms of freedom to contract should be balanced against the concern that 'the wealth currently tied up in the matrimonial home does not become economically sterile'.[9] This paper analyzes the tension between freedom and regulation of third party guarantees, between the desire to permit efficiency increasing investment projects being undertaken and also to protect guarantors who for whatever reason the law might regard as more susceptable to coercive pressures within a relationship.[10] A ban on such guarantees would freeze forever all assets held in domestic use while unfettered freedom exposes a subset of guarantor's to intolerable risk of primary asset loss.[11] The optimal guarantee contract will trade off these concerns.

Aware of legal concern about 'coercion', banking associations in the United Kingdom and United States have drawn up conventions which branch managers must take into ac-

[7]See for example Fehlberg (1997).

[8]See the surveys of cases and jurisdiction by (for example) Fehlberg (1995) and Trebilcock and Ballantyne-Elliot (1998?).

[9]See Barclays Bank Plc v O'Brien [1994] 1 AC 180. The instance of signing of third party guarantees appears to be classifiable into two broad categories - those where the guarantee is for a start-up and those where the guarantee is for the refinancing of an already existing investment project. The Barclays Bank case concerned the latter, and such cases do appear to offer cause for greater concern than do the former type of cases (see ??).

[10]The contractual legal doctrines protecting disadvantaged persons in common law countries fall under the rubric of equity. For a summary of equitable doctrines in contract see ?? for Britain/Canade and ?? for the United States.

[11]Currently in the United States residential property is worth ??. Approximately ?? of that is encumbered with securities etc [look up]. In the United Kingdom the figures are [].

count when presenting third-party guarantees for signing.[12] Such conventions include *[fill in here]*. The paradox appears to be that those aspects of a domestic relationship whose presence is likely to lead a court to suspect coercion in the signing of a guarantee are precisely those that make the guarantee valuable from a bank's perspective. Thus one bank manager reported in a survey study his belief that 'any borrower who is undergoing difficult financial times is far more likely to repay a debt which is secured on his or her home so that itself is a factor in assessing risk' while another reported that the family home in particular was an important 'motivational asset'.[13] As the author of the study concluded after her survey of lending institutions, "private commitments enhanced public enforceability":

> Lenders acknowledged the problems inherent in taking security from a person in an intimate relationship with the debtor, but they also emphasized the importance to them in commercial terms of the surety's emotional investments in both the *relationship* with the debtor and the *home* (where relevant). [emphasis original]

Economists familiar with the literature on financial contracting and the 'income diversion' stories therein (see chapter 5 of Hart (1995)) will be unsurprised by these expressions by lenders of the desirability of any factor that forces the borrower to pay out funds rather than default, both in times of financial distress and even in good times.

While data on the incidence of third party guarantees is not available (and not easily made available), the significance of internal financing for young or small businesses is known. Thus Petersen and Rajan (1994), summarizing the survey data reported in NFIB (1983), note that (at page 8):

> The smallest 10 percent of firms in our sample borrow about 50 percent of their debt from banks. Another 27 percent comes from the firm's owners and their families. The table [referring to Table II on page 9 of their article] shows that the fraction from personal (owner and family) sources declines to 10 percent for the largest 10 percent of firms in our sample.

[12] See for the US [. . .] and for the UK BBA (1994) and for Canada [. . .].

[13] See Fehlberg (1997) at page 204.

On the same page they go on to note that "The youngest firms (age less than or equal to 2 years) rely most heavily on loans from the owner and his or her family" and again on page 10 "[F]irms follow a 'pecking order' of borrowing over time, starting with the closest sources (family) and then progressing to more arm's length sources."[14] This data would seem to suggest that, if data on the incidence of third party guarantees were available, it would most likely be concentrated among young and/or small firms. They are likely to play an important role in many start-up companies.

Obviously the first best can be achieved if comprehensive contracts could be written ex ante. But the type of relationships modelled in this paper are not in reality governed by comprehensive contracts or indeed often by any formal contract at all. Indeed, in some common law jurisdictions pre-marital contracts are not permitted as a matter of public policy, and even in those jurisdictions where they are, their content is heavily circumscribed again on public policy grounds.[15] Even if they are permitted, there may be adverse signalling reasons preventing their widespread adoption.[16] In this paper it is assumed that, in addition to the usual non-contractability of the income flow within the financial contracting literature, the relationship between guarantor and guarantee is also non-contractable. As will be seen, this 'incompleteness' in the relational contract opens the door to the possibility of ex post renegotiation and hence to the possibility of that 'coercion' or hold-up that exercised the minds of common law courts during the nineties. Paradoxically, it will be seen that what is 'bad' for the guarantor (coercion) is 'good' from the point of view of overall societal welfare (more projects are financed ex ante), though only up to a point.

[14] The actual figures are: "The youngest 10 percent of firms in our sample borrow an amount equal to 0.32 of their book assets, while the oldest 10 percent of firms in our sample borrow only 0.15. The smallest 10 percent of firms in our sample borrow 0.22 of their book assets while the largest 10 percent of firms in our sample borrow 0.30 of their book assets." (at page 10, footnote 8).

[15] See ?? for the US situation and ?? for the UK. For Canada see ??.

[16] See for example Spier (1992). An obvious reason why complete guarantees are rarely seen (that is, contracts which not only involve clauses concerning the loan but also the domestic relationship which forms the context of the request for the guarantee) is the non-verifiable nature of many of the relationship variables, and it is known that when some are unverifiable, it may be in the contractual parties' interests to leave other, verifiable variables unspecified also (see Holmstrom and Milgrom (1992) and Bernheim and Whinston (1995)).

Relationship to literature The paper contributes to that recent literature which analyzes financial decisions from the 'incomplete contracting' perspective inaugurated by Grossman and Hart (1986) and applied to the financial contracting setting by Aghion and Bolton (1992), Hart and Moore (1994) and Hart and Moore (1998).[17] In these papers (let's call it the standard framework) a financially constrained entrepreneur seeks funds from an investor in order to exploit an investment opportunity. The funds are used to buy project assets which in turn generate return streams. The theory assumes that, at the time the loan contract is written, the parties to the contract are not able enforceably to condition on all future states of the world (especially return streams), so that the contract instead must specify who gets control of the project assets in the event the entrepreneur defaults. Because loan repayment cannot be conditioned on return streams (meaning that any contractually specified repayment amount can be renegotiated), default can occur strategically and not just because returns are low. To minimize the incentives for such strategic default the investor must liquidate part of the project assets in the event of non-strategic default, even though such liquidation is ex post inefficient. The reason such liquidation has the right incentive effects is because future project return streams (which accrue only to the entrepreneur) depend on the entrepreneur controlling the project assets. It is this which gives the asset control decision in the standard framework an important 'leverage' effect between entrepreneur and investor. The current paper differs crucially from this standard framework in that the asset on which the security is taken is not a project asset but a 'relationship' asset independent of the entrepreneur's business. Consequently, signing a security shifts the leverage from the investor/entrepreneur relationship to the entrepreneur/guarantor relationship.

This indicates another aspect of the current paper that differs from the standard framework, in that the model involves three agents rather than two. Papers extending the standard framework to more than one investor include Bolton and Scharfstein (1996), Dewatripont and Maskin (1995), Dewatripont and Tirole (1994) and Berglöf and von Thadden

[17] For a summary of the standard framework and related literature see chapter 5 of Hart (1995). For another early example utilizing a similar style of incomplete contracts model, but analyzing predation in industrial organization theory instead, see Bolton and Scharfstein (1990).

(1994). The first two involve multiple investors with the same asset claim while the second two explore the effects of having different investors hold different asset claims. In both types of paper the purpose of multiple investors and/or multiple claims is to harden the budget constraint of the entrepreneur and consequently ease the threshold borrowing condition for the investors, thus enabling more loans to be provided (and more investments made). Indeed, anything (such as more agents involved in the bargaining, or asymmetric information in the ex post bargaining) which makes renegotiation harder to achieve will have this effect on the entrepreneur, with the cost of course that the possibility of efficient renegotiation (that is, when default has been necessary) is lost.

Outline of paper Section 2 outlines the basic structure of the model and solves for the optimal contract. Section 3 outlines the ex post bargaining convention adopted. Section 5 characterizes the conditions under which we are likely to observe the two types of spousal guarantee in the general model and explores the relationship between coercion and investment. Section 6 concludes.

2 The model

2.1 Basic set-up

At date 0 a bank (denoted B), an entrepreneur (denoted E) and a guarantor (denoted G) convene to sign a guaranteed loan contract to enable the wealth-constrained entrepreneur to invest in a long-term profitable project.[18] For concreteness one can think of the project as arising from the operation of a close (family-run) corporation which the entrepreneur heads. The project lasts two periods. The project provides non-negative returns of R_1 at date 1 and $R_2 = r$ at date 2 which in the first instance accrue to the entrepreneur.[19] The entrepreneur has zero initial wealth and we assume that $K > 0$ is the project's initial cost.[20]

[18] Hereafter the entrepreneur is referred to generically as 'he' and the guarantor as 'she'.

[19] These returns are specific to the entrepreneur - that is, neither the bank nor the guarantor can obtain these returns from the project without the entrepreneur. However, we do not model the process by which the entrepreneur generates these returns, assuming instead that they are exogenously given.

[20] The guarantor also has zero liquid wealth.

The amount $B = K$ is borrowed by the entrepreneur from the bank at date 0 for a promise to repay P (interpreted as both principal and interest) at date 1.[21] There exists a 'savings account' belonging to the entrepreneur. An important assumption regarding this 'savings account' (and any amount deposited therein) is that it is untouchable by the bank, even in the event that the entrepreneur defaults on the repayment of P. It is also assumed that the amounts R_1 and r are *uncontractable* (that is, 'observable' but not 'verifiable' in the language of Grossman and Hart (1986)) so that the entrepreneur is able without penalty to deposit these returns as they accrue into this account.[22] At least within the context of family businesses a reason for such untouchability lies in the ability of entrepreneurs potentially to divert business profits into family gifts and trusts, thus providing a de facto form of limited liability to the close corporation.

Consequently the bank requires security for B. There exist two types of asset which might act as security. The first (call it asset A) is a business asset that will be bought with the borrowed funds. It lasts only one period (that is, by date 2 it has been exhausted). This asset is essential to the production process: in combination with the entrepreneur's skill it produces the return stream over the two periods. If the asset is liquidated at date 1, then the date 2 return can not be realized. If part of the asset is liquidated at date 1 then only part of the date 2 return can be realized. We will denote this lesser date 2 return amount by αr, where $\alpha \in [0, 1)$.

The second asset which could act as a security (call it asset B) is a shared non-liquid relationship asset which is completely independent of the business and with a date 1 market value of z. Thus z might represent the market value of the family home. At date 2 (when

[21] The date 0 contract to be signed between the bank and the entrepreneur is a *standard debt contract*, namely (B, P). That is, the bank agrees to lend B at date 0 for a promise by the entrepreneur to repay the *non-contingent* amount P at date 1. This paper does not consider the issue of whether other, more elaborate, types of debt contract would be pareto superior to the standard debt contract examined here: for example contracts utilizing options to own a là Nöldeke and Schmidt (1995), or contracts utilizing, in the tradition of Maskin (1999), ex post message games such as is examined in the appendix to Aghion and Bolton (1992) or in the latter half of Hart and Moore (1998) (where necessary and sufficient conditions for a standard debt contract to be optimal are derived).

[22] This is the 'diversion' or 'stealing' assumption of Hart and Moore (1998), a possibly extreme but nonetheless useful assumption designed to capture the more realistic phenomenon of managerial discretion in the use and disbursement of corporate funds.

the model ends) this relationship asset is sold and consumed by the entrepreneur and/or guarantor according to their exogenously determined share of the asset. Thus let S^E denote the entrepreneur's date 2 share of the relationship asset (or the date 2 sale proceeds thereof). Hence the guarantor's share is $(1-S^E)$. If the relationship asset is the asset used as security and date 1 liquidation occurs, then less than z remains to the entrepreneur/guarantor at date 2 for consumption. We will denote this lesser amount by λz, where $\lambda \in [0, 1)$. This modelling assumption is meant to capture the fact that the relationship asset is worth more when maintained as a relationship asset than when in the possession of the bank. Specifically, it captures the fact that a relationship asset like a family home provides a value to its occupants not encapsulated in liquidated sale price alone. Call the combined assets AB. A security on the assets (either or both) is verifiable. In fact we will assume that a security is always over the combined assets, though ex post the choice of which asset to foreclose is left to the parties.

We now make the following assumptions regarding the relationship between these asset parameters.

Assumption 1 $\quad z > r$

Assumption 2 $\quad \Lambda \equiv \alpha - \lambda > 0$

Assumption 1 is intuitive in the context of spousal guarantees where the relationship asset is invariably a residential home and the investment project desired to be financed is a start-up or a refinancing agreement or some other small business example as the case law indicates. In the short run at least it acknowledges that the value of a relationship asset like a family home is likely to be much greater than any immediate returns from the project. Note that Λ is a measure of relative inefficiency, helping to decide which asset to foreclose (A or B) at least social cost. In principle it could have either sign, and the assumption that it is always strictly positive captures the fact that in this paper we are modelling normal, close relationships.

All variables are assumed certain except for the date 1 return R_1.[23] The uncertain

[23]It is consistent with the model to interpret, if one wishes, the date 2 variables as uncertain also (from

return R_1 is binary with a date 0 commonly known distribution given by:[24]

$$R_1 = \begin{cases} 0 & \text{with probability } 1-\theta, \\ x & \text{with probability } \theta. \end{cases}$$

Since the purpose of the paper is to model uncertainty we assume that common knowledge beliefs over R_1 are non-degenerate, or:

Assumption 3 $\theta \in (0, 1)$.

The date 0 contract stipulates the date 1 payment P to be as follows: at date 1 the entrepreneur must repay P_0 when $R_1 = 0$ and P_x when $R_1 = x$ (clearly P_0 will be less than or equal to P_x). The *actual* payment made by the entrepreneur at date 1 is denoted $\hat{P}$. We can without loss restrict this date 1 action set to be the same as the date 0 contractually mandated (though unenforceable) repayment schedule, thus $\hat{P} \in \{P_0, P_x\}$.

The most general type of security specifies that when the entrepreneur makes a date 1 payment $\hat{P}$, the bank has the right to liquidate some fraction of the asset(s) $AB(\hat{P}) \leq 1$ with probability $\beta(\hat{P}) \leq 1$.[25] The date 0 contract therefore will specify that when the entrepreneur makes the payment P_x when $R_1 = x$ then the bank has the right to liquidate L_x with probability β_x, and when the entrepreneur makes the repayment P_0 when $R_1 = 0$ then the bank has the right to liquidate L_0 with probability β_0. Thus the L_i's refer to the liquidation value in the hands of the bank depending on which payment was made. The value of this liquidation will also depend on which asset (or both) was used as security. The details of this will be covered in section 3. For now we leave it unspecified.

We denote by y_i (where $i = 0$ or x) the amount the entrepreneur promises to pay the guarantor at date 2 (conditional on the entrepreneur's date 1 payment) in return for her permitting the relationship asset to be utilized as security. Note that y_i need not

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the perspective of dates 0 and 1), but since any such uncertainty will enter into the model at date 1 as expectations there is no loss in assuming that they are certainty equivalents.

[24] Note that there is no loss of generality in confining attention to a two-state date 1 return, since even if R_1 is an interval (say $\mathbb{R}^+$) it is clear that it is never optimal for the entrepreneur to make a partial payment, so that default in that more general case would be defined as not paying anything at all at date 1.

[25] This general contractual form is taken from Bolton and Scharfstein (1990) and Bolton and Scharfstein (1996).

be interpreted as an explicit payment arising out of the guarantee contract but can be interpreted more expansively as the promise of a 'standard of living' arising out of the relationship.

All agents are risk neutral. The interest rate and the discount factors of the agents are all normalized to zero. Finally, the project is ex ante viable or productive, that is $\theta x + r > K$. Thus, we have biased the spousal guarantee problem in favor of financing indubitably worthwhile investments.

The timeline is as follows. At date 0 the three agents convene to sign a contract denoted by: $\Gamma = [B, \beta_0, \beta_x, P_0, P_x, y_0, y_x]$.[26] At date 1 the entrepreneur either pays the amount owed (if possible) or defaults (strategically or of necessity). In the event of default the bank takes control of the assets AB and with probability β liquidates them (or a fraction of them) for the value L. However, the loan might be renegotiated owing to the ex post inefficiency of liquidating AB. If renegotiation occurs, then a new contract is agreed upon at date 1 (in which the old contract acts as the disagreement point in the event of the ex post bargaining breaking down). At date 2 the final payouts are made between entrepreneur and guarantor (under either the old or new contract) and AB (or what is left of it) is sold and consumed by the guarantor and/or entrepreneur according to their date 2 shares. The timeline is depicted in Figure 1.

2.2 Discussion of basic set-up

If a comprehensive contract could be signed, then given the assumptions on the productivity of the project the entrepreneur would have no difficulty getting a bank to finance the project, and the first-best would be achieved. Note therefore that securing the loan with AB would not be necessary and, if nonetheless undertaken, liquidation would never be part of a first-best outcome. However, the inability to contract (that is, enforceably condition) on the return stream (combined with the untouchability of the entrepreneur's date 1 savings account into which the entrepreneur can siphon project returns) means that, without a

[26] Although Γ contains clauses for $P \in \{P_0, P_x\}$ and $y \in \{y_0, y_x\}$, that is, for the date 1 repayment to the bank and date 2 payment to the guarantor conditional on the date 1 return, these clauses (unlike the others in Γ) are of course not enforceable owing to the assumption that the date 1 return space and the relationship between the entrepreneur and guarantor are both non-contractible.

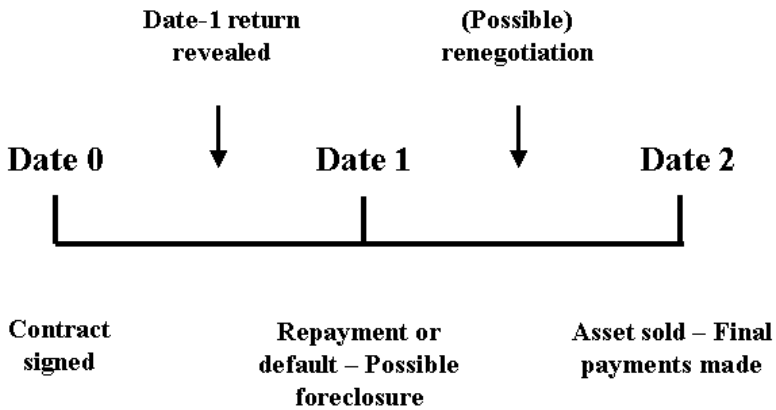

Figure 1: Spousal Guarantee Timeline

mechanism to enforce date 1 repayment, no bank will lend to the entrepreneur in spite of the overall viability of the project. The usual mechanism in the literature is a security over the project asset A that will be bought with the borrowed funds. Since the entrepreneur values continuance of the project, liquidation of A at date 1 gives the bank leverage over the entrepreneur to pay the loan out of the date 1 return stream. However this type of leverage would not be available in a model where no assets were bought with the borrowed funds (a special case of the current model, considered in section 5). In such cases the value of securing the relationship asset B comes into play. However, this then switches the leverage problem from the bank/entrepreneur relationship to the entrepreneur/guarantor relationship.

Although the three agents sign the contract Γ at date 0, in reality this contract is comprised of two separate, overlapping contracts: the debt contract between the entrepreneur and the bank $(B, \hat{P} \in \{P_0, P_x\})$ and the 'contract' between the entrepreneur and the guarantor $(\beta \in \{\beta_0, \beta_x\}, y \in \{y_0, y_x\})$.[27] This overlappingness is due to the fact that a

[27] Legally, a guarantee is an agreement (or signed deed) between the guarantor and *bank* rather than

third-party guarantee is invariably signed within the context of a larger (long-term, relational) contract obtaining between the entrepreneur and guarantor.[28] In reality a guarantee contract usually involves explicit clauses about B, β and $\hat{P}$. It does not usually involve an explicit clause concerning y. Certainly the bank has no interest at stake with respect to such a clause, and since the legal fiction is that the contract is between the bank and the guarantor (rather than between the entrepreneur and the guarantor, or among all three agents in the model), spousal guarantees are radically incomplete from the guarantor's point of view.

2.3 The optimal spousal guarantee

In this subsection we set out the linear program that the parties solve at date 0 and use it to characterize the optimal contract. Under this date 0 contract $\Gamma = [\beta_0, \beta_x, P_0, P_x, y_0, y_x]$ the entrepreneur's expected payoff is:

$$\begin{aligned}\theta[x - P_x + (1-\beta_x)r - y_x + (1-\beta_x)S^E z] \\ + (1-\theta)[-P_0 + (1-\beta_0)r - y_0 + (1-\beta_0)S^E z]\end{aligned} \tag{1}$$

and the bank's expected payoff is

$$\theta[P_x + \beta_x L_x] + (1-\theta)[P_0 + \beta_0 L_0] - K \tag{2}$$

the guarantor's expected payoff is

$$\begin{aligned}\theta[y_x + (1-\beta_x)(1-S^E)z] \\ + (1-\theta)[y_0 + (1-\beta_0)(1-S^E)z] - (1-S^E)z\end{aligned} \tag{3}$$

In order to ensure that the entrepreneur does not default when $R_1 = x$ we also need the following 'renegotiation constraint'

$$\begin{aligned}x - P_x + (1-\beta_x)r - y_x + (1-\beta_x)S^E z \\ \geq x - P_0 + (1-\beta_0)r - y_0 + (1-\beta_0)S^E z + \beta_0 g_E\end{aligned} \tag{4}$$

between the guarantor and *entrepreneur*.

[28] On marriage as a relational contract see Scott and Scott (1998). 'Relational' contracts is the name given by legal scholars to ??, often called implicit or self-enforcing contracts by economists.

where where the LHS is taken from the LHS of (1) and where g_E is the entrepreneur's surplus arising out of any renegotiation when he strategically defaults and the bank then has the right to foreclose on the asset(s) with probability β_0.[29] This amount will be determined in section 3 below. For now we leave it unspecified. In addition, we have the following miscellaneous constraints:

$$P_0 \leq 0 \text{ and } P_x \leq x \tag{5}$$

$$0 \leq y_0 \text{ and } 0 \leq y_x \tag{6}$$

$$0 \leq \beta_0, \beta_x \leq 1 \tag{7}$$

where equations (5) and (6) are the 'limited liability' constraints for the entrepreneur and guarantor respectively, and equation (7) is the feasibility constraint on the probabilities β_0 and β_x. Equations (1) - (7) define the maximization problem at date 0. Specifically, the problem is for the entrepreneur to maximize (1) subject to the 'individual rationality' constraints (2) and (3) for the bank and guarantor respectively, as well as subject to the renegotiation constraint (4) and the entrepreneur's and guarantor's limited liability constraints (5) and (6) and the probability feasibility constraints (7). We will call this linear programing problem (★).

The following proposition characterizes the optimal contract.

Proposition 1 (Characterization) *In the optimal contract*

(i) $P_0 = 0$,

(ii) $\beta_x = 0$,

(iii) both the bank's and guarantor's individual rationality constraints bind, and

[29]Note that if $R_1 = 0$ it is not feasible for the entrepreneur to 'default' in that income state and pay P_x rather than P_0, because such a payment could only be made if $P_x \leq 0$, which would mean the bank does not recover its loan. Thus we need not consider the 'other' renegotiation constraint ensuring that the entrepreneur pays P_0 when $R_1 = 0$.

(iv) the renegotiation constraint binds.

In order to prove this proposition, we break it up into a number of lemmas. The proofs of these four lemmas are in Appendix B.

Lemma 1 $P_0 = 0$.

Lemma 2 *Both the bank's and the guarantor's individual rationality constraints ((2) and (3) respectively) are binding at an optimum.*

Lemma 3 *It can never be optimal to foreclose on the relationship asset when $R_1 = x$; that is, $\beta_x = 0$.*

The proof of lemma (2) simply involves the entreprenuer, who has all the ex ante bargaining power, paying both the bank and guarantor as little as possible. Lemma (3) arises from the need to provide the entrepreneur with incentives to pay out date 1 returns to the bank. Foreclosing on the secured asset(s) gives the entrepreneur the wrong incentives. Lemmas (1) to (3) are used to prove the following lemma.

Lemma 4 *If (as assumed in subsection 2.1) beliefs about the viability of the investment project θ are non-degenerate (that is, they lie in the open interval $(0, 1)$ rather than the closed interval), then the renegotiation constraint (4) is binding at an optimum.*

The proof is by contradiction. The renegotiation constraint must bind in order to provide the incentive for the entrepreneur to repay the debt in the good income state, since no other reason exists for him to repay the loan. If the entrepreneur defaults when $R_1 = x$ the bank cannot convince a court that $R_1 \neq 0$.

Proof of Proposition 1 (Characterization). The proof of this proposition follows immediately from lemmas 1 to 4. ■

In addition, from the proof of lemma 4 we have the following corollary.

Corollary 1 *In the optimal contract β_0 is bounded away from zero.*

Proof. See Appendix B. ■

Even though asset foreclosure is inefficient ex post when $R_1 = 0$, nonetheless it will occur. This is the inefficiency owing to the twin effects of limited liability and contractual incompleteness. It can be seen that this model is characterized by the fact that there is liquidation when date 1 firm performance is poor, in spite of the fact that this liquidation is ex post inefficient. The firm is not liquidated because poor date 1 returns indicate poor date 2 returns (they are uncorrelated) but rather to ensure that date 1 repayments are made when firm performance is good.

We now attempt to find this β_0. Proposition 1 enables us to simplify the contracting problem considerably. It will be convenient to define and use $\Delta y \equiv y_x - y_0$ as well as the expectation $\bar{y} \equiv \theta y_x + (1-\theta)y_0$ instead of working with the individual y_i's. Utilising proposition 1 and this new definition we can simplify the maximization problem (★) to get the following. Choose $[P_x, \beta_0, \Delta y, \bar{y}]$ to maximize

$$\theta[x - P_x + r + S^E z] + (1-\theta)(1-\beta_0)[r + S^E z] - \bar{y} \tag{8}$$

subject to the following constraints

$$P_x = \frac{1}{\theta}[K - (1-\theta)\beta_0 L_0] \tag{9}$$

$$\bar{y} = (1-\theta)\beta_0(1 - S^E)z \tag{10}$$

$$P_x = \beta_0(r + S^E z - g_E) - \Delta y \tag{11}$$

$$P_x \in [0, x] \tag{12}$$

$$\bar{y} \geq 0 \tag{13}$$

$$\beta_0 \in [0, 1] \tag{14}$$

Now substitute (9) and (10) into (8) to get the entrepreneur's expected payoff from the contract

$$\theta x - K + (r + S^E z) - \beta_0(1-\theta)(r + z - L_0) \tag{15}$$

where the first two terms are the net present value of the project in the first best case of no liquidation, while the last term is the expected efficiency loss from the incompleteness of the contract. This last will be referred to often in the remainder of the paper, so we will denote it by $EL \equiv \beta_0(1-\theta)(r+z-L_0)$. Note that the payment to the guarantor does not enter into the entrepreneur's payoff directly. At best such payments only have an indirect effect, via the renegotiation constraint. Now also substitute (9) into (11) to get

$$\beta_0 = \frac{K + \theta\Delta y}{\theta(r + S^E z - g_E) + (1-\theta)L_0} \tag{16}$$

The simplified problem is now to choose $[\beta_0, \Delta y]$ to maximize (15) subject to (16) and (14). Call this simplified optimization problem (★★). Since (15) is linear in β_0, this is equivalent to minimizing β_0 subject to (16) and (14). This is achieved by setting $\Delta y = 0$. Consequently we have

$$\beta_0 = \frac{K}{\theta(r + S^E z - g_E) + (1-\theta)L_0} \tag{17}$$

which will be a solution provided that the RHS is not greater than one.

3 Ex post hold-up and bargaining

In our model entrepreneurial date 1 default is either *necessary* or *strategic*. In both cases the bank takes possession of the secured asset AB. Foreclosure of those assets, however, is not automatic in the case of strategic default - since foreclosure is inefficient, scope for ex post renegotiation arises. After a necessary default there can be no renegotiation because both the entrepreneur and the guarantor are wealth constrained, and the offer of a claim to the date 2 return is not credible. After a strategic default renegotiation is possible. The entrepreneur has the means to pay the promised amount (in exchange for a reduction in β_0), or, alternatively formulated, can agree to 'buy back' the seized asset(s) (and so negate the need for sale by the bank, i.e, $\beta_0 = 0$).

Table 1: **Table of ex post Values**

Panel A		
Available Asset	Foreclosure Value	
$i = A$	$L_0^A = \alpha r$	
$i = B$	$L_0^B = \lambda z$	
$i = AB$	$L_0^{AB} = \alpha r + \lambda z$	
Panel B		
Available Asset	Total Ex Post Surplus Value	
$i = A$	$\Pi_A = r + z - (\alpha r + z)$	$= r - L_0^A$
$i = B$	$\Pi_B = r + z - (r + \lambda z)$	$= z - L_0^B$
$i = AB$	$\Pi_{AB} = r + z - (\alpha r + \lambda z)$	$= r + z - (L_0^A + L_0^B)$
Panel C		
Available Asset	Bargained Share of Surplus	
$i = A$	$g_E^A = \tau(1-\alpha)r$	$= \tau(r - L_0^A)$
$i = B$	$g_E^B = \tau(1-\lambda)z$	$= \tau(z - L_0^B)$
$i = AB$	$g_E^{AB} = \tau[(1-\alpha)r + (1-\lambda)z]$	$= \tau[r + z - (L_0^A + L_0^B)]$

Up to now we have said nothing about the the values of L_0 and g_E. The foreclosure value L_0 arises as a result of a necessary default and the entrepreneur's renegotation surplus g_E arises as a result of a strategic default. The foreclosure value depends on whether either the project asset A, the relationship asset B, or both assets AB, is available for foreclosure under the security. These values are shown in Panel A of Table 1. At date 1 a strategic default will lead to the bank seizing (taking control of) the secured asset i (where $i = A$, B or AB). This does not mean that the asset is sold. Since sale of the asset is socially inefficient there exists scope for ex post renegotiation. Denote by Π_i the social surplus salvaged by the parties when the foreclosure of asset i is prevented via renegotation. These different amounts are shown in Panel B of Table 1. The entrepreneur's surplus from renegotiation in the event of a strategic default (g_E) depends on the assumptions about the ex post bargaining process to be made in this section. In the literature a number of bargaining conventions are used, the most common being the Nash bargaining solution where the parties equally split the surplus. Because in sections 4 and **??** we wish to explore the effects of differing bargaining power, here we use a 'shortcut' form of the generalised Nash bargaining solution, borrowed from Hart and Moore (1998), in which the entrepreneur has all the ex post bargaining power with probability τ (and where we are only interested in the entrepreneur's bargaining power because we are only interested in his share of the ex post surplus). In Appendix A we show that, for this model at least, this type of shortcut bargaining convention is without loss of generality. This bargaining convention produces values of g_E^i shown in Panel C of Table 1.

Recall that the security is over all assets (that is, AB). We define partial liquidation as involving the decision only asset A or only assset B will be foreclosed by the bank. Total liquidation involves the decision to liquidate both assets. In the sequel we are interested in comparing partial liquidations only. For that we need the follwing definintion.

Definition 1 $\quad c(\lambda) \equiv \frac{1}{\lambda} - 1$ *and* $\kappa(\tau, c) \equiv \tau c(\lambda)$.

Note that $c(\lambda)$ can be interpreted as a measure of the 'closeness' of, or amenity arising out of, the relationship between entrepreneur and guarantor. This is better appreciated after rearrangement, which gives $\lambda(c) \equiv \frac{1}{1+c}$ (which exists since $c(\lambda)$ is monotone decreasing). The closer the relationship (c increasing) the greater the inefficiency from foreclosing on the relationship asset (λ decreasing).[30] We will call $\kappa(\tau, c)$ the amount of 'coercion' within the relationship. As is to be expected, coercion is increasing in both entrepreneur bargaining power and relationship closeness.

We assume that securing oneself up to the eyeballs is not something that either the entrepreneur or guarantor want. It is not the case that for all parameter values partial liquidation is preferred to total liquidation, as the following proposition shows.

Proposition 2 (Partial Liquidation) *Let* $\hat{\theta} \equiv \frac{1 - c(\lambda)}{1 + \kappa(\tau, c)}$. *Then partial liquidation is preferred to total liquidation when* $\theta > \hat{\theta}$*; otherwise total liquidation is preferred.*

[30] Justification for this interpretation linking relationship closeness and asset inefficiency is found in the common law marital property doctine of 'tenancy in the entirety', a category of property ownership that can only exist between married couples, and only while the couple remains married - it involves the fiction that husband and wife are one, which operationalizes as each party having a non-severable interest over the *entire* property in question (such as a family home). Justification for such a rule within the Grossman and Hart (1986) property rights literature can be found in Cai (2003). Common law property regimes exist in the UK and UK-colonized countries like Canada, Australia, South Africa and New Zealand, as well as in most states of the United States.

Proof. See Appendix B. ■

The lower is θ the more risky the investment project. When the project is risky, understandably the entrepreneur must choose to permit the bank to foreclose on all available assets. Put another way, the more assets one has available to act as collateral, either the greater the loan one can take out or the riskier the project one can finance. By confining ourselves to situations where only one asset at most is permitted to be foreclosed, we are limiting the riskiness of projects that can be financed by spousal guarantees. The fact that such a self imposed limitation may not be such a concern given the focus this paper is shown by the following corollary.

Corollary 2 *For κ large enough, total liquidation is never preferred regardless of the project's risk profile.*

Proof. See Appendix B. ■

We can now write β_0 and the contractual efficiency loss, EL, for the two cases we confine ourselves to in this paper (setting for notational convenience $\rho \equiv r + S^E z$)

$$\beta_0^A = \frac{K}{\theta(\rho - \tau(1-\alpha)r) + (1-\theta)\alpha r} \tag{18}$$

$$\beta_0^B = \frac{K}{\theta(\rho - \tau(1-\lambda)z) + (1-\theta)\lambda z} \tag{19}$$

and

$$\begin{aligned} EL^A &= \beta_0^A(1-\theta)(r+z-\alpha r) \\ &= \frac{r(1-\alpha)+z}{\theta(\rho-\tau(1-\alpha)r)+(1-\theta)\alpha r} K(1-\theta) \end{aligned} \tag{20}$$

$$\begin{aligned} EL^B &= \beta_0^B(1-\theta)(r+z-\lambda z) \\ &= \frac{z(1-\lambda)+r}{\theta(\rho-\tau(1-\lambda)z)+(1-\theta)\lambda z} K(1-\theta) \end{aligned} \tag{21}$$

4 A special case

It is instructive to consider the special case, important in practice, in which there exists a relationship asset but no project asset. Considering this special case gives us a flavor of the welfare consequences for an economy relying on one or the other type of guarantee exclusively.

4.1 No project asset

4.1.1 Shared ownership

In this version of the model the entrepreneur maximises his utilty

$$\theta[x - P_x + r - y_x + (1-\beta_x)S^E z] + (1-\theta)[-P_0 + r - y_0 + (1-\beta_0)S^E z]$$

subject to the individual rationality constraint of the bank

$$\theta[P_x + \beta_x L_x] + (1-\theta)[P_0 + \beta_0 L_0] - K$$

and the individual rationality constraint of the guarantor

$$\theta[y_x + (1-\beta_x)(1-S^E)z] + (1-\theta)[y_0 + (1-\beta_0)(1-S^E)z] - (1-S^E)z$$

the renegotiation constraint for the entrepreneur

$$x - P_x + r - y_x + (1-\beta_x)S^E z \geq x - P_0 + r - y_0 + (1-\beta_0)S^E z + \beta_0 g_E$$

and the feasibility constraints

$$P_0 \leq 0 \text{ and } P_x \leq x$$

$$0 \leq y_0 \text{ and } 0 \leq y_x$$

$$0 \leq \beta_0, \beta_x \leq 1$$

Results similar to those of subsection **??** obtain, and we use those results to find the efficiency loss EL and the optimal β_0

$$EL^{ra} = (1-\theta)\beta_0(z - L_0) \tag{22}$$

$$\beta_0^{ra} = \frac{K}{\theta[S^E z - g_E] + (1-\theta)L_0} \tag{23}$$

which will be a solution provided that the RHS of (23) is not greater than one and where the superscript 'ra' denotes 'relationship asset'. The security is over the relationship asset B and the project returns accrue to the entrepreneur exogenously and independent of the existence of the relationship asset. Necessary default provides the bank a return of λz while a strategic default provides the entrepreneur with

$$g_E = \tau(1-\lambda)z \tag{24}$$

We write β_0 in terms of the model's exogenous parameters

$$\beta_0^{ra} = \frac{K}{\theta[S^E z - \tau(1-\lambda)z)] + (1-\theta)\lambda z} \tag{25}$$

4.1.2 Guarantor wholly owns relationship asset

In the case of no project asset, the further special subcase of $S^E = 0$ is worth examining because it has arisen often in equitable case law regarding third party guarantees, the paradigmatic example being grandparents guaranteeing a loan for a grandchild.[31] Substituting this change into (25) gives

$$\beta_0^{s} = \frac{K}{-\theta\tau(1-\lambda)z + (1-\theta)\lambda z} \tag{26}$$

where the superscript 's' denotes the entrepreneur 'share' of the the relationship asset.

4.2 Welfare comparison

Comparing (25) and (26) we have the following proposition.

Proposition 3

(i) $\beta_0^s \geq \beta_0^{ra}$,

(ii) $EL^s \geq EL^{ra}$.

[31] See, for example, the Australian case of ?? where *[etc]*.

Proof. See Appendix B. ■

Among the factors courts look at when adjudicating on the enforcability of third party guarantees has been whether the guarantee co-shares the secured asset. The assumption has been that a greater ownership stake in the relationship asset being used as security is to be viewed with less suspicion. Proposition 3 confirms that judicial intuition. The less the entrepreneur shares in the relationship asset the less leverage the threat to foreclose on it has on him, so that the probability of foreclosing needs to rise to compensate, as part (i) informs us. As we will see in proposition 4 in section 5, this actually makes a third party guarantee more attractive to him (compared with using any of his own assets, assuming he had any). Unfortunately, it also means that the cost of financial distress is unavoidably higher in the case where the entrepreneur does not partly own the secured asset and so involves greater social loss, as part (ii) informs us. The β_0's can be considered in a population-wide sense, rather than as the probability of foreclosure in an individual case, as the proportion of loans in an economy which foreclose. An economy which was forced to rely solely on third party rather than personal guarantees would see a larger amount of asset foreclosure.

5 Personal versus Third-Party Guarantees

In order to investigate the conditions under which we might anticipate the use of either personal or third party guarantees using the general model of sections 2 and 3, we need to compare contractual inefficiencies in these two different scenarios. First note that assumptions 1 and 2 combined imply that $\nabla \equiv r(1-\alpha)-z(1-\lambda) < 0$. Also define $\psi \equiv EL^A - EL^B$. The entrepreneur is indifferent between using the project asset or the relationship asset for foreclosure when $\psi = 0$. That is (from (20) and (21) in section 3)

$$\psi \equiv \frac{r(1-\alpha)+z}{\theta(\rho-\tau(1-\alpha)r)+(1-\theta)\alpha r}K(1-\theta)$$
$$-\frac{z(1-\lambda)+r}{\theta(\rho-\tau(1-\lambda)z)+(1-\theta)\lambda z}K(1-\theta)=0$$

or

$$\frac{r(1-\alpha)+z}{\theta(\rho-\tau(1-\alpha)r)+(1-\theta)\alpha r}-\frac{z(1-\lambda)+r}{\theta(\rho-\tau(1-\lambda)z)+(1-\theta)\lambda z}=0$$

This implies that

$$[r(1-\alpha)+z][\theta(\rho-\tau(1-\lambda)z)+(1-\theta)\lambda z]$$

$$-[z(1-\lambda)+r][\theta(\rho-\tau(1-\alpha)r)+(1-\theta)\alpha r]=0$$

which can be written as

$$\theta\left\{r(1-\alpha)[(\rho-\tau(1-\lambda)z)-\lambda z]-z(1-\lambda)[(\rho-\tau(1-\alpha)r)-\alpha r]\right\}$$

$$-rz(\alpha-r)=0$$

which simplifying gives

$$\psi\equiv\theta\left\{\rho\nabla-rz\lambda[1-\alpha\kappa(\tau,c)]\right\}-rz\Lambda=0 \tag{27}$$

We have the following two propositions. All proofs can be found in Appendix B and all proofs involve comparing the expected efficiency loss from contractual incompleteness in the two cases of when the project asset is foreclosed and when the relationship asset is foreclosed. The first concerns the conditions determining whether personal or third-party guarantees are likely to be used.

Proposition 4 (Guarantee Conditions)

(i) Entrepreneur Bargaining Power *The optimal guarantee involves foreclosing the relationship asset (third party guarantees) when the entrepreneur has more bargaining power (τ high) and foreclosing the project asset (personal guarantees) when the entrepreneur has less bargaining power (τ low).*

(ii) Relationship closeness *The optimal guarantee involves foreclosing the project asset (personal guarantees) when relationship closeness is low ($c(\lambda)$ high) and foreclosing the relationship asset (third party guarantees) when relationship closness is high ($c(\lambda)$ low).*

(iii) Entrepreneur Relationship Asset Share *The optimal guarantee involves foreclosing the project asset (personal guarantees) when entrepreneur share of the relationship asset is high (S^E high) and foreclosing the relationship asset (third party guarantees) when entrepreneur share of the relationship asset is low (S^E low).*

Proof. See Appendix B. ■

Parts (i) and (ii) of proposition 4 imply that personal guarantees are used when coercion is low and third-party guarantees are used when coercion is high, thus corroborating the statements from lenders quoted in section 1 of the paper. Third party guarantees have value only when they are able to act as substitute leverage for that lost between the entrepreneur and bank when the option of securing project assets is waived. Part (iii) may appear at first glance counter-intuitive, but there are two effects at work. The first effect is that a lower S^E decreases the capacity of the relationship asset to act as leverage, which would seem to make foreclosing on the project asset instead more favorable, but the intuition is that this lower ownership share is counterbalanced by a greater β_0, which raises the cost of fiinancial distress and is more inefficient.[32] This second effect dominates the first, thus producing the result that lower ownership share by the entrepreneur in the relationship asset makes him more likely to want to use such an outside asset as security rather than his own.

Finally, we arrive at the main result of the paper linking cercion within a relationship and investment risk. In order to prove that proposition we need this one.

Proposition 5 (Asset share, Coercion and Project Risk Reversal) *There exist* $\bar{S}^E \in [0,1]$, $\kappa^* \in [0,\infty)$ *and* $\tilde{\theta} \in (0,1)$ *such that*

Case 1: $S^E > \bar{S}^E$

For all κ the optimal guarantee involves foreclosing the project asset (personal guarantee) when default risk is low ($\theta > \tilde{\theta}$) and foreclosing the relationship asset (third party guarantee) when default risk is high ($\theta < \tilde{\theta}$).

[32] This is seen more clearly in proposition 3 in subsection **??**, which consider welfare effects explicitly.

Case 2: $S^E < \bar{S}^E$

Subcase (i): $\kappa < \kappa^*$ When coercion in a relationship is low ($\kappa < \kappa^*$), the optimal guarantee involves foreclosing the project asset (personal guarantee) when default risk is low ($\theta > \tilde{\theta}$) and foreclosing the relationship asset (third party guarantee) when default risk is high ($\theta < \tilde{\theta}$).

Subcase (ii): $\kappa > \kappa^*$ When coercion in a relationship is high ($\kappa > \kappa^*$), the optimal guarantee involves foreclosing on the relationship asset (third party guarantee) when default risk is low ($\theta > \tilde{\theta}$) and foreclosing on the project asset (personal guarantee) when default risk is high ($\theta < \tilde{\theta}$).

Proof. See Appendix B. ■

Note that when the entrepreneur has a large share of the relationship asset there is a sense in which securing that asset is closer to a personal guarantee than a third party guarantee. If the intent of this paper is to discover what extra benefits in terms of opening up access to finance for otherwise unfinanced project possibilities then the case in which the entrepreneur has only a small or zero share in the secured asset is more instructive. This situation is represented by case 2 in proposition 5.

Both in Case 1 and in subcase (i) of case 2 spousal guarantees are used for high risk projects rather than low risk projects, something that accords with our prior knowledge in that they are used primarily for start-ups or refinancing. But note that in subcase (ii) the situation reverses so that under certain conditions (when entrepreneurial asset share is low and coercion is high) spousal guarantees can be used to support low risk projects. There is a sense in which this subcase is paradigmatic of third-party guarantees, in that it involves a From the result of subcase (ii) it would appear that coercion can only have positive benefits, so that courts or policy makers who intuit something not always right about these guarantees act upon misplaced concern. But case 2 of proposition 5 is only half the picture - increased coercion not only effects risk profile (which side of the cutoff θ is on) but also effects the cutoff itself. Support for judicial concern regarding spousal guarantees is provided in the following proposition.

Proposition 6 (Coercion and Investment Risk) *In subcase (ii) of case* 2 *of proposition 5, when coercion is large enough, third party guarantees are almost always preferred to personal guarantees regardless of project risk profile.*

Proof. See Appendix B. ∎

Hence in subcase (ii) of case 2, some coercion is a good thing but too much is a bad thing. The parameter values in question are of course a simultaneous excess of coercion and lowness of entrepreneurial share of the relationship asset. We know that there are many examples in the case law where entrepreneurs possessed small or zero share in the asset used as collateral. In the case of marriages however, marital property laws guarantee both spouses an equal share in marital assets, perhaps leading to the surmise that in that particular case no concern is warranted. But in a different model S^E could paramaterize not asset share but asset importance or asset preference. In that case, if a husband places less importance upon the relationship asset than does the wife, the result becomes applicable in the case of spousal guarantees also. Differences in outside earning capacity make it likely that (especially in the case where relationship ardour is cooling) wives are more likely to *[etc]*. If a primary policy concern when judging the enforceability of such guarantees is the promotion of closeness within a relationship, then *[etc]*.

6 Conclusion

This paper analyzes a form of secured transactions that has received much judicial attention during the last fifteen years. The main theme of the paper is that the relationship between guarantor and guarantee, represented in this paper as coercion and relative asset share, has both socially beneficial and detrimental effects. Third party guarantees involve an inherent tension between the good and bad effects of coercion within relationships. The good effect allows otherwise wealth constrained individuals to finance projects that would not otherwise obtain financing, involving an efficiency loss to society (given the assumption in this paper that such projects are socially beneficial). The mechanism by which this is achieved is precisely via the exploitation of that relationship connection that exists between guarantor and guarantee - a connection which reassures banks that the loan will be repaid. However, that connection can also serve as a means by which guarantors sign security agreements even when their better judgement is that the project is high risk. Indeed, it was shown that in the paradigmatic third party guarantee case, where such guarantees are likely to have their most socially beneficial impact (in terms of opening financial access to low risk projects), too much coercion leads to the situation that any project is supported regardless of risk profile or merit.

A Generalized Nash bargaining

In this appendix we show that the shortcut bargaining convention adopted in section 3 of the paper is without loss of generality. We know that g_E denotes the share of ex post surplus obtained by the entrepreneur during renegotation. Correspondingly, let g_B denote the share of the ex post surplus obtained by the bank and g_G the share of the ex post surplus obtained by the guarantor. Obviously $g_E + g_B + g_G = \Pi_i$. Let the bargaining power for each of the three agents in the model (entrepreneur, bank and guarantor) be respectively (τ_E, τ_B, τ_G). The generalized Nash bargaining problem takes the following form

$$\max_{g_E, g_B, g_G} \phi \equiv (g_E)^{\tau_E}(g_B)^{\tau_B}(g_G)^{\tau_G}$$

subject to

$$g_E + g_B + g_G = \Pi_i$$

The first order conditions are

$$\frac{-\phi\tau_B}{\Pi_i - g_E - g_G} + \frac{\phi\tau_E}{g_E} = 0$$

and

$$\frac{\phi\tau_G}{g_G} - \frac{\phi\tau_B}{\Pi_i - g_E - g_G} = 0$$

which after manipulation gives (since we are only interested in the entrepreneur's share)

$$g_E^i = \frac{\tau_E}{\tau_E + \tau_B + \tau_G}\Pi_i \tag{A.1}$$

To get a single parameter summarizing the relative bargaining strength of the entrepreneur and the guarantor, we divide the RHS of (A.1) by τ_E and then define $\tau_0 \equiv \frac{\tau_B + \tau_G}{\tau_E}$ and

$\tau \equiv \frac{1}{1+\tau_0}$. This then gives us

$$g_E^i = \tau \Pi_i$$

which is the form of entrepreneurial surplus share we use in section 3. An increase in τ represents a shift in power away from the entrepreneur while a decrease represents a shift in power in favor of the entrepreneur. The entrepreneur's share of the ex post renegotiaiton surplus increases as his power within the relationship increases.

B Mathematical proofs

Proof of Lemma 1. Since the entrepreneur is wealth-constrained we have the following feasibility constraint $P_0 \leq 0$. So either $P_0 = 0$ or $P_0 < 0$. Assume the latter. This means that the bank (it can't be the guarantor, who also has zero (liquid) wealth) pays the entrepreneur something when $R_1 = 0$. But then it would be more socially efficient (since foreclosing on either the project or relationship asset is always inefficient) to increase P_0 and so reduce β_0. Hence $P_0 = 0$. ■

Proof of Lemma 2. The bank's individual rationality constraint binds at an optimum since, if it did not, it would be possible to decrease P_x and consequently raise the entrepreneur's payoff. Such a change would not effect the guarantor's payoff and would slacken the reneogotiation constraint.

The guarantor's individual rationality constraint binds at an optimum since if it did not, it would be possible to decrease y_x and consequently raise the entrepreneur's payoff. Such a change would not effect the bank's payoff and would slacken the reneogotiation constraint. ■

Proof of Lemma 3. Suppose β_x is strictly positive. Then it is possible to reduce β_x by some infinitesimal amount ϵ without changing the bank's and guarantor's payoffs provided that we simultaneously ensure that P_x is increased by ϵL_x and y_x is decreased by $\epsilon(1-S^E)z$. With these changes, the entrepreneur's payoff changes by $\theta\epsilon[r+z-L_x]$ and the LHS of the renegotiation constraint changes by $\epsilon[r+z-L_x]$. These changes are positive by assumption (the liquidation value cannot be greater than the value of the combined assets) which contradicts the initial assumption that a strictly positive β_x could be part of an optimum. ∎

Proof of Lemma 4. Suppose to the contrary that (4) is slack. We can solve for the optimal contract assuming this and show that the solution violates (4). Given lemmas 1 - 3, the optimization problem (★) can be reformulated as chossing $[P_x, \beta_0, y_x, y_0]$ to maximize

$$\theta[x - P_x + r - y_x + S^E z] + (1-\theta)[(1-\beta_0)r - y_0 + (1-\beta_0)S^E z] \tag{B.1}$$

subject to:

$$\theta P_x + (1-\theta)\beta_0 L_0 - K = 0 \tag{B.2}$$

$$\theta[y_x + (1-S^E)z] + (1-\theta)[y_0 + (1-\beta_0)(1-S^E)z] - (1-S^E)z = 0 \tag{B.3}$$

$$x - P_x + r - y_x + S^E z \geq x + (1-\beta_0)r - y_0 + (1-\beta_0)S^E z + \beta_0 g_E \tag{B.4}$$

$$P_x \leq x \tag{B.5}$$

$$0 \leq y_0 \text{ and } 0 \leq y_x \tag{B.6}$$

$$0 \leq \beta_0 \leq 1 \tag{B.7}$$

For now we ignore the renegotiation constraint (B.4). Define $\Delta y \equiv y_x - y_0$ and the expectation by $\bar{y} \equiv \theta y_x + (1-\theta)y_0$. Rearranging (B.1) in terms of $\bar{y}$ gives

$$\theta[x - P_x + r + S^E z] + (1-\theta)[(1-\beta_0)r + (1-\beta_0)S^E z] - \bar{y} \tag{B.8}$$

and rearranging (B.3) in terms of $\bar{y}$ gives

$$\bar{y} = (1-\theta)\beta_0(1-S^E)z \tag{B.9}$$

Rearranging (B.2) we get

$$P_x = \frac{1}{\theta}[K - (1-\theta)\beta_0 L_0] \tag{B.10}$$

The feasibility constraint on the y_i's now becomes

$$\bar{y} \geq 0 \tag{B.11}$$

Now substituting (B.9) and (B.10) into (B.8) gives the new objective function in terms of β_0 alone, which (after some manipulation and collecting the β_0 terms) gives

$$\theta x - K + r + S^E z - \beta_0(1-\theta)(r + z - L_0) \tag{B.12}$$

This objective function is linear in β_0 so we have a corner solution. The feasibility constraint on β_0 means that the entrepreneur's payoff is maximized when $\beta_0 = 0$ (because L_0 can never be greater than $r + z$ by assumption). Substituting $\beta_0 = 0$ into (B.9) and (B.10) gives respectively $\bar{y} = 0$ (which does not violate (B.11)) and $P_x = \dfrac{K}{\theta}$ (which does not violate (B.5)). Note that the condition $\bar{y} = 0$ is true in three cases: either $\theta = 0$ and $y_0 = 0$, or $\theta = 1$ and $y_x = 0$, or $\theta \in (0, 1)$ and both the y_i's are equal to zero. By assumption 3 θ is bounded away from zero and one, so we need only consider the case $y_x = y_0 = 0$. Note that this implies $\Delta y = 0$. Now rearrange the renegotiation constraint (B.4) in terms of Δy to give (after manipulation)

$$\Delta y < \beta_0[r + S^E z - g_E] - P_x \tag{B.13}$$

where the inequality is strict because we are assuming that this constraint is slack. Finally, we substitute the obtained values for P_x and β_0 into (B.13) to obtain $\Delta y < -\dfrac{K}{\theta}$ which gives the required contradiction. ■

Proof of Corollary 1. From the proof of Lemma 4 it can be seen that β_0, the solution to the relaxed maximization problem, cannot be the solution to the complete mazimization problem (★), which therefore must be strictly different from zero. ■

Proof of Proposition 2 (Partial Liquidation). In order to encapsulate in one notation all three cases we will need to define $\omega \equiv r + z$, $\omega_A \equiv r$ and $\omega_B \equiv z$. Let L_0^i be the foreclosure value when the asset $i = \{A\}$, $\{B\}$, $\{AB\}$ is sold by the bank after a necessary default. Correspondingly, g_E^i is the entrepreneur's ex post surplus following a strategic default when asset i is foreclosed by the bank. We let $-i$ denote the set of assets not in i. Using this more general notation the optimization problem (★★) from section **??** can be written as

$$\theta x - K + (r + S^E z) - \beta_0(1-\theta)(\omega - \omega_{-i} - L_0^i) \tag{B.14}$$

$$\beta_0 = \frac{K}{\theta(r + S^E z - g_E^i) + (1-\theta)L_0^i} \tag{B.15}$$

Equation (B.14) is written in terms of $\omega - \omega_{-i}$ because when asset i is foreclosed, ω_{-i} is still available for consumption at date 2. We will denote the efficiency loss owing to the incompleteness of the lending contract when asset i is foreclosed as

$$EL^i = \beta_0(1-\theta)(\omega - \omega_{-i} - L_0^i)$$

Hence the efficiency loss can be written as

$$EL^i = \frac{\omega - \omega_{-i} - L_0^i}{\theta(\rho - g_E^i) + (1-\theta)L_0^i}(1-\theta)K \tag{B.16}$$

where $\rho \equiv r + S^E z$ for notational convenience. Partial foreclosure is preferred to total foreclosure when both $EL^A < EL^{AB}$ and $EL^B < EL^{AB}$. Consider first the case of $EL^A < EL^{AB}$ (the case of $EL^B < EL^{AB}$ is analogous and consequently omitted). In this case we wish to show

$$\frac{\omega - \omega_B - L_0^A}{\theta(\rho - g_E^A) + (1-\theta)L_0^A}(1-\theta)K < \frac{\omega - L_0^{AB}}{\theta(\rho - g_E^{AB}) + (1-\theta)L_0^{AB}}(1-\theta)K$$

which is the same as showing (referring to Table 1 in section 3)

$$\begin{aligned} &\frac{r - L_0^A}{\theta(\rho - \tau(r - L_0^A)) + (1-\theta)L_0^A} \\ &< \frac{r + z - (L_0^A + L_0^B)}{\theta(\rho - \tau(r + z - (L_0^A + L_0^B))) + (1-\theta)(L_0^A + L_0^B)} \end{aligned} \tag{B.17}$$

The RHS of (B.17) can be rearranged as follows

$$\frac{(r - L_0^A) + (z - L_0^B)}{\theta(\rho - \tau((r - L_0^A) + (z - L_0^B))) + (1-\theta)L_0^A + (1-\theta)L_0^B}$$

which can be further rearranged to give

$$\frac{(r - L_0^A) + (z - L_0^B)}{\theta(\rho - \tau(r - L_0^A)) + (1-\theta)L_0^A + (1-\theta)L_0^B - \theta\tau(z - L_0^B)} \tag{B.18}$$

Define the LHS of (B.17) for convenience by $\Sigma \equiv \frac{\Sigma^0}{\Sigma_0}$. Then (B.18) can be written as

$$\frac{\Sigma^0 + (z - L_0^B)}{\Sigma_0 + (1-\theta)L_0^B - \theta\tau(z - L_0^B)}$$

or (again using Table 1 in section 3)

$$\frac{\Sigma^0 + z(1-\lambda)}{\Sigma_0 + (1-\theta)\lambda z - \theta\tau z(1-\lambda)} \tag{B.19}$$

Inequality (B.17) can now be written as (substituting (B.19) into the RHS)

$$\frac{\Sigma^0}{\Sigma_0} - \frac{\Sigma^0 + z(1-\lambda)}{\Sigma_0 + z[(1-\theta)\lambda - \theta\tau(1-\lambda)]} < 0$$

Now $(1-\lambda)$ is always strictly positive. Consequently, showing that the inequality holds depends on the sign and/or magnitude of $(1-\theta)\lambda - \theta\tau(1-\lambda)$. If it is zero or negative then the inequality holds immediately. This is true if

$$\theta \geq \frac{1}{1 + \tau\left(\frac{1}{\lambda} - 1\right)} \tag{B.20}$$

If it is positive then the inequality only holds if the numerator is greater than the denominator, or

$$(1-\lambda) - [(1-\theta)\lambda - \theta\tau(1-\lambda)] > 0$$

which implies

$$\theta > \frac{1 - \left(\frac{1}{\lambda} - 1\right)}{1 + \tau\left(\frac{1}{\lambda} - 1\right)} \equiv \hat{\theta} \tag{B.21}$$

where the last equivalence follows from definition 1 in section 3. Since λ by definition is never strictly one, it follows that the RHS of (B.21) is always strictly less than the RHS of (B.18). Going through analagous steps will show that $EL^B < EL^{AB}$ whenever

$$\theta > \frac{1 - \left(\frac{1}{\alpha} - 1\right)}{1 + \tau\left(\frac{1}{\alpha} - 1\right)}$$

Hence the final part of the proof involves showing that the cut-off θ when asset A is foreclosed is less than the cut-off θ when asset B is foreclosed. We know that $c'(x) < 0$. Since $\theta'(c) = -\dfrac{(1+\tau)}{[1+\tau c]^2} < 0$, then the fact that $\hat{\theta}$ is the cut-off point follows immediately from assumption 2. ■

Proof of Corollary 2. As $\kappa \to \infty$ (or $\lambda \to 0$ *[check this last with L'Hospital's Rule]*), $\hat{\theta} \to 0$. The assertion in the corollary then follows from proposition 2, since partial liquidation is preferred when $\theta > \hat{\theta}$. ■

Proof of Proposition 4 (Guarantee Conditions).

(i) Entrepreneur Bargaining Power From (27) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$\tau = \frac{rz\Lambda + \theta rz\lambda - \theta\rho\nabla}{\theta rz\alpha(1-\lambda)} \equiv \tilde{\tau}$$

provided that $\tilde{\tau} \in [0,1]$. Taking the derivative of ψ with respect to τ we get

$$\frac{d\psi}{d\tau} = \theta rz\alpha(1-\lambda) > 0$$

so that ψ is increasing in τ for all values of τ. It follows that the entrepreneur forecloses on the project asset when $\tau < \tilde{\tau}$ and forecloses on the relationship asset when $\tau > \tilde{\tau}$.

(ii) Relationship Closeness From (27) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$c(\lambda) = \frac{(1-\alpha)(z+\theta\rho)r + \theta rz}{\theta\rho[z - r(1-\alpha)] + rz\alpha(1-\theta\tau)} \equiv \tilde{c}(\lambda)$$

Taking the derivative of ψ with respect to $c(\lambda)$ we get

$$\frac{d\psi}{dc(\lambda)} = \theta\rho[z - r(1-\alpha)] + rz\alpha(1-\theta\tau) > 0$$

so that ψ is increasing in $c(\lambda)$ for all values of $c(\lambda)$. It follows that the entrepreneur forecloses on the project asset when $c(\lambda) < \tilde{c}(\lambda)$ and forecloses on the relationship asset when $c(\lambda) > \tilde{c}(\lambda)$.

(iii) Entrepreneur Relationship Asset Share From (27) it can be seen that the entrepreneur is indifferent between foreclosing on the project or relationship asset when

$$S^E = \frac{rz\Lambda + \theta rz\lambda(1-\alpha\kappa(\tau,c)) - \theta\nabla r}{z\theta\nabla} \equiv \tilde{S}^E$$

provided that $\tilde{S}^E \in [0,1]$. Taking the derivative of ψ with respect to S^E we get

$$\frac{d\psi}{dS^E} = z\theta\nabla < 0$$

so that ψ is decreasing in S^E for all values of S^E. It follows that the entrepreneur forecloses on the project asset when $S^E > \tilde{S}^E$ and forecloses on the relationship asset when $S^E < \tilde{S}^E$. ■

Proof of Proposition 5 (Asset Share, Coercion and Project Risk Reversal). *[Need to rewrite this]* Equation (27) implies

$$\theta = \frac{rz\Lambda}{\rho\nabla - rz\lambda[1-\alpha\kappa(\tau,c)]} \equiv \tilde{\theta} \tag{B.22}$$

Taking the derivate of ψ with respect to θ we get

$$\frac{d\psi}{d\theta} = \rho\nabla - rz\lambda[1-\alpha\kappa(\tau,c)]$$

which (since $\nabla < 0$ by assumption) is positive or negative depending solely on the magnitude of κ. When $\kappa = 0$ so that there is no coercion, $\dfrac{d\psi}{d\theta} < 0$, which implies that for $\theta > \hat{\theta}$ the entrepreneur forecloses on the project asset (asset A) and for $\theta < \hat{\theta}$ the entrepreneur forecloses on the relationship asset (asset B). However

$$\frac{d^2\psi}{d\kappa d\theta} = rz\lambda\alpha \geq 0$$

implying that when coercion is large enough, $\dfrac{d\psi}{d\theta} > 0$, implying that the situation reverses where for $\theta > \hat{\theta}$ the entrepreneur forecloses on the relationship asset (asset B) and for $\theta < \hat{\theta}$ the entrepreneur forecloses on the project asset (asset A). The turning point occurs where

$$\kappa(\tau,\lambda) = \frac{rz\lambda - \rho\nabla}{\alpha rz\lambda} \equiv \kappa^* \tag{B.23}$$

■

Proof of Proposition 6 (Coercion and Investment Risk). The proposition follows from the fact that

$$\frac{d\tilde{\theta}}{d\kappa} = \frac{-(rz)^2\Lambda\alpha\lambda}{\{\rho\nabla - rz\lambda[1-\alpha\kappa(\tau,c)]\}^2} < 0 \text{ for all } \kappa$$

so that, in the limit as $\kappa \to \infty$, $\tilde{\theta} \to 0$. The statement of the proposition then follows from proposition 5, subcase (ii) since in that case $\theta > \tilde{\theta} \to 0$ is trivially true. ■

Proof of Proposition 3.

(i) Since the numerators of β_0^s and β_0^{ra} are identical we need only compare denominators. The smaller the denominator, the higher its respective β_0. Denote these denominators by π^i (for $i = s, ra$). Comparing π^{ra} and π^s we have that

$$\pi^{ra} - \pi^s = \theta S^E z \geq 0$$

(ii) Follows immediately from (i), since EL^i is increasing in β_0^i (for $i = s$, ra). ■

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